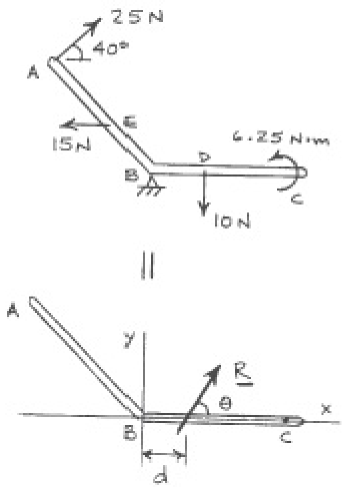
FINAL EXAM Winter 2010.

Answer 1:

(*a*)



Have $\mathbf{R} = \Sigma\mathbf{F}$

$$\mathbf{R} = (25\text{ N})(\cos 40°\mathbf{i} + \sin 40°\mathbf{j}) - (15\text{ N})\mathbf{i} - (10\text{ N})\mathbf{j}$$

$$= (4.1511\text{ N})\mathbf{i} + (6.0696\text{ N})\mathbf{j}$$

$$R = \sqrt{(4.1511)^2 + (6.0696)^2}$$

$$= 7.3533\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{6.0696}{4.1511}\right)$$

$$= 55.631°$$

or $\mathbf{R} = 7.35\text{ N} \measuredangle 55.6°$ ◀

(*b*) From $M_B = \Sigma M_B = dR_y$

where

$$M_B = -\left[(25\text{ N})\cos 40°\right]\left[(0.375\text{ m})\sin 50°\right]$$

$$-\left[(25\text{ N})\sin 40°\right]\left[(0.375\text{ m})\cos 50°\right]$$

$$+(15\text{ N})\left[(0.150\text{ m})\sin 50°\right] - (10\text{ N})(0.150\text{ m})$$

$$+ 6.25\text{ N}\cdot\text{m}$$

$$\therefore \quad M_B = -2.9014\text{ N}\cdot\text{m}$$

and

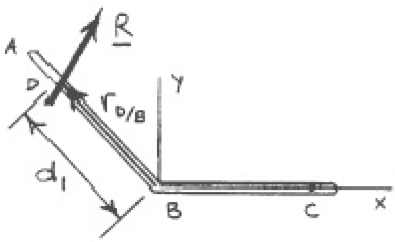
$$d = \frac{M_B}{R_y}$$

$$= \frac{-2.9014\text{ N}\cdot\text{m}}{6.0696\text{ N}}$$

$$= 0.47802\text{ m}$$

or $d = 478\text{ mm}$ to the left of B ◀

(*c*)



From $\mathbf{M}_B = \mathbf{r}_{D/B} \times \mathbf{R}$

$$-(2.9014\ \text{N}\cdot\text{m})\mathbf{k} = (-d_1 \cos 50°\mathbf{i} + d_1 \sin 50°\mathbf{j}) \times [(4.1511\ \text{N})\mathbf{i} + (6.096\ \text{N})\mathbf{j}]$$

$$-(2.9014\ \text{N}\cdot\text{m})\mathbf{k} = -(7.0814 d_1)\mathbf{k}$$

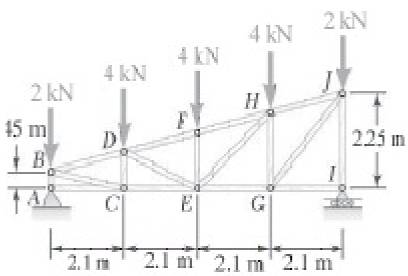
$$\therefore\ d_1 = 0.40972\ \text{m}$$

or $d_1 = 410$ mm from B along line AB

or 34.7 mm above and to left of A ◀

Answer 2:

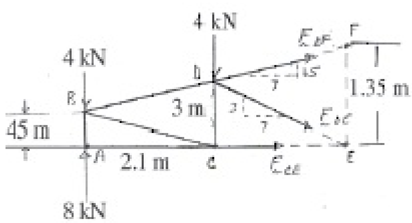
FBD Truss:



$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$

By load symmetry, $\mathbf{A}_y = \mathbf{I}_y = 8 \text{ kN} \uparrow$

FBD Section:



$$\circlearrowleft \Sigma M_D = 0: \quad (2.1 \text{ m})(2 \text{ kN} - 8 \text{ kN}) + (.9 \text{ m})(F_{CE}) = 0$$

$$F_{CE} = 14 \text{ kN} \qquad\qquad F_{CE} = 14.00 \text{ kN T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_E = 0: \quad (2.1 \text{ m})\left[1(4 \text{ kN}) + 2(2 \text{ kN} - 8 \text{ kN})\right]$$

$$(1.35 \text{ m})\frac{7}{\sqrt{51.25}}F_{DF} = 0$$

$$F_{DF} = \frac{8\sqrt{51.25}}{4.5} \text{ kN} \qquad\qquad F_{DF} = 12.73 \text{ kN C} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: 8 \text{ kN} \; - 2 \text{ kN} - 4 \text{ kN} + \frac{1.5}{\sqrt{51.25}}\frac{8\sqrt{51.25}}{4.5} \text{ kN} - \frac{3}{\sqrt{58}}F_{DE} = 0$$

$$F_{DE} = -1.692 \text{ kN} \qquad\qquad F_{DE} = 1.692 \text{ kN C} \blacktriangleleft$$

Answer3:

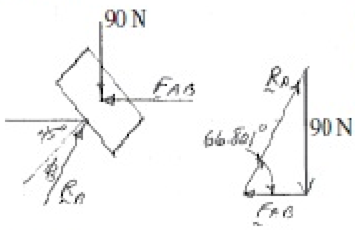
Not available

Answer 4:

Note: Rod is a two force member. For impending slip the reactions are at angle

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.40) = 21.801°$$

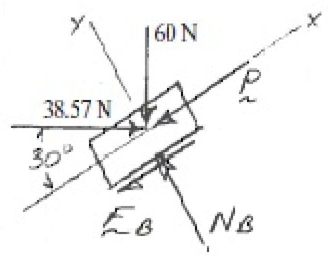
FBD A:



Consider first impending slip to right

$$F_{AB} = \frac{90\ \text{N}}{\tan 66.801} = 38.57\ \text{N}$$

FBD B:



↖ $\Sigma F_y = 0$: $\quad N_B - (38.57\ \text{N})\sin 30° - (60\ \text{N})\cos 30° = 0$

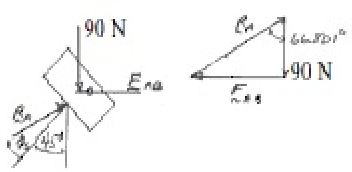
$$N_B = 71.25\ \text{N}, \quad F_B = \mu_s N_B = 0.40(71.25\ \text{N})$$

$$F_B = 28.5\ \text{N}$$

↗ $\Sigma F_x = 0$: $\quad -28.5\ \text{N} + (38.57\ \text{N})\cos 30° - (60\ \text{N})\sin 30° - P = 0$

$$P_{min} = -25.1\ \text{N}$$

FBD A:

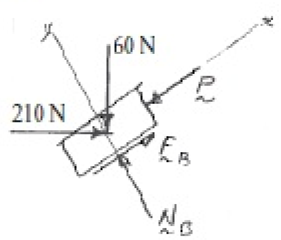


Next consider impending slip to left

$$F_{AB} = (90\ \text{N})\tan 66.801° = 210\ \text{N}$$

↖ $\Sigma F_y = 0$: $\quad N_B - (210\ \text{N})\sin 30° - (60\ \text{N})\cos 30° = 0, \quad N_B = 157\ \text{N}$

FBD B:



$$F_B = \mu_s N_B = 0.4(157\ \text{N}) = 62.8\ \text{N}$$

↗ $\Sigma Fx = 0$: $\quad 62.8\ \text{N} + (210\ \text{N})\cos 30° - (60\ \text{N})\sin 30° - P = 0$

$$P_{max} = 214.6\ \text{N}$$

equilibrium for $-25.1\ \text{N} \le P \le 214.6\ \text{N}$ ◀

Answer 5:

Place the origin at A with x axis horizontal to the right and y axis vertically upward.

Horizontal motion: $v_x = v_0 \sin\beta \qquad x = (v_0 \sin\beta)t$

At point B, $x_B = R\cos\beta = (v_0 \sin\beta)t_B$

Solve for t_B. $t_B = \dfrac{R}{v_0 \tan\beta}$

Vertical motion: $v_y = v_0 \cos\beta$

$$y = (v_0 \cos\beta)t - \frac{1}{2}gt^2$$

At point B, $y_B = -R\sin\beta = \dfrac{R\cos\beta}{\tan\beta} - \dfrac{gR^2}{2v_0^2 \tan^2\beta}$

Simplifying, $R\left(\sin\beta + \dfrac{\cos\beta}{\tan\beta}\right) = \dfrac{R}{\sin\beta} = \dfrac{gR^2}{2v_0^2 \tan^2\beta}$

Solve for v_0. $v_0^2 = \dfrac{gR\sin\beta}{2\tan^2\beta}$

$$= \frac{gR\cos^2\beta}{2\sin\beta}$$

$$v_0 = \cos\beta\sqrt{\frac{gR}{2\sin\beta}} \blacktriangleleft$$