

University of Ottawa
Department of Mathematics and Statistics

MAT 2377 3X: Probability and Statistics for Engineers
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Final Exam
July 2010

Surname _____ First Name _____

Student # _____

Instructions:

- You have 3 hours to complete this exam.
- Record your answers in the table below.
- Only TI 30 calculators or equivalent models are permitted.
- Do not detach the staple.
- Write your student number at the top of each page in the space provided.
- There are 22 multiple choice questions to answer.

GOOD LUCK!

Question	Answer	Question	Answer	Question	Answer
1	D	11	D	21	A
2	A	12	B	22	B
3	A	13	A		
4	C	14	A		
5	B	15	A		
6	D	16	C		
7	A	17	D		
8	D	18	C		
9	B	19	D		
10	C	20	D		

Question 1. In a study relating the degree of warping, in mm, of copper plate (y) to temperature (x), in C, the following summary statistics were calculated for $n=42$ observations:

$$\sum_{i=1}^{42} (x_i - \bar{x})^2 = 98,775, \quad \sum_{i=1}^{42} (y_i - \bar{y})^2 = 19.10, \quad \sum_{i=1}^{42} y_i x_i = 1401.31$$

$$\bar{x} = 26.36, \quad \bar{y} = 0.5188,$$

- a) Compute the regression line for predicting warping from temperature.
 b) At what temperature will we predict the warping to be 0.5 mm?

- A) a) $\hat{y} = 0.308 + 0.008x$, b) 0.312
 B) a) $\hat{y} = 0.008 + 0.308x$, b) 24
 C) a) $\hat{y} = 0.308 + 0.008x$, b) 0.008
 D) a) $\hat{y} = 0.308 + 0.008x$, b) 24
 E) a) $\hat{y} = 0.505 + 0.717x$, b) 21

Question 2. Of all failures of a certain type of computer hard drive, 20% are from physical damages and 80% are caused by virus. The chance of repairing the hard drive after physical damage is 20%, while 50% of drives affected by virus are repaired. You purchase a repaired hard drive, what is the probability that it has had a physical damage?

- A) 9% $P(\text{p.d.}) = 0.2$ $P(V) = 0.8$ $P(r|p.d.) = 0.2$ $P(r|v) = 0.5$
 B) 44% $P(p.d|r) = \frac{P(p.d)P(r|p.d.)}{P(p.d)P(r|p.d.) + (0.8)(0.5)} = 0.09$
 C) 20%
 D) 4%
 E) 80%

Question 3. There is 25% chance that a fuse of a certain type is defective. *Approximate* the probability of having more than or equal to 15 defective fuses in a box containing 30 fuses.

- A) 0.0016 $0.25 \ P(X \geq 15) \quad 1 - P(X < 15) = 1 - P(X \leq 14)$
 $\frac{30 - 14(0.25)}{\sqrt{(14)(0.25)(0.75)}}$
 B) 0.0052
 C) 0.0008
 D) 0.9984
 E) 0.0005

Question 4. The time for a super glue to set can be treated as a random variable having a normal distribution with mean 30 seconds. Find its standard deviation if the probability is 0.0002 that it will take on a value greater than 39.2 seconds.

- A) 0.55
 B) 0.38
 C) 2.6
 D) 32.6
 E) 0.67

Question 5. Two random variables X and Y are given with the following joint probability mass function:

x	y	$f_{XY}(x, y)$
1	2	1/4
0	1	1/8
3	0	1/4
6	7	3/8

- a) Determine $cov(X, Y)$. b) Are X and Y independent?
- A) a) 5.6875, b) They are independent.
 B) a) 5.6875, b) They are not independent.
 C) a) 13, b) They are not independent.
 D) a) 0, b) They are independent.
 E) None of the preceding.

Question 6. Consider two random variables X and Y with the following joint mass function:

x	0	1	2	3
y	-1	1	1	2
f(x,y)	1/8	1/4	1/2	1/8

Determine $E(X|Y = 1)$.

- A) 0.165
- B) 0.25
- C) 1
- D) 1.667
- E) 1.5

Question 7. The mean weight loss of $n = 16$ grinding balls after a certain length of time in mill slurry is 3.42 grams with a standard deviation of $s = 0.68$ grams. Assuming normality, construct a two-sided 99% confidence interval for the true mean weight loss of such grinding balls.

- A) $2.92 \leq \mu \leq 3.92$
- B) $2.98 \leq \mu \leq 3.86$
- C) $2.92 \leq \mu \leq 3.86$
- D) $3.90 \leq \mu \leq 3.92$
- E) None of the preceding.

Question 8. If a radar covers on average an area of 512 square km with a standard deviation of $\sigma = 30.5$ square km, what is the probability that the average area covered by a sample of 40 radars will be anywhere from 510.07 and 520.5 square km?

- A) 1
- B) 0
- C) 0.384
- D) 0.616
- E) 0.717

Question 9. A normal population has mean 100 and variance 25. How large must the sample size, n , be if we want the standard error of the sample average to be approximately 2?

- A) $n = 157$
- B) $n = 7$
- C) $n = 101$
- D) $n = 14$
- E) $n = 20$

Question 10. The diameter of a certain bullet is known to have a normal distribution with $\sigma = 0.01$ cm. A random sample of size 10 yields an average diameter of 1.50 cm. Find a 99% two sided confidence interval on the mean bullet diameter.

- A) $1.474 \leq \mu \leq 1.526$
- B) $0 \leq \mu \leq 1.508$
- C) $1.492 \leq \mu \leq 1.508$
- D) $1.492 \leq \mu \leq 1.526$
- E) $1.472 \leq \mu \leq 1.526$

Question 11. The life time in hours of a 75-watt light bulb is known to be normally distributed. A random sample of 20 bulbs has a mean life of $\bar{x} = 1014$ hours and sample standard deviation of $s=25$ hours. Construct a 95% lower confidence bound on the mean life.

- A) $1002.23 \leq \mu$
- B) $970 \leq \mu$
- C) $1004.80 \leq \mu$
- D) $1004.33 \leq \mu$
- E) $1004.44 \leq \mu$

Question 12. The melting point of each of 16 samples of certain brand of vegetable oil was determined, resulting in $\bar{x} = 94.32$. Assume that the distribution of melting point is normal with $\sigma = 1.2$. Test $H_0 : \mu = 95$ versus $H_1 : \mu \neq 95$ with $\alpha = 0.01$. State your conclusion and determine the P-value.

- A) Reject H_0 , P-value=0.012
- B) Can not reject H_0 , P-value=0.023
- C) Reject H_0 , P-value=0.023
- D) Can not reject H_0 , P-value=0.012
- E) Reject H_0 , P-value=0.025

Question 13. Before a substance can be deemed safe for landfilling, its chemical properties must be characterized. A report claims that in a sample of six replicates of sludge from a waste water treatment plant, the mean pH was 6.68 with a standard deviation of $s = 0.2$. Test $H_0 : \mu = 7$ versus $H_1 : \mu < 7$. State your decision, using a test with .01 level of significance. Determine the P-value of the test.

- A) Reject H_0 , $0.005 < P - value < 0.01$
- B) Can not reject H_0 , $0.005 < P - value < 0.01$
- C) Reject H_0 , $0.01 < P - value < 0.025$
- D) Can not reject H_0 , $0.01 < P - value < 0.025$
- E) None of the preceding.

Question 14. It is known that the steering computers of a spaces shuttle make a mistake with probability 0.001 in each move. The moves are considered independent. Determine the mean number of moves until the first mistake.

- A) 1000
- B) 999
- C) 1
- D) 0.0001
- E) 500

Question 15. In the previous question, suppose that there has been no mistake in the first 1000 moves, what is the probability that the first error happens at move number 1003?

- A) 998×10^{-6}
- B) 10^{-6}
- C) Almost 1.
- D) 500×10^{-6}
- E) None of the preceding.

Question 16. A class has 50 students. Suppose that there are 35 male and 15 female students. If we select 5 students randomly, what is the probability that all selected students are male?

- A) 0.0014
- B) 0.7
- C) 0.1532
- D) 0.5
- E) 0.65

Question 17. Let the continuous random variable X be the number of accidents on a highway during one year. Suppose that X has the following density function,

$$f(x) = cx(1 + x) \text{ for } 0 < x < 1.$$

Determine the mean number of accidents.

- A) 1.2
- B) 0.5
- C) 0.25
- D) 0.7
- E) None of the preceding.

Question 18. Flights arrive at the XYZ county's airport according to a Poisson process. On average there are 2 arrivals per hour. What is the probability that there are strictly less than 3 arrivals in one hour?

- A) 0.857
- B) 0.135
- C) 0.677
- D) 0.299
- E) 0.955

Question 19. Continuing with the previous question. Let X be the time between two subsequent arrivals. If a flight arrives at 8:45 a.m. what is the probability that the next flight arrives before 9:15 a.m.?

- A) 0.5
- B) 0.321
- C) 0.236
- D) 0.632
- E) 0.711

Question 20. Assume that we have a sample of size 10 from a population $N(4, 9)$. Denote by \bar{X} and S^2 , the sample mean and sample variance, respectively. Find c such that

$$P\left(\frac{\bar{X} - 4}{S/\sqrt{10}} \leq c\right) = 0.01.$$

- A) 1.833
- B) -1.833
- C) 2.821
- D) -2.821
- E) 1.645

Question 21. Preventing fatigue crack propagation in aircraft structures is an important element of aircraft safety. An engineering study to investigate fatigue crack in $n = 9$ cyclically loaded wing boxes reported the following crack lengths (in mm):

2.13, 2.96, 3.02, 1.82, 1.15, 1.37, 2.04, 2.47, 2.60.

Calculate the sample mean and sample standard deviation.

- A) $\bar{x} = 2.173, s = 0.660$
- B) $\bar{x} = 19.56, s = 0.656$
- C) $\bar{x} = 2.173, s = 0.583$
- D) $\bar{x} = 19.56, s = 0.271$
- E) $\bar{x} = 2.173, s = 45.953$

Question 22. A certain rocket is manufactured with a range of at least 1400 meters. A country's military will not purchase the rockets unless experimental evidence indicates conclusively that the range specification has been met, i.e. they test $H_0 : \mu = 1400$ versus $H_1 : \mu \neq 1400$. Suppose the range for rockets is normally distributed with $\sigma = 60$. They accept H_0 if $1350 < \bar{x} < 1450$. For a random sample of size $n = 20$, calculate the probability of type I error.

- A) 0.000096
- B) 0.000192
- C) 0.324444
- D) 0.5
- E) 0.999808