# COMP-424 - Assignment 3 Solutions

1. [25 points] **First-order Logic**

   (a) [10 points] Formalize the following text in first-order logic.

   Dogs obey their owners all the time.

   $$\forall d \forall o \forall t Dog(d) \wedge Time(t) \wedge Owner(o,d) \rightarrow Obeys(d,o,t) \quad (1)$$

   Cats obey their owners some of the time.

   $$\forall c \forall o \exists t Cat(c) \wedge Time(t) \wedge Owner(o,c) \wedge Obeys(c,o,t) \quad (2)$$

   An animal is either a dog or a cat, but not both.

   $$\forall a Animal(a) \rightarrow (Cat(a) \wedge \neg Dog(a)) \vee (\neg Cat(a) \wedge Dog(a)) \quad (3)$$

   Cats purr.

   $$\forall c Cat(c) \rightarrow Purr(c) \quad (4)$$

   Dogs do not purr.

   $$\forall d Dog(d) \rightarrow \neg Purr(d) \quad (5)$$

   Barney is an animal that purrs.

   $$Animal(Barney) \wedge Purr(Barney) \quad (6)$$

   (b) [10 points] Using resolution and unification, show that Barney is not a dog.
   Note that (6) can be written as two separate clauses: $Animal(Barney)$ and $Purr(Barney)$. Suppose by contradiction that $Dog(Barney)$. We can write (5) as:

   $$\neg Dog(d) \vee \neg Purr(d)$$

   By resolution and unification of the last two clauses, we get $\neg Purr(Barney)$, which generates a contradiction.

   (c) [5 points] Can you prove formally if Barney obeys its owner all the time? If yes, show a proof. If no, explain why not.

   No. From what you have shown and from (3) we can obtain $Cat(Barney)$, but you cannot show if Barney is one of the cats that obey their owners or not.

2. [15 points] **Abellard and Eloise**

   Let $x$, $y$, $z$ be natural numbers. Show whether the following statements are true or false. If a statement is true, specify what Eloises winning strategy is. If the statement is false, show the series of values chosen by Abellard to which Eloise cannot respond.

(a) $\forall x \exists y \exists z \; x = y + z$

True: choose $y = x$, $z = 0$.

(b) $\forall x \exists y \forall z \; x = y + z$

False: Pick $y = x + 1$. Then the equality cannot be satisfied for $z \in \mathbf{N}$.

(c) $\forall x \forall y \exists z \; x = y + z$

False: If $y > x$, no such setting exists.

(d) $\exists x \forall y \exists z \; x = y + z$

False: see above

(e) $\exists x \exists y \forall z \; x = y + z$

False: if we pick some arbitrary $x$ and $y$, the above is not true for any $z \neq x - y$.

(f) Suppose that instead of natural number, we have integers. Which of the sentences change their truth values? Explain your answer.

(c) becomes true, as we can pick $z = x - y$. (d) also becomes true, as we can pick $x = 0$ and $z = -y$.

3. [30 points] **Planning**

You are planning to make a cake for your friend's birthday. In order to make cake, you need to have the ingredients, a clean bowl and a clean pan. To clean an object, you need to wash it in the sink. For this, the sink needs to be empty. After an object gets washed, it is taken out of the sink and the sink becomes empty. If the bowl is clean, you can mix the ingredients in the bowl. As a result, they will turn into batter, and the bowl will be full and dirty. In order to have a cake, you need to pour the batter in a clean pan and bake it. When investigating, you see that you have the ingredients, but the bowl is dirty in the sink and the pan is dirty in the kitchen.

(a) Write a STRIPS-style description of this problem

To be posted later

(b) Develop the planning graph and extract a solution to the problem. Explain if this solution is unique.

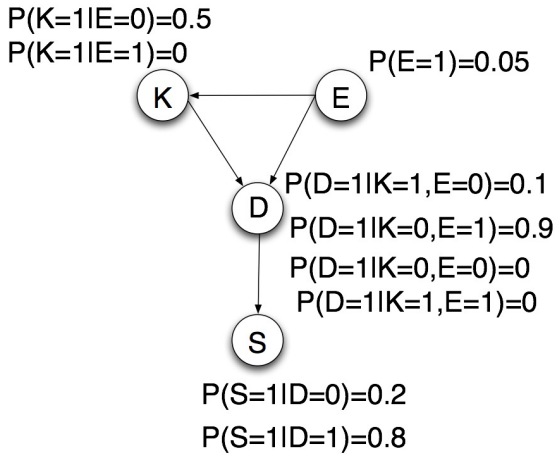
To be posted later

4. [30 points] **Bayes nets**

The starship Enterprise has detected a magnetic disturbance and is trying to figure out its cause. A disturbance could be caused by a cloaked Klingon ship, or by a star explosion. The probability of a Klingon ship in this area is 50%. The probability of a star explosion is 5%. A cloaked ship causes a disturbance 10% of the time. A star explosion causes a disturbance 90% of the time. Klingons never hang out close to stars who are about to explode, because they can detect them very well. The Enterprise's magnetic sensor is not very reliable: it will give the wrong reading with 20% probability.

(a) Draw a Bayes net corresponding to this problem. Specify the variables, the graph structure, and all the parameters.

Let $E$ be the r.v. that means there is a start explosion, $K$ indicates Klingon Ship, $D$ is the r.v. equal to 1 if there is a disturbance and $S$ is a r.v. =1 if the sensor reads a disturbance. the Bayes net is as follows:




(b) What are all the conditional independencies implied by the Bayes net?
$S \perp K|D$, $S \perp E|D$.

(c) Compute the conditional probabilities of the two possible causes for the Enterprise's sensor reading, and give the most plausible explanation.

$$\begin{aligned}
P(D=1|S=1) &= \frac{P(D=1,S=1)}{P(S=1)} \\
&\propto \sum_{k\in\{0,1\}}\sum_{e\in\{0,1\}} P(D=1,S=1,K=k,E=e) \\
&= P(E=0)P(K=1|E=0)P(D=1|E=0,K=1)P(S=1|D=1) \\
&+ P(E=1)P(K=0|E=1)P(D=1|E=1,K=0)P(S=1|D=1)
\end{aligned}$$

( other terms are 0)

$$= 0.95*0.5*0.1*0.8+0.05*1*0.9*0.8 = 0.074$$

$$\begin{aligned}
P(D=0|S=1) &= \frac{P(D=0,S=1)}{P(S=1)} \\
&\propto \sum_{k\in\{0,1\}}\sum_{e\in\{0,1\}} P(D=0,S=1,K=k,E=e) \\
&= P(E=0)P(K=1|E=0)P(D=0|E=0,K=1)P(S=1|D=0) \\
&+ P(E=1)P(K=0|E=1)P(D=0|E=1,K=0)P(S=1|D=0)
\end{aligned}$$

( other terms are 0)

$$= 0.95*0.5*0.9*0.2+0.05*1*0.1*0.2 = 0.0856$$

Hence, the most plausible explanation is that there is no real disturbance ($D=0$) even though the sensor reading is true.