**School of Mathematics and Statistics**
**Carleton University**
**Math. 1005A, Winter 2011**

# TEST 4

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

**Print Name :** Solutions

**Student Number:**

**Tutorial Section (A1, A4, ...):**

## PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Which of the following series converges to a finite value?

   (a) $\sum_{n\geq 1} \frac{1}{n}$, (b) $\sum_{n\geq 1} \frac{1}{\sqrt{n}}$, (c) $\sum_{n\geq 1} \frac{1}{n+1}$, (d) $\sum_{n\geq 1} \frac{1}{n^2}$.

2. **[2 marks]** Which of the following series diverges to $+\infty$?

   (a) $\sum_{n\geq 2} \frac{3}{(n-1)^2}$, (b) $\sum_{n\geq 1} \frac{2}{3^n}$, (c) $\sum_{n\geq 1} \frac{1}{(n+1)^{1.002}}$, (d) $\sum_{n\geq 1} \frac{8}{n+10}$.

3. **[2 marks]** Which of the following series **IS** a geometric series?

   (a) $\sum_{n\geq 1} \frac{2^n}{5^n}$, (b) $\sum_{n\geq 1} \frac{1}{(n+2)^5}$, (c) $\sum_{n\geq 1} \frac{2}{n^n}$, (d) $\sum_{n\geq 1} \frac{4}{n}$.

4. **[2 marks]** What can you say about the following series,

$$\sum_{n\geq 1} \frac{1}{n(n+1)}?$$

   (a) It diverges to $+\infty$, (b) It converges to the value $+1$,
   (c) It converges to the value $+1/2$, (d) It converges to the value $-1$.

5. **[2 marks]** Answer TRUE or FALSE:

   The series $\sum_{n\geq 1} \sin(\frac{1}{n})$ converges to a finite number.

   (a) TRUE, (b) FALSE

1. d
2. d
3. a
4. b
5. b

## PART II: Show all work here and give details. No additional pages will be accepted

6. **[10 marks]** Use any test to determine the convergence or divergence of the series

$$\sum_{n\geq 1} \frac{n^2}{2+n^2} \cos(\frac{1}{n}).$$

**Note:** You get 5 marks for judicious (clever) guessing, and another 5 marks for supporting your guess. In either case, give details (be specific).

Let $a_n = \frac{n^2}{2+n^2} \cos(\frac{1}{n})$.

(2)

Guess: For large values of $n$ (or as $n\to\infty$), $\cos(\frac{1}{n}) \to 1$ (1) and $\boxed{\frac{n^2}{2+n^2} \to 1}$ (1) $\therefore$ $a_n \to 1$ as $n\to\infty$ and so by the divergence test, $\sum a_n$ $\boxed{\text{diverges}}$ (1)

Justification ...

Use limit comparison test! (LCT)

$a_n = \frac{n^2}{2+n^2}\cos\left(\frac{1}{n}\right)$ ← 1 → $a_n = \frac{n^2}{2+n^2}\cos\frac{1}{n}$

$b_n = \frac{n^2}{2+n^2}$ ← 1 → $b_n = \cos\frac{1}{n}$

2 $\Rightarrow \frac{a_n}{b_n} = \cos\frac{1}{n} \to 1$ as $n\to\infty$ ← 1 → $\Rightarrow \frac{a_n}{b_n} = \frac{n^2}{2+n^2} \to 1$ as $n\to\infty$ (by L'Hospital's rule)

$\therefore$ by LCT $\sum a_n, \sum b_n$ converge or diverge together. ← $\frac{1}{2}$ → $\therefore$ by LCT $\sum a_n, \sum b_n$ converge or diverge together

But $b_n \to 1$ (by L'Hospital's Rule) ← $\frac{1}{2}$ → But $b_n \to 1$ as $n\to\infty$

$\therefore \sum b_n$ diverges ← $\frac{1}{2}$ → $\therefore \sum b_n$ diverges

$\therefore \sum a_n$ diverges ← $\frac{1}{2}$ → $\therefore \sum a_n$ diverges

7. **[10 marks]** Use any test to determine the convergence or divergence of the series

$$\sum_{n\geq 1} \frac{2^n+1}{n^3\,2^n}.$$

**Note:** You get 5 marks for judicious (clever) guessing, and another 5 marks for supporting your guess. In either case, give details (be specific).

Guess: Let $a_n = \frac{2^n+1}{n^3 2^n}$ $\Rightarrow a_n = \left(\frac{2^n+1}{2^n}\right)\frac{1}{n^3} = \boxed{\left(1+\frac{1}{2^n}\right)\frac{1}{n^3}}$ ← 1

So, for large values of $n$ (or as $n\to\infty$), $\boxed{a_n \approx \frac{1}{n^3}}$ ← 1

$\sum \frac{1}{n^3}$ converges ← 1 ($\because$ it is a p-series with $p=3$).

$\therefore \sum a_n$ converges. ← 2

Justification (use LCT)!

$a_n = \frac{2^n+1}{n^3 2^n}$ ← 1

$b_n = \frac{1}{n^3}$ ← 1

$\Rightarrow \frac{a_n}{b_n} = \frac{2^n+1}{2^n} = 1+\frac{1}{2^n}$ ← 1

$\to 1$ as $n\to\infty$.

$\therefore$ by LCT both $\sum a_n, \sum b_n$ converge or diverge together ← $\frac{1}{2}$

But $\sum b_n = \sum \frac{1}{n^3}$ is a p-series with $p=3$. ← $\frac{1}{2}$

$\therefore \sum b_n$ converges ← $\frac{1}{2}$

$\therefore \sum a_n$ converges. ← $\frac{1}{2}$