$\therefore\ c_1' = -\cos x \sin x$, $c_2'(x) = \cos^2 x$.

$\therefore\ c_1(x) = -\int \cos x \sin x\, dx$,

$= \dfrac{\cos^2 x}{2} + c_1$ (1)

OR

$= -\dfrac{\sin^2 x}{2} + c_1$.

$c_2(x) = \int \cos^2 x\, dx$

$= \frac{1}{2}\int (1 + \cos 2x)\, dx$ (1)

$c_2(x) = \frac{1}{2}x + \frac{1}{4}\sin 2x + c_2$

$\therefore\ \boxed{y(x) = \left(\dfrac{\cos^2 x}{2} + c_1\right)\cos x + \left(\frac{1}{2}x + \frac{1}{4}\sin 2x + c_2\right)\sin x}$ (1)

N.B. $\left(-\dfrac{\sin^2 x}{2} + c_1\right)\cos x + \left(\frac{1}{2}x + \cdots\right)\sin x$ is also correct!

7. [10 marks] Find the general solution of the given differential equation using any method whatsoever:

$$y'' + 3y' + 2y = e^x.$$

aux. eq: $\lambda^2 + 3\lambda + 2 = 0 \Rightarrow (\lambda+1)(\lambda+2) = 0 \Rightarrow$ $\lambda = -1$ (1), $\lambda = -2$ (1)

$\therefore\ y_h(x) = c_1 e^{-x} + c_2 e^{-2x}$ (1) (1)

$\begin{cases} y_1(x) = e^{-x} \\ y_2(x) = e^{-2x} \end{cases}$

$\therefore\ W = y_1 y_2' - y_1' y_2 = -e^{-3x}$ (1)

Here $f = e^x$.

$\therefore\ y(x) = c_1(x)\, e^{-x} + c_2(x)\, e^{-2x}$ where

$c_1'(x) = -\dfrac{f y_2}{W}$, $c_2'(x) = \dfrac{f y_1}{W}$

(1) (1)

ie. $c_1'(x) = -\dfrac{e^x \cdot e^{-2x}}{-e^{-3x}}$

$= e^{2x}$

$\boxed{c_1(x) = \dfrac{e^{2x}}{2} + c_1}$ (1)

ie.

$c_2'(x) = \dfrac{e^x \cdot e^{-x}}{-e^{-3x}}$

$= -e^{3x}$

$\therefore\ \boxed{c_2(x) = -\dfrac{e^{3x}}{3} + c_2}$ (1)

$\therefore$ general solution is:

$$\boxed{y(x) = \left(\frac{e^{2x}}{2} + c_1\right)e^{-x} + \left(c_2 - \frac{e^{3x}}{3}\right)e^{-2x}}$$

(1)

School of Mathematics and Statistics
Carleton University
Math. 1005A, Winter 2011

TEST 2

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name : Solutions

Student Number:

Tutorial Section (A1, A4, ...):

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** What is the number of linearly independent solutions of a second order linear differential equation?
   (a) 1, (b) 3, (c) 2, (d) 4.

2. **[2 marks]** Determine the auxiliary equation of the differential equation
$$4\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2.3y = 0.$$
   (a) $4\lambda^2 - 2\lambda + 2.3 = 0$, (b) $4\lambda^2 - 2\lambda + 1 = 0$, (c) $4\lambda^2 - 4.3\lambda + 2.3 = 0$, (d) $\lambda^2 - 2\lambda + 2.3 = 0$

3. **[2 marks]** Determine the general solution $y(x)$ of the differential equation
$$y'' + 2y = 0,$$
   for $x > 0$.
   (a) $y(x) = c_1 \sin 2x + c_2 \cos 2x$, (b) $y(x) = c_1 e^{\sqrt{2}x} + c_2 e^{-\sqrt{2}x}$, (c) $y(x) = 2c_1 + c_2 \cos\sqrt{2}x$, (d) $y(x) = c_1 \sin\sqrt{2}x + c_2 \cos\sqrt{2}x$

4. **[2 marks]** Find the general solution of the non-homogeneous differential equation $y'' + 4y = 2$.
   (a) $y(x) = c_1 \sin 2x + c_2 \cos 2x$, (b) $y(x) = c_1 \sin 2x + c_2 \cos 2x + 1/2$ (c) $y(x) = c_1 \sin 2x + c_2 \cos 2x - 2$, (d) $y(x) = c_1 \sin\sqrt{2}x + c_2 \cos\sqrt{2}x$

5. **[2 marks]** Answer TRUE or FALSE: The general solution of the third order linear differential equation $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ contains a term of the form xe^x.
   (a) TRUE, (b) FALSE

1 c)
2 a)
3 d)
4 b)
5 a)

PART II: Show all work here and give details. No additional pages will be accepted

6. **[10 marks]** a) Find the general solution of the differential equation
$$y'' + y = \cos x.$$

auxiliary eq: $\lambda^2 + 1 = 0$ (1) $\lambda = i, \lambda = -i$ (1)

$\therefore y_h(x) = c_1 \cos x + c_2 \sin x$ (1) (1) $\quad \{ y_1(x) = \cos x,\ y_2(x) = \sin x$

$W = y_1 y_2' - y_1' y_2 = \cos^2 x + \sin^2 x = 1$ (1)

Here $f = \cos x$.

$\therefore y(x) = c_1(x) y_1(x) + c_2(x) y_2(x)$.

where $c_1'(x) = -\frac{f y_2}{W}$ (1), $c_2'(x) = \frac{f y_1}{W}$ (1)