THE UNIVERSITY OF BRITISH COLUMBIA

2011/2012 WINTER SESSION

SESSIONAL EXAMINATION

PHYSICS 216 MECHANICS I

7.00 pm to 9.30 pm Thursday, 12 April, 2012

This exam consists of 5 pages including this one. Please check that you have a complete exam.

Please write your name and student number on the exam answer booklet.

Write all work to be marked in the answer booklets provided.

This is a closed book exam. You are allowed to use writing and drawing instruments, a calculator (without internet connection capability) and the 8 ½ inch by 11 inch information sheet that you will have prepared.

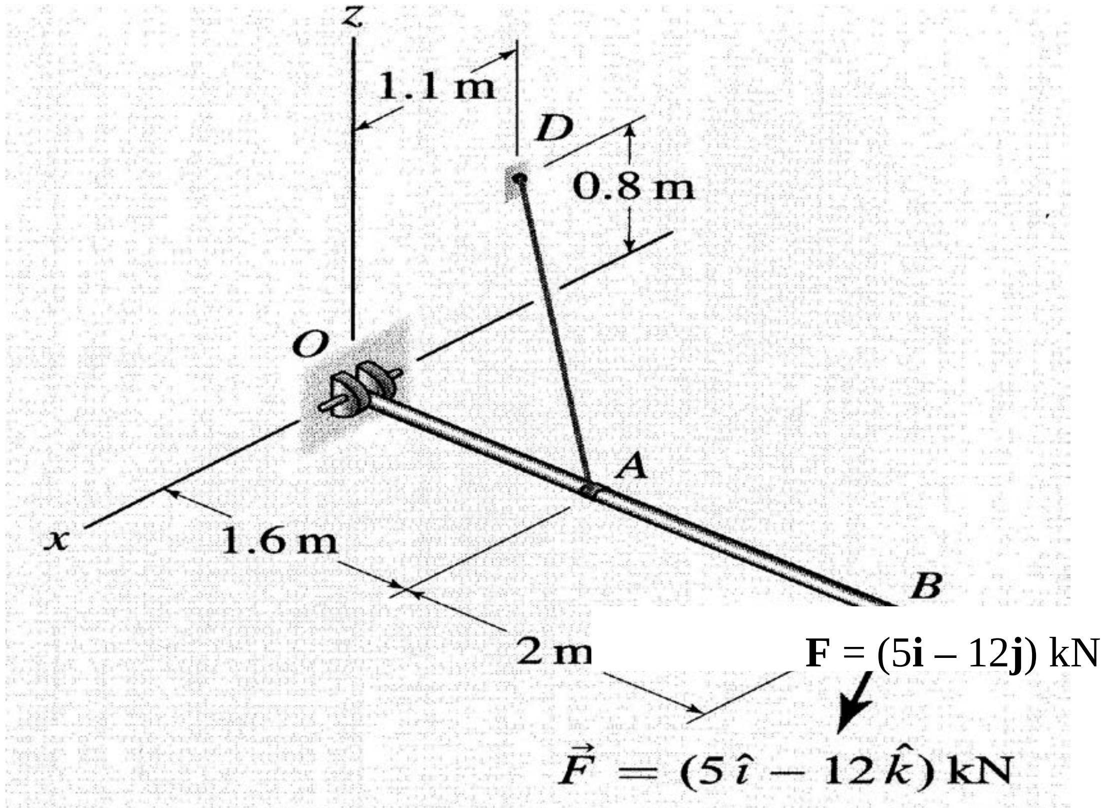
You must hand in BOTH your information sheet (that should be signed) and your answer booklet(s) in order for your exam to be marked.

The exam consists of four questions each worth 25 marks. Within each question, the probable marks are given as indicated.

Answers should be given to 3 significant figures and with correct units.

Question 1

The boom OAB is supported by a single smooth pin at point O and a cable at point A. A force **F** = (5**i** – 12**j**) kN is applied at point B.

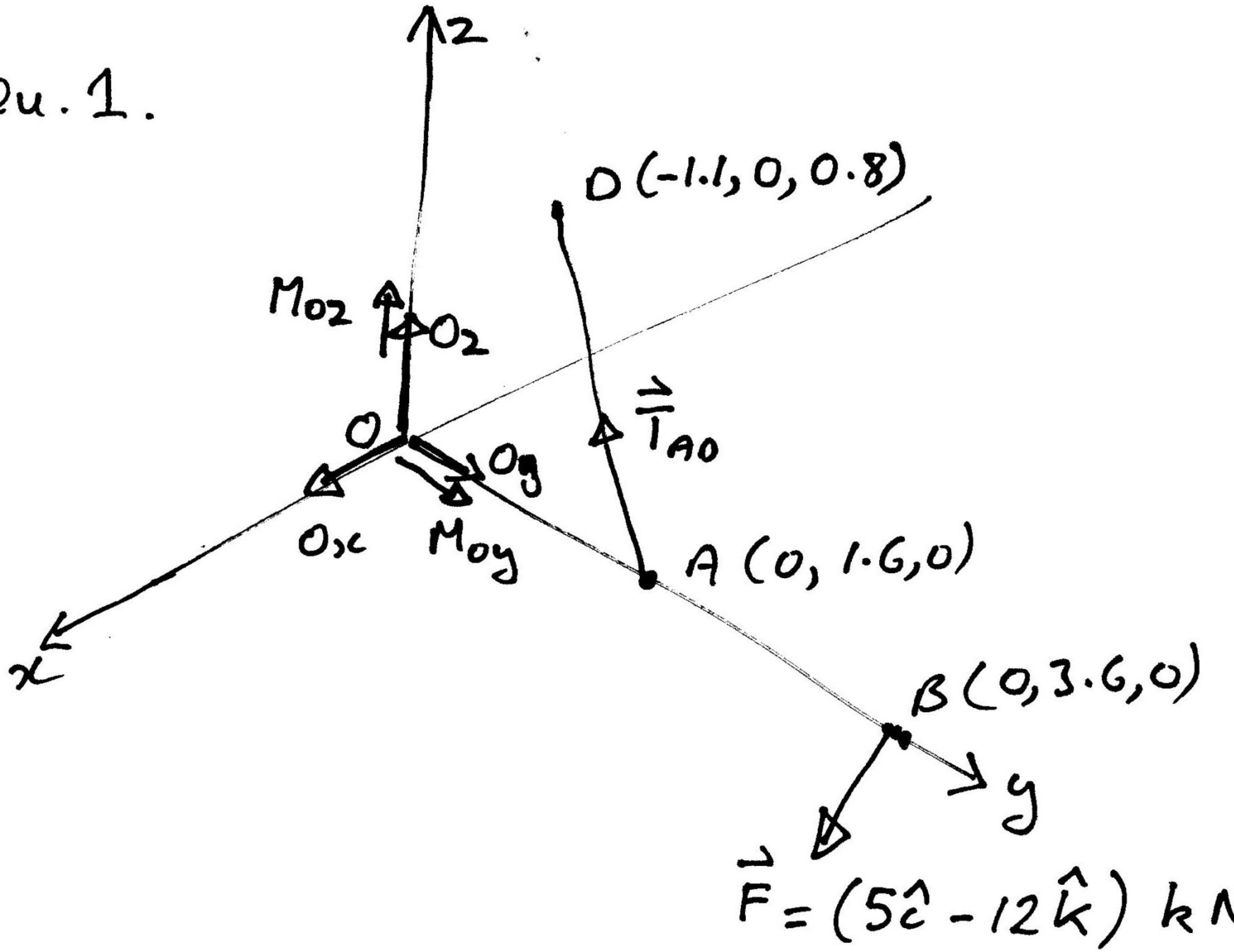


(a) Draw a clear free body diagram for the structure.

(b) Express the tension in cable AD as a Cartesian vector.

(c) Determine the tension in cable AD and the reaction components at the support O.

Qu. 1.

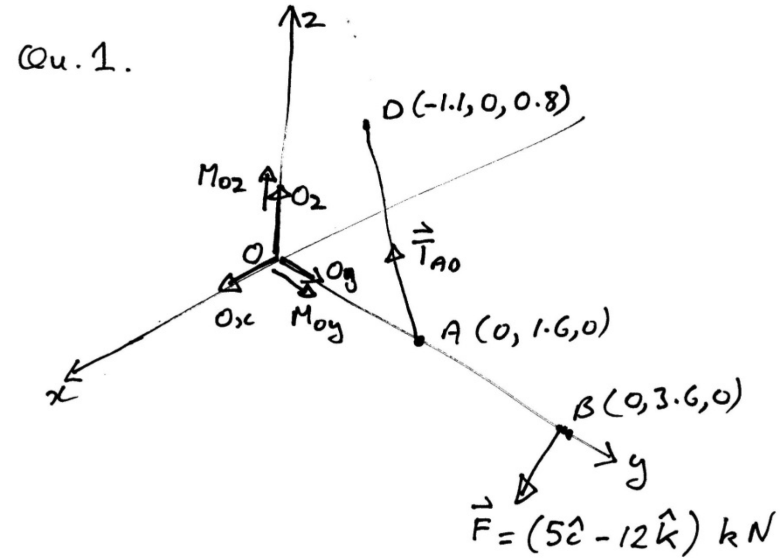


z
D (-1.1, 0, 0.8)
M$_{Oz}$
O$_z$
$\vec{T}_{AD}$
O
O$_y$
O$_x$
M$_{Oy}$
A (0, 1.6, 0)
x
B (0, 3.6, 0)
y
$\vec{F} = (5\hat{i} - 12\hat{k})$ kN

For cable AD,

$$\vec{T}_{AD} = T_{AD} \left\{ \frac{(-1.1)\hat{i} + (-1.6)\hat{j} + (0.8)\hat{k}}{\left[(-1.1)^2 + (-1.6)^2 + (0.8)^2\right]^{1/2}} \right.$$

$$= T_{AD}\left(-0.5238\,\hat{i} - 0.7612\,\hat{j} + 0.3810\,\hat{k}\right)$$



Take moments about point O :

$$\sum \vec{M}_O = \vec{r}_{OA} \times \vec{T}_{AD} + \vec{r}_{OB} \times \vec{F} + M_{Oy}\hat{j} + M_{Oz}\hat{k}$$

$$= 0$$

$$\vec{r}_{OA} \times \vec{T}_{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1.6 & 0 \\ -0.5238 & -0.7612 & 0.3810 \end{vmatrix} T_{AD}$$

$$= \left(0.6096\,\hat{i} + 0.8381\,\hat{k}\right) T_{AD}$$

$$\vec{r}_{OB} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3.6 & 0 \\ 5 & 0 & -12 \end{vmatrix}$$

$$= -43.2\,\hat{i} - 18.00\,\hat{k}$$

From $\hat{i}$ components :

$$0.6096\, T_{AD} - 43.2 = 0$$

$$T_{AD} = 70.87$$

$$\boxed{T_{AD} = 70.9\,kN}$$

From $\hat{j}$ components :

$$\boxed{M_{oy} = 0}$$

From $\hat{k}$ components :

$$0.8381\, T_{AD} - 18.00 + M_{oz} = 0$$

$$M_{oz} = 41.39$$

$$\boxed{M_{oz} = 41.4\ kN.m}$$

Considering force eq^{n}s :

$$\sum F_x = (-0.5238)\,T_{AD} + 5 + O_x = 0$$

$$O_x = (0.5238)(70.87) - 5$$

$$= 32.12$$

$$\boxed{O_x = 32.1 \text{ kN}}$$

$$\sum F_y = (-0.7612)(70.87) + O_y = 0$$

$$O_y = 53.95$$

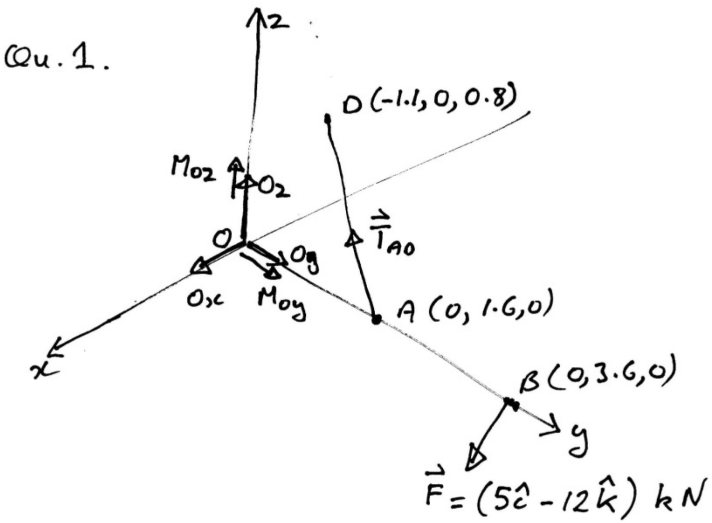
$$\boxed{O_y = 54.0 \text{ kN}}$$

$$\sum F_z = (0.3810)(70.87) - 12 + O_z$$

$$O_z = -15.00$$

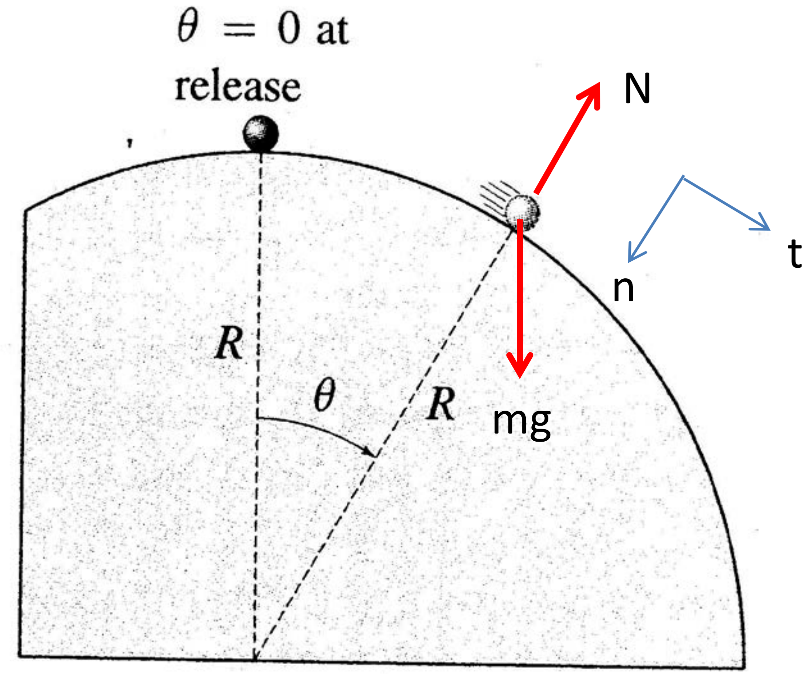
$$\boxed{O_z = -15.0 \text{ kN}}$$

Qu. 1.



Question 2

(ia) A very small sphere, that can be treated as a particle, is at rest at the top of a frictionless semicylindrical surface. It is given a very small nudge (that you can ignore) so that it slides down the surface. Determine the angle θ at which the particle separates from the surface.



Conservation of energy:

$mg\,(R - R\cos\theta) = \frac{1}{2} mv^2$

$v^2 = 2g\,(R - R\cos\theta)$

Force in n-direction produces centripetal acceleration:

$mg\cos\theta - N = mv^2/R$

Particle separates from surface when N = 0, so that

$g\cos\theta = v^2/R = 2g\,(R - R\cos\theta)\,/\,R = 2g\,(1 - \cos\theta\,)$

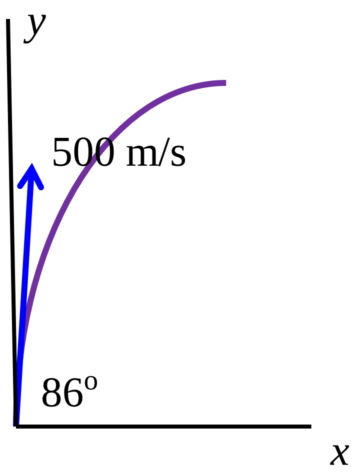
Therefore, $3\cos\theta = 2$ $\qquad\theta = \boxed{\cos^{-1}(2/3)}$

(ib) If the sphere was not so small, and it rolled without slipping down the surface, would the angle at which it left the surface be smaller or larger than for the particle in (a). Why?

Part of the kinetic energy would be rotational, so that the translational velocity, v, would be less than in (a). Therefore, θ would have to be larger.

Question 2

(ii) A projectile is fired vertically with a speed of 500 m/s at an angle of 86^o to the horizontal.
Determine the radius of curvature of the projectile's path at its maximum height.
Neglect air resistance.



At maximum height, $v_y = 0$ and $v_x = 500 \cos 86^o$

The centripetal acceleration (downward) is g, so that

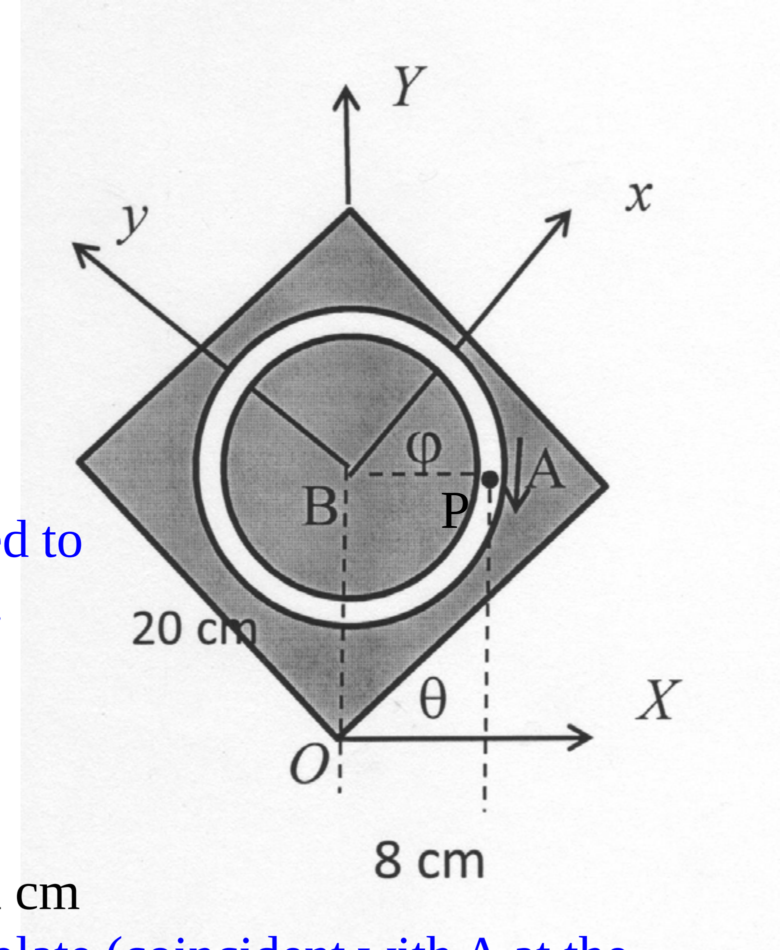
$$m v_x^2 / \rho = mg$$

$$\rho = v_x^2 / g = (500 \cos 86^o)^2 / (9.81) = \boxed{124 \text{ m}}$$

Question 2

(iii) Particle *A* moves in a smooth circular groove of 8 cm radius with angular velocity $\omega = \dot{\varphi}$ with respect to the square grooved plate of side 20 cm. At the same time, the grooved plate rotates about its corner *O* with angular velocity $\Omega = \dot{\theta}$.

Determine the absolute velocity of *A* in the position shown for which $\theta = 45^o$ and $\varphi = 45^o$ if, at this instant, $\Omega = 3$ rad/s and $\omega = -5$ rad/s.



Reference frame X-Y is fixed. Frame x-y is attached to the plate with origin at B and rotates with the plate.

The relative velocity equation is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{\Omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{A/B,xy}$$

Point *B* moves in a circle about O:

$$v_B = -r_B \Omega \mathbf{i} = 10\sqrt{2}\,(3)\,\mathbf{i} = 42.43\,\mathbf{i} \text{ cm}$$

The term $\mathbf{\Omega} \times \mathbf{r}_{A/B}$ is the velocity of point P on the plate (coincident with A at the instant considered) relative to *B*. Note that line PB rotates at angular velocity Ω :

$$\mathbf{\Omega} \times \mathbf{r}_{A/B} = r\,\Omega\,(\mathbf{k} \times \mathbf{i}) = (8)\,(3) = 24\,\mathbf{j} \text{ cm/s}$$

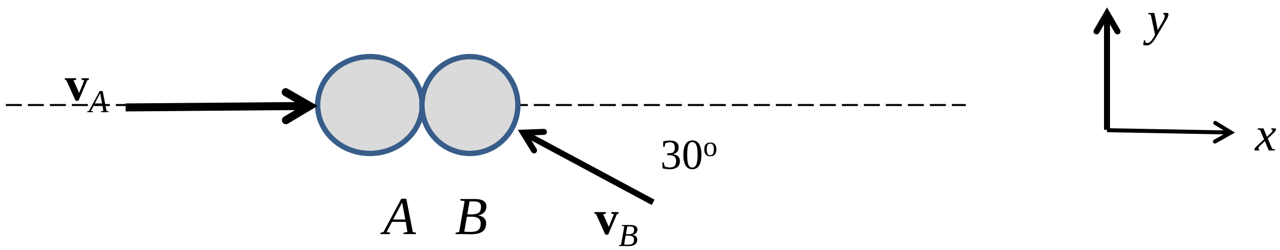
The term $\mathbf{v}_{A/B,xy}$ is: $\mathbf{v}_{A/B,xy} = -r\omega\,\mathbf{j} = (8)\,(5) = -40\,\mathbf{j}$

Therefore, $\mathbf{v}_A = 42.3\,\mathbf{i} + 24\,\mathbf{j} - 40\,\mathbf{j}$

$= 42.3\,\mathbf{i} - 16\,\mathbf{j}$ cm/s

Question 3

(i) Two identical discs *A* and *B* sliding on a smooth surface with initial velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ collide in such a way that the line joining their centres is in the direction of $\mathbf{v}_A$.



The speeds are v_A = 4 m/s and v_B = 6 m/s, and the coefficient of restitution is e = 0.6. Ignoring any friction, determine:

(a) the velocity of each ball immediately after the impact ;

Conservation of momentum in x-direction: $mv_{Ax,1} + mv_{Bx,1} = mv_{Ax2} + mv_{Bx2}$

$$4 - 6 \cos 30^o = v_{Ax,1} + v_{Bx,1}$$

$$v_{Ax,2} + v_{Bx,2} = -\ 1.196 \ \ldots\ldots (1)$$

Coefficient of restitution: $e = 0.6 = (v_{Bx,2} - v_{Ax,2}) / (4 + 6 \cos 30^o)$

$$v_{Bx,2} - v_{Ax,2} = 5.518 \ \ldots\ldots\ (2)$$

Conservation of momentum in y-direction: $mv_{By,1} = mv_{By,2}$

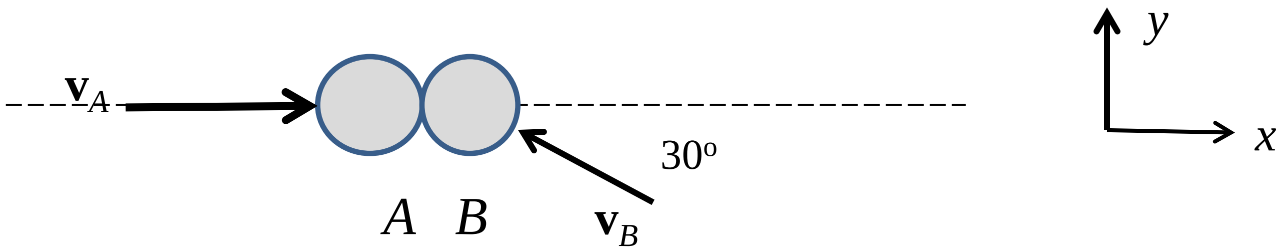
$$v_{By,2} = 6 \sin 30^o = 3.0 \ \ldots (3)$$

From Eq^{n}s (1), (2) and (3), $v_{A,x2} = -\ 3.357$; $v_{B,x2} = 2.161$; $v_{By,2} = 3.0$; $v_{B,x2} = 2.161$

These values yield $v_{A,2} =$ **3.36 m/s** ; $v_{B,2} =$ **3.70 m/s**

Question 3

(i) Two identical discs *A* and *B* sliding on a smooth surface with initial velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ collide in such a way that the line joining their centres is in the direction of $\mathbf{v}_A$.



(b) the ratio of the final and initial kinetic energies?

Initial K.E. is $T_1 = \frac{1}{2} m v_{A1}^2 + \frac{1}{2} m v_{B1}^2$

Final K.E. is $T_2 = \frac{1}{2} m v_{A2}^2 + \frac{1}{2} m v_{B2}^2$

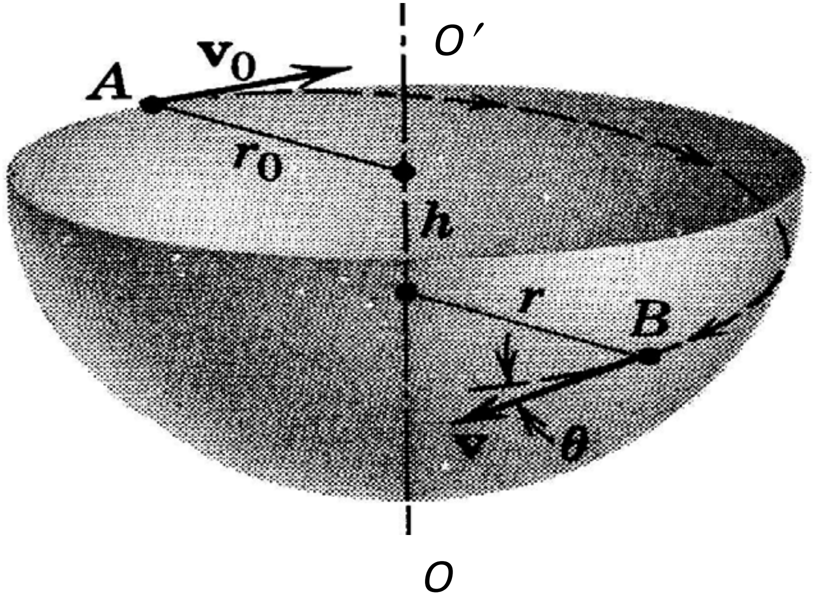
$$T_2 / T_1 = (v_{A2}^2 + v_{B2}^2) / (v_{A1}^2 + v_{B1}^2)$$

$$= (3.36^2 + 3.70^2) / (4^2 + 6^2)$$

$$= \boxed{0.48}$$

Question 3

(ii) A particle of mass m is given an initial velocity v_0 = 0.9 m/s tangential to the horizontal rim of a smooth hemispherical bowl at the radius r_0 = 10 cm from the vertical centre line OO′. Later, the particle slides through point B a distance h = 6 cm below A . There is no friction. Determine the magnitude of its velocity **v** at point B and the angle θ that **v** makes with the horizontal tangent to the bowl through B.



The only forces acting on the particle are the weight and normal reaction exerted by the surface of the bowl. Neither force creates a moment about axis *OO'*, so that angular momentum is conserved about that axis.

$$m\, v_0\, r_0 = m\, v\, r \cos\theta$$

$$v\, r \cos\theta = (0.9)\,(0.1) = 0.09 \; ..... \; (1)$$

Also, energy is conserved: $\frac{1}{2} m\,(v^2 - v_0^2) - mgh = 0$

$$v = (v_0^2 + 2\,g\,h)^{1/2} = [0.9^2 + (2)(9.81)(0.06)]^{1/2} = 1.410 \text{ m/s}$$

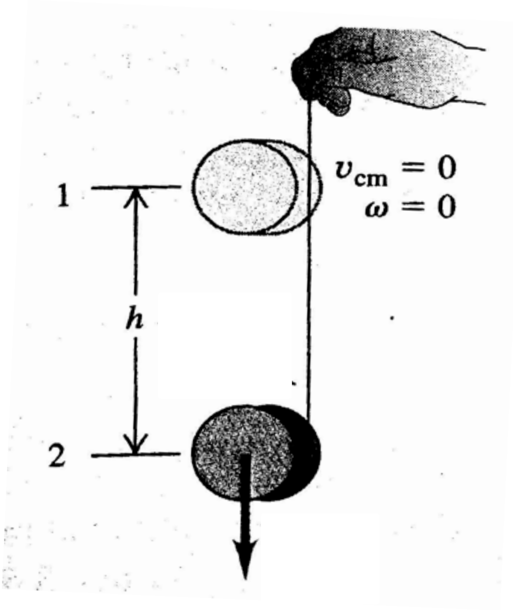
$r^2 = r_0^2 - h^2 = 0.1^2 - 0.06^2$; $r = 0.08$ m

From eq. (1), $\cos\theta = (0.09) / [(1.41)\,(0.08)] = 0.798$

$$\theta = \boxed{37.1^o} \quad \text{and} \quad v = \boxed{1.41 \text{ m/s}}$$

<u>Question 4</u>.

(ia) A string is wrapped many times around a solid cylinder of mass m and radius R. (A rudimentary yo-yo). The end of the string is held stationary and the cylinder is released from rest. The string unwinds, but does not slip, and drops a distance h. In terms of the given parameters determine the speed of the centre of mass of the cylinder and the tension in the string.



Net force acting down is $mg - T = m\,a_G$

Yo-yo does not slip: $\alpha = a_G / R = (g - T/m)/R$

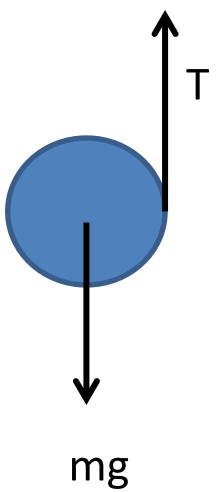
$\Sigma M = I\alpha$

$TR = \frac{1}{2} mR^2 (g - T/m)/R$

$T = \frac{1}{2} mg - \frac{1}{2} T$; $T = \boxed{1/3\ mg}$

Conservation of Energy

Note that the end of the string does not move so string does no work.



KE $T_1 = 0$; $T_2 = \frac{1}{2} mv_G^2 + \frac{1}{2} I\omega^2$ V

$= \frac{1}{2} mv_G^2 + \frac{1}{2} (\frac{1}{2} mR^2)\omega^2$

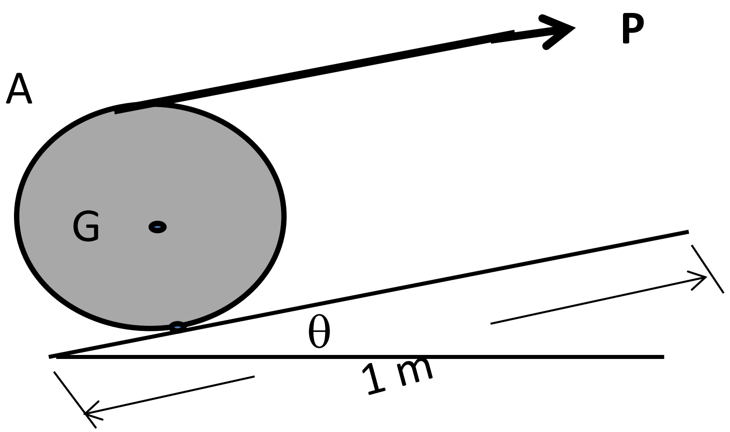
$= \frac{1}{2} mv_G^2 + 1/4\ I\ v_G^2 = \frac{3}{4} m v_G^2$

PE: $V_1 - V_2 = mgh$

$\frac{3}{4} m v_G^2 = mgh$; $v_G = \boxed{(4gh/3)^{1/2}}$

Question 4.

(ib) The same cylinder is now pulled up a plane, inclined at angle $\theta = \sin^{-1}(0.23)$ to the horizontal, by a force $P = \frac{1}{4}$ mg applied to the cord parallel to the plane. The cylinder rolls without slipping. What is the speed of point A compared to that of the centre of mass G? Determine the angular velocity of the cylinder in terms of g and R after the centre of mass has moved from rest a distance of 1m up the incline.



Cylinder rolls without slipping. Point of contact of cylinder with plane is IC.

$v_A = 2\,v_G$

Work - Energy:

$T_1 = 0\,;\; T_2 = ½\, m v_G^2 + ½\, I_G\, \omega^2 = ½\, m v_G^2 + ½\,(½\, m R^2)\,\omega^2 = ½\, m v_G^2 + 1/4\, m\, v_G^2$

$= ¾\, m\, v_G^2$

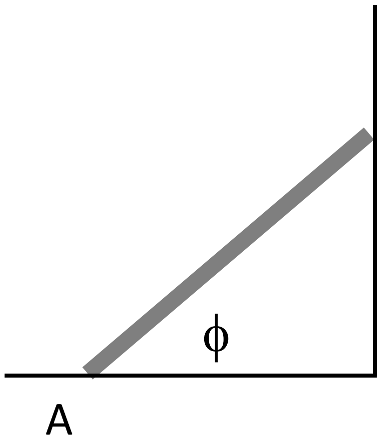
$\Delta V = mg \sin\theta$

$P\,(1) = ¼\, mg = mg \sin\theta + ¾\, m\, v_G^2\;;\quad v_G^2 = 4/3\,(0.25 - 0.23)\, g\,;\quad v_G = 0.0267\, g$

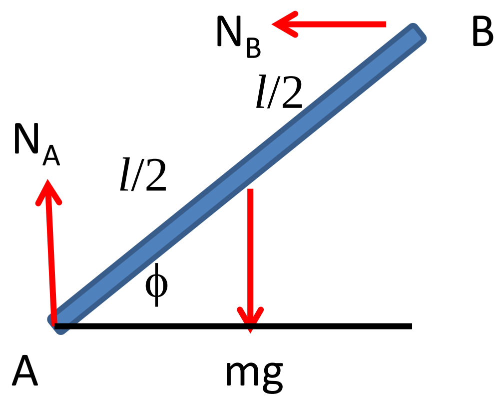
$\omega = v_G / R = 0.0267\, g/R$

Question 4.

(ii) A slender, uniform rod AB has a mass *m* and length *l*. It is supported as it leans with end B against a smooth wall and end A on a smooth floor. It is released from rest and starts to slide down. At a certain point in the motion when $\phi = \phi_0$, end B loses contact with the wall. Determine the angular acceleration of the rod at this instant.



When $N_B = 0$, tip of rod B leaves wall.
At this instant, take moments about A:



$$\Sigma M_A = mg\,(l/2)\cos\phi_0 = I_A\,\alpha = [(I_G + m(l/2)^2]\,\alpha$$

$$\alpha = mg\,(l/2)\cos\phi_0 \,/\, [(ml^2/12 + m(l/2)^2]$$

$$\alpha = \boxed{3g/2l\cos\phi_0}$$