

Problem Set 1

1.1) known

$$F = 30 \text{ lbf}$$

$$W = 10 \text{ lbf}$$

$$F - W = ma = (30 - 10) \text{ lbf} = \left(\frac{10 \text{ lbf}}{32.2 \text{ ft/s}^2} \right) a$$

$$a = 64.4 \text{ ft/s}^2$$

1.2) known

$$F = 30 \text{ lbf}$$

$$g = 5.31 \text{ ft/s}^2$$

$$F - W = ma \quad (30 - 1.65) \text{ lbf} = \left(\frac{1.65 \text{ lbf}}{5.31 \text{ ft/s}^2} \right) a$$

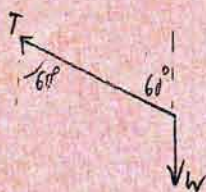
$$a = 91.23 \text{ ft/s}^2$$

1.3) known

$$L = 1 \text{ m}$$

$$m = 1 \text{ kg}$$

$$\theta = 60^\circ$$



$$W = mg = 9.81 \text{ m/s}^2 \quad \Sigma F_y = 0 \quad T \cos 60^\circ - W = 0$$

$$T = 19.62 \text{ N}$$

$$a = \frac{T}{m} = \frac{19.62 \text{ N}}{1 \text{ kg}} = 19.62 \text{ m/s}^2$$

$$r = L \sin 60^\circ = 0.866 \text{ m}$$

$$a = \omega^2 r \quad \omega^2 \frac{a}{r} = \frac{19.62 \sin 60^\circ}{0.866}$$

$$\omega = 4.43 \text{ rad/s} \cdot (\text{rev}/2\pi \text{ rad}) \cdot (60 \text{ s/min}) = 42.3 \text{ RPM}$$

1.4) F, F, F, T

1.5) d

1.6)	psia	psig	psfa	psfg	kPa(a)	kPa(g)
	14.696	0	2116.2	0	101.33	0
	39.696	25	5716.2	3600	273.70	172.37
	6.840	-7.856	985	-1131.2	47.16	-54.17
	29.39	14.694	4232.2	2116	202.64	101.31
	14.695	0.001	2116.1	0.1	101.32	0
	14502.4	145037.7	20887551.4	20885435.2	100101.33	10 ⁵

1.7) Known

$$m = 3000 \text{ kg}$$

$$d = 0.25 \text{ m}$$

$$r = 0.125 \text{ m}$$

$$F = mg = (3000 \text{ kg})(9.81 \text{ m/s}^2) = 29.43 \text{ kN}$$

$$A = \pi r^2 = 0.0491 \text{ m}^2$$

$$P = \frac{F}{A} = \frac{29.43 \text{ kN}}{0.0491 \text{ m}^2} = 600 \text{ kPa}$$

1.8) Known

$$P_{02} = 100 \text{ kPa (g)}$$

$$h = 12000 \text{ m}$$

$$P_{atm} = 19.399 \text{ kPa (a)} = 2.814 \text{ psia}$$

$$P_A = 8 \text{ psig} + 2.814 \text{ psia} = 10.814 \text{ psia} = \boxed{74.6 \text{ kPa (a)}}$$

$$P_{02} = 100 \text{ kPa (a)} + 74.6 \text{ kPa (a)} = \boxed{174.6 \text{ kPa (a)}}$$

1.9) Known

$$P_1 = 20 \text{ psig} = 34.7 \text{ psia}$$

$$V_2 = \frac{1}{3} V_1$$

$$PV = nRT \quad m_1 R_1 T_1 = m_2 R_2 T_2$$

$$P_1 V_1 = P_2 V_2$$

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{(34.7 \text{ psia})(3)}{(1)} = 104.1 \text{ psia} = 89.4 \text{ psig}$$

1.10) Known

$$h_1 = 266 \text{ m}$$

$$h_2 = 915 \text{ m}$$

$$T = -20^\circ\text{C} = 253^\circ\text{K}$$

$$P = 100 \text{ kPa}$$

$$R = 0.287 \text{ kJ/kgK}$$

$$B = 0.00650 \text{ k/m}$$

$$a) \rho = \frac{P}{RT} = \frac{100 \text{ kPa}}{(0.287 \text{ kJ/kgK})(253 \text{ K})} = \frac{1.377 \text{ kN}\cdot\text{kg/m}^3}{\text{kN}\cdot\text{m}} = \boxed{1.377 \text{ kg/m}^3}$$

$$b) -\rho g = \frac{dP}{dy}$$

$$\int dP = -\int \rho g dy$$

$$\Delta P = (-1.377 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(915 \text{ m} - 266 \text{ m})$$

$$= -8767 \text{ kg/m}\cdot\text{s}^2 = \boxed{-8767 \text{ Pa}}$$

$$c) -\rho g = \frac{dP}{dy} \rightarrow \frac{-\rho g}{RT} = \frac{dP}{dy} \rightarrow \int_{P_1}^{P_2} \frac{dP}{P} = \int_{h_1}^{h_2} \frac{-g}{RT} dy \rightarrow \ln\left(\frac{P_2}{P_1}\right) = \frac{-g}{RT} (h_2 - h_1)$$

$$\rightarrow \ln(P_2) = \frac{-g}{RT} (h_2 - h_1) + \ln(P_1) = \frac{-(9.81 \text{ m/s}^2)(915 - 266 \text{ m})}{(287 \text{ J/kgK})(253 \text{ K})} + \ln(100000)$$

$$\ln(P_2) = 11.42524$$

$$P_2 = e^{11.42524}$$

$$= 91605 \text{ Pa}$$

$$\Delta P = P_2 - P_1 = (91605 - 100000) \text{ Pa} = \boxed{-8395 \text{ Pa}}$$

$$d) T = T_0 - B(y - y_0) = 253 - 0.00650 \text{ K/m} (915 - 266) \text{ m} = 248.78 \text{ K}$$

$$P_2 = P_0 \left[\frac{T_0 - B(z - z_0)}{T_0} \right]^{\frac{g}{BR}} = (100000 \text{ Pa}) \left(\frac{248.78 \text{ K}}{253 \text{ K}} \right)^{\frac{9.81 \text{ m/s}^2}{(0.00650 \text{ K/m})(287 \text{ J/kgK})}} = 91524.8 \text{ Pa}$$

$$\Delta P = P_2 - P_1 = (91524.8 - 100000) \text{ Pa}$$

$$= \boxed{-8475 \text{ Pa}}$$

$$\rho = \frac{P}{RT} = \frac{91524.8 \text{ Pa}}{(287 \text{ J/kgK})(248.78 \text{ K})} = \boxed{1.282 \text{ kg/m}^3}$$

1.11) Known

$$h_2 = 10 \text{ km}$$

$$P_2 = 100 \text{ MPa}$$

$$P_1 = 0.1 \text{ MPa}$$

$$k = 2100 \text{ MPa}$$

$$SG = 1.03$$

$$h_1 = 0 \text{ km}$$

$$\rho_1 = SG(1000 \text{ kg/m}^3)$$

$$= 1030 \text{ kg/m}^3$$

$$m = \rho V$$

$$k = -V \left(\frac{\partial P}{\partial V} \right) = \frac{-\Delta P / \Delta V}{\Delta V} = \frac{-\Delta P V}{\Delta V} = \frac{-\Delta P (m/\rho_1)}{(m/\rho_2) - (m/\rho_1)}$$

$$\rightarrow = \frac{-\Delta P (m/\rho_1)}{(m/\rho_1 - m/\rho_2)/\rho_2 \rho_1} = \frac{m(-\rho_2 \rho_1 \rho_2)}{\rho_2 \rho_1 (\rho_1 - \rho_2)} = \frac{-\Delta P \rho_2}{\rho_1 - \rho_2}$$

$$\rightarrow \rho_1 - \rho_2 = \frac{-\Delta P \rho_2}{k} \rightarrow \rho_2 \left(\frac{\Delta P}{k} - 1 \right) = -\rho_1$$

$$\rightarrow \rho_2 = \frac{-\rho_1}{(\Delta P/k - 1)} = \frac{-1030 \text{ kg/m}^3}{\left(\frac{(100 - 0.1) \text{ MPa}}{2100 \text{ MPa}} - 1 \right)} = \boxed{1081 \text{ kg/m}^3}$$

1.12) Known

$$\mu = 2 \times 10^{-5} \text{ Ns/m}^2$$

$$l = 0.05 \text{ m}$$

$$r_p = 0.025 \text{ m}$$

$$r_c = 0.02505 \text{ m}$$

$$v = 12 \text{ m/s}$$

$$A_p = 7.8540 \times 10^{-3} \text{ m}^2$$

$$A_c = 7.8697 \times 10^{-3} \text{ m}^2$$

$$\mathcal{T}_p = \mu \frac{dv}{dy} A_p = \frac{(2 \times 10^{-5} \text{ Ns/m}^2)(12 \text{ m/s})(7.8540 \times 10^{-3} \text{ m}^2)}{(0.02505 - 0.025) \text{ m}}$$

$$= \boxed{3.77 \text{ N}}$$

1.13) Known

$$R = 0.06 \text{ m}$$

$$r = 0.03 \text{ m}$$

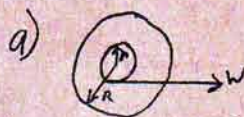
$$L = 0.15 \text{ m}$$

$$t = 0.001 \text{ m}$$

$$m = 0.03 \text{ kg}$$

$$\omega = 30 \text{ RPM} = \pi \text{ rad/s}$$

$$W = mg = 0.2943 \text{ N}$$



$$M_{\text{pulley}} = M_p = rW = 0.03 \text{ m}(0.2943 \text{ N}) = 0.008829 \text{ Nm}$$

$$\mathcal{T} = \mu \frac{dv}{dy} \quad v = R\omega \quad A_{\text{cylinder}} = A_c = RL$$

$$M_L = \mathcal{T} R A_c = \mu R \frac{dv}{dy} A_c = 2\mu R^2 \omega \pi R L / t = 2\mu R^3 L \pi \omega / t$$

$$M_L - M_p = 0 \quad M_L = M_p \quad 2\mu R^3 L \pi \omega / t = M_p$$

$$\mu = \frac{M_p t}{2 R^3 L \pi \omega} = \frac{(0.008829 \text{ Nm})(0.001 \text{ m})}{2(0.06 \text{ m})^3(0.15 \text{ m})(\pi)(\pi \text{ rad/s})} = \boxed{1.38 \times 10^{-2} \text{ Ns/m}^2}$$

$$1.13.) \text{b) } M_{L, \text{Hom}} = M_b = \int r dF = \int r \mu (rw/t) (2\pi r) dr = \int_0^R (2\mu\omega\pi r^3/t) dr = (2\pi\mu\omega/t) \int_0^R r^3 dr \\ = \mu\omega\pi R^4/2t$$

$$M_p - M_L - M_b = 0 \quad M_p = M_L + M_b$$

$$0.008829 \text{ N}\cdot\text{m} = 2\mu R^3 L \pi \omega / t + \mu\omega\pi R^4 / 2t$$

$$\rightarrow \mu = \frac{(0.008829 \text{ N}\cdot\text{m})}{\omega\pi (2R^3 L/t + R^4/2t)} = \frac{(0.008829 \text{ N}\cdot\text{m})}{(\pi)(\pi \text{ rad/s}) [2(0.06\text{m})^3(0.15\text{m})/0.001\text{s} + (0.06\text{m})^4/2(0.001\text{s})]} \\ = \frac{0.008829 \text{ N}\cdot\text{m}}{0.7035 \text{ m}^3/\text{s}} = \boxed{1.26 \times 10^{-3} \text{ N}\cdot\text{s}/\text{m}^2}$$

Problem Set 2

2.1) known

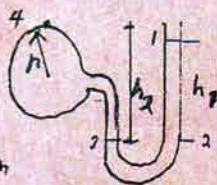
$$h_1 = 0.15 \text{ m}$$

$$h_2 = 0.05 \text{ m}$$

$$S.G. = 13.6$$

$$P_1 = 0 \text{ kPa (g)}$$

$$r = 0.03 \text{ m}$$



$$\begin{aligned} P_2 &= P_1 + \rho_{H_2O} g h_1 \\ &= 1000 \text{ kg/m}^3 (13.6) (9.81 \text{ m/s}^2) (0.15 \text{ m}) \\ &= 20012.4 \text{ kg/ms}^2 \\ &= 20.0 \text{ kPa} \end{aligned}$$

Note: $\text{kg/ms}^2 = \text{Pa}$

$$\begin{aligned} P_2 &= P_3 \quad P_4 = P_3 - \rho_{H_2O} g h_2 \\ &= 20.0 \text{ kPa} - 1000 \text{ kg/m}^3 (9.81 \text{ m/s}^2) (0.08 \text{ m}) \\ &= 20.0 \text{ kPa} - 784.8 \text{ kg/ms}^2 \\ &= 20.0 \text{ kPa} - 0.784 \text{ kPa} \\ &= \boxed{19.2 \text{ kPa}} \end{aligned}$$

2.2) Known

$$h_1 = 0.20 \text{ m}$$

$$h_2 = 0.10 \text{ m}$$

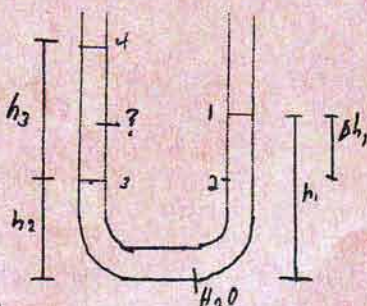
$$h_3 = 0.15 \text{ m}$$

$$\Delta h_1 = 0.10 \text{ m}$$

$$\Delta h_2 = 0.15 \text{ m}$$

$$P_1 = 0 \text{ kPa (g)}$$

$$P_2 = ?$$



$$\begin{aligned} P_2 &= P_1 + \rho_{H_2O} g \Delta h_1 \\ &= 0 + 1000 \text{ kg/m}^3 (9.81 \text{ m/s}^2) (0.10 \text{ m}) \\ &= 981 \text{ kg/ms}^2 \\ &= 0.981 \text{ kPa} \end{aligned}$$

$$P_2 = P_3$$

$$\begin{aligned} P_3 &= P_4 + \rho_{H_2O} g \Delta h_3 \\ P_4 &= \frac{P_3 - P_1}{g \Delta h_3} = \frac{981 \text{ Pa} - 0}{(9.81 \text{ m/s}^2) (0.15 \text{ m})} = 666.7 \text{ kg/m}^3 \end{aligned}$$

$$S.G. = \rho / 1000 \text{ kg/m}^3 = 666.7 \text{ kg/m}^3 = \boxed{0.667}$$

2.3) known

$$h_1 = 12 \text{ m}$$

$$h_2 = 6 \text{ m}$$

$$h_3 = 4 \text{ m}$$

$$h_4 = 0.9 \text{ m}$$

$$\rho_{oil} = 2950 \text{ kg/m}^3$$

$$\rho_{ALC} = 630 \text{ kg/m}^3$$

$$P_1 = 259.8 \text{ kPa}$$

$$\rho_{H_2O} = 998 \text{ kg/m}^3$$

$$\begin{aligned} P_2 &= P_1 - \rho_{ALC} g \Delta h_1 \\ &= 259800 \text{ Pa} - 630 \text{ kg/m}^3 (9.81 \text{ m/s}^2) (6 - 4) \text{ m} \\ &= 247439 \text{ Pa} \end{aligned}$$

$$\begin{aligned} P_3 &= P_2 + \rho_{oil} g h_4 \\ &= 247439 \text{ Pa} + 2950 \text{ kg/m}^3 (9.81 \text{ m/s}^2) (0.90 \text{ m}) \\ &= 273484.6 \text{ Pa} \end{aligned}$$

$$\begin{aligned} P_4 &= P_3 + \rho_{H_2O} g \Delta h_2 \\ &= 273484.6 \text{ Pa} + 998 \text{ kg/m}^3 (9.81 \text{ m/s}^2) (12 - 4 - 0.9) \text{ m} \\ &= 342996 \text{ Pa} \end{aligned}$$

$$\boxed{P_4 = 343 \text{ kPa}}$$

2.4) Known

$$P_1 = 50 \text{ psig} = 7200 \text{ psfg}$$

$$h_1 = 20 \text{ ft}$$

$$h_2 = 20 \text{ in} = 1.67 \text{ ft}$$

$$h_3 = 30 \text{ in} = 2.5 \text{ ft}$$

$$h_u = 5 \text{ ft}$$

$$SG_{Hg} = 13.6$$

$$SG_{oil} = 0.8$$

$$P_2 = P_1 - \rho_{oil} g \Delta h_1$$

$$= 7200 \text{ psfg} - 0.8 (1.9379 \text{ slugs/ft}^3) (32.2 \text{ ft/s}^2) (20 - 1.67) \text{ ft}$$

$$= 7200 \text{ psfg} - 915.0 \text{ slugs/ft}^2$$

$$= 6285 \text{ psfg}$$

Note: slugs/ft² = psf

$$P_3 = P_2 + \rho_{Hg} g h_3$$

$$= 6285 + 13.6 (1.9379 \text{ slug/ft}^3) (32.2 \text{ ft/s}^2) (2.5 \text{ ft})$$

$$= 6285 \text{ psfg} + 2122 \text{ slug/ft}^2$$

$$= 8407 \text{ psfg}$$

$$P_4 = P_3 - \rho_{H_2O} g \Delta h_2$$

$$= 8407 - 1.9379 \text{ slug/ft}^3 (32.2 \text{ ft/s}^2) (2.5 + 1.67 + 5) \text{ ft}$$

$$= 8407 - 5725 \text{ slug/ft}^2$$

$$= 7835 \text{ psfg}$$

$$P_A = 7835 \text{ psfg} / 144 \text{ ft}^2/\text{in}^2$$

$$= 54.4 \text{ psig} + 14.80 \text{ psia}$$

$$= 69.2 \text{ psia}$$

$$P_A = 69.2 \text{ psia}$$

2.5) Known

$$SG_{Hg} = 13.6$$

$$A_R = 25 A_T$$

$$\Delta h_T = 0.10 \text{ m}$$

$$\Delta V_R = \Delta V_T$$

$$A_R \Delta h_R = A_T \Delta h_T$$

$$25 A_R \Delta h_R = A_T \Delta h_T$$

$$\Delta h_R = \frac{\Delta h_T}{25} = \frac{0.10}{25} = 4.0 \times 10^{-3}$$

$$\Delta h = \Delta h_R + \Delta h_T \sin \theta$$

$$= 4.0 \times 10^{-3} + 0.1 \sin 16^\circ = 0.0299 \text{ m}$$

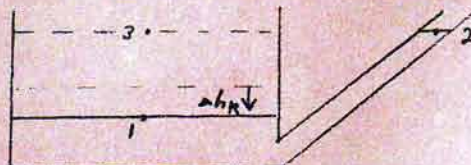
$$\Delta_1 P_2 = \rho_{Hg} g \Delta h = 13.6 (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) (0.0299 \text{ m}) = 3989 \text{ Pa}$$

$$\Delta_2 P_3 = \Delta_1 P_2 - \rho_{H_2O} g \Delta h = 3989 \text{ Pa} - 1000 \text{ kg/m}^3 (9.81 \text{ m/s}^2) (0.0299 \text{ m})$$

$$= 3989 \text{ Pa} - 293$$

$$= 3696 \text{ Pa}$$

$$\Delta P = 3.70 \text{ kPa}$$



2.6) Known

$$\Delta h = 500 \text{ m}$$

$$S.G._{\text{salt}} = 1.015$$

$$S.G._{H_2O} = 1.0$$

$$T = 4^\circ\text{C} = 277 \text{ K}$$

$$R = 287$$

Salt water @ 500 m

$$P_2 = P_1 + \rho_{\text{salt}} g \Delta h$$

$$= 1.015 (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) (500 \text{ m})$$

$$= 4978575 \text{ Pa}$$

Pump @ Surface

$$P_1 = P_2 - \rho_{H_2O} g \Delta h$$

$$= 4978575 \text{ Pa} - 1000 \text{ kg/m}^3 (9.81 \text{ m/s}^2) (500 \text{ m})$$

$$= 73575 \text{ Pa}$$

$$P_{\text{pump}} = 73.6 \text{ kPa (9)}$$

Air compressor @ Surface

$$\frac{P_0}{P} = e^{-(z-z_0) \frac{g}{RT}} \quad P = P_0 e^{(z-z_0) \frac{g}{RT}} = 4978575 e^{-500 (9.81 / (287 \cdot 277))}$$

$$= 4978575 e^{-0.061699}$$

$$= 4680690 \text{ Pa}$$

$$P_{\text{Air}} = 4681 \text{ kPa}$$

2.7) Known

$$\Delta h = 0.15 \text{ m}$$

$$l = 0.25 \text{ m}$$

$$\Delta P_h = \rho g h$$

$$\Delta P_e = \rho a l$$

$$\Delta P_h = \Delta P_e$$

$$\rho g h = \rho a l$$

$$a = gh / l$$

$$= (9.81 \text{ m/s}^2) (0.15 \text{ m}) / 0.25 \text{ m}$$

$$= 5.9 \text{ m/s}^2$$

$$a = 5.9 \text{ m/s}^2 \text{ Left}$$

2.8) Known

$$\omega = 600 \text{ Rev} = 20\pi \text{ rad/s}$$

$$\rho = 1.2 \text{ kg/m}^3$$

$$P_{\text{axis}} = 99125 \text{ Pa}$$

$$a_n = \omega^2 r$$

$$\rho = 1.2 \text{ kg/m}^3$$

$$r = 1.0 \text{ m}$$

$$\begin{aligned} a) \Delta P &= \int_a^r \rho a_n dr \\ &= \int_a^r \rho \omega^2 r dr \\ &= \frac{1}{2} \rho \omega^2 r^2 \end{aligned}$$

$$P_r = P_{\text{axis}} + \frac{1}{2} \rho \omega^2 r^2$$

$$b) P_r = P_{\text{axis}} + \frac{1}{2} \rho \omega^2 r^2$$

$$= 99125 + \frac{1}{2} (1.2 \text{ kg/m}^3) (20\pi)^2 (1.0 \text{ m})^2$$

$$= 101494 \text{ Pa}$$

$$P_r = 101494 \text{ Pa}$$

$$2.9) \quad P = P_{atm} + \rho g z \quad (1)$$

$$\frac{dP}{dr} = \rho a_r = -\rho r \omega^2$$

$$P - P_0 = \int_0^r \rho r \omega^2 dr$$

$$P - P_0 = \frac{1}{2} \rho \omega^2 r^2$$

$$P - P_{atm} = \rho g z_0 + \frac{1}{2} \rho \omega^2 r^2$$

$$P = P_{atm} + \rho g z_0 + \frac{1}{2} \rho \omega^2 r^2 \quad (2)$$

$$(1) = (2) \quad \rho g z = \rho g z_0 + \frac{1}{2} \rho \omega^2 r^2$$

$$z = z_0 + \omega^2 r^2 / 2g$$

$$b) \quad z = \frac{\omega^2 r^2}{2g}$$

$$(1) \quad \frac{dz}{dr} = r \omega^2 / g \quad (2) \quad \frac{dz}{dr} = \tan 45^\circ = 1$$

$$r \omega^2 / g = 1 \quad \omega = \sqrt{g/r} = \sqrt{9.81/0.05} = 14.0 \text{ rad/s} = 134 \text{ RPM}$$

Problem Set 3

3.1) known

$$R_1 = 0.075 \text{ m}$$

$$R_2 = 0.125 \text{ m}$$

$$\bar{v}_i = 10 \text{ m/s}$$

$$a) \quad Q=0 = \sum \bar{v}_i A_i = \bar{v}_i A_i - \bar{v}_e A_e$$

$$0 = (10 \text{ m/s}) [\pi (0.075 \text{ m})^2] - \bar{v}_e [\pi (0.125 \text{ m})^2]$$

$$\boxed{\bar{v}_e = 3.6 \text{ m/s}}$$

$$b) \quad Q=0 = \int_0^R \bar{v}_i \cdot dA = \bar{v}_i A_i - \int_0^{R_2} \bar{v}_e dA$$

$$0 = \bar{v}_i A_i - \int_0^{R_2} \bar{v}_{cl} [1 - (r^2/R^2)] 2\pi r dr$$

$$\bar{v}_i A_i = \int_0^{R_2} \bar{v}_{cl} 2\pi r dr - \int_0^{R_2} \bar{v}_{cl} (r^2/R^2) dr = \bar{v}_{cl} \left[\pi R_2^2 - \pi R_2^3 / (2R^2) \right]$$

$$\bar{v}_{cl} = \frac{2\bar{v}_i A_i}{\pi R_2^2} = \frac{2\pi (10 \text{ m/s}) (0.075 \text{ m})^2}{\pi (0.125 \text{ m})^2} = 7.2 \text{ m/s}$$

$$\boxed{\bar{v}_{cl} = 7.2 \text{ m/s}}$$

3.2) known

$$Q_{in} = 500 \text{ Usgal/min} = 1.114 \text{ ft}^3/\text{s}$$

$$R_1 = 5 \text{ ft}$$

$$R_2 = 1.25 \text{ in} = 0.1042 \text{ ft}$$

$$\bar{v}_e = \sqrt{2gh}$$

$$a) \quad Q=0 = Q_{in} - \bar{v}_e A_e \rightarrow (\sqrt{2gh}) (\pi R_2^2) = Q_{in}$$

$$\rightarrow h = \frac{(Q_{in}/\pi R_2^2)^2}{2g} = \frac{[(1.114 \text{ ft}^3/\text{s})/\pi (0.1042 \text{ ft})^2]^2}{2(32.2 \text{ ft/s}^2)} = 16.6 \text{ ft}$$

$$\boxed{h = 16.6 \text{ ft}}$$

$$b) \quad h = 5 \text{ ft} \quad Q = \frac{dV}{dt} = \frac{\pi r^2 dh}{dt} = Q_{in} - \bar{v}_e A_e$$

$$\frac{dh}{dt} = \frac{[Q_{in} - (\pi R_2^2) \sqrt{2gh}]}{\pi R_1^2}$$

$$= \frac{[1.114 \text{ ft}^3/\text{s} - (\pi [0.1042 \text{ ft}]^2) \sqrt{2(32.2 \text{ ft/s}^2)(5 \text{ ft})}]}{\pi (5 \text{ ft})^2}$$

$$= (0.5019 \text{ ft}^3/\text{s}) / 25\pi \text{ ft}^2$$

$$= 0.00639 \text{ ft/s} = 0.383 \text{ ft/min}$$

$$\boxed{dh/dt = 0.383 \text{ ft/min}}$$

3.3) Known

$$R_1 = 25 \text{ mm} = 0.025 \text{ m}$$

$$R_2 = 300 \text{ mm} = 0.30 \text{ m}$$

$$h = 10 \text{ mm} = 0.01 \text{ m}$$

$$\bar{v}_i = 10 \text{ m/s}$$

$$Q = 0 = \bar{v}_i A_i - \bar{v}_e A_e$$

$$0 = (10 \text{ m/s}) [\pi (0.025 \text{ m})^2] - \bar{v}_e [2\pi (0.30 \text{ m})(0.01 \text{ m})]$$

$$\bar{v}_e = \frac{0.0196 \text{ m}^2/\text{s}}{0.0188 \text{ m}^2} = 1.04 \text{ m/s}$$

$$\boxed{\bar{v}_e = 1.04 \text{ m/s}}$$

3.4) Known

$$\bar{v}_1 = 100 \text{ ft/s}$$

$$\bar{v}_3 = 19 \text{ ft/s}$$

$$\rho = 64.4 \text{ lbm/ft}^3$$

$$= 2 \text{ slug/ft}^3$$

$$A_1 = 0.1 \text{ ft}^2$$

$$A_3 = 1.0 \text{ ft}^2$$

$$A_2 = A_3 - A_1 = 0.9 \text{ ft}^2$$

$$a) Q = 0 = \bar{v}_1 A_1 + \bar{v}_2 A_2 - \bar{v}_3 A_3$$

$$\bar{v}_2 = \frac{\bar{v}_3 A_3 - \bar{v}_1 A_1}{A_2} = \frac{(19 \text{ ft/s})(1.0 \text{ ft}^2) - (100 \text{ ft/s})(0.1 \text{ ft}^2)}{(0.9 \text{ ft}^2)} = 10 \text{ ft/s}$$

$$\boxed{\bar{v}_2 = 10 \text{ ft/s}}$$

$$b) \dot{m}_2 = \rho \bar{v}_2 A_2 = (2 \text{ slug/ft}^3)(10 \text{ ft/s})(0.9 \text{ ft}^2) = 18 \text{ slug/s}$$

$$\boxed{\dot{m}_2 = 18 \text{ slug/s}}$$

3.5) Known

$$r_1 = 0.15 \text{ m}$$

$$r_2 = 0.08 \text{ m}$$

$$\bar{v}_1 = 2 \text{ m/s}$$

$$P_1 = 100 \text{ kPa (g)}$$

$$P_{\text{atm}} = 101 \text{ kPa (a)}$$

$$F_x = (\dot{m}\bar{v})_{\text{out}} + (\dot{m}\bar{v})_{\text{in}}$$

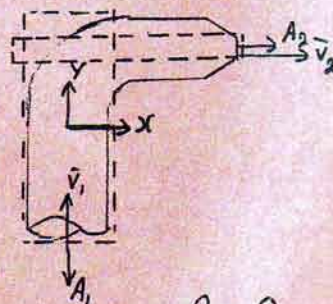
$$= \rho \bar{v}_2 A_2 \bar{v}_2 = \rho A_2 \bar{v}_2^2$$

$$\Sigma F_x = F_x + P_2 A_2 - P_{\text{atm}} A_2 \quad P_2 = P_{\text{atm}}$$

$$F_x = -(1000 \text{ kg/m}^3) [\pi (0.08 \text{ m})^2] (7.03 \text{ m/s})^2$$

$$= -994 \text{ kg m/s}^2$$

$$\boxed{F_x = -994 \text{ N}}$$



$$Q_1 = Q_2$$

$$\bar{v}_1 A_1 = \bar{v}_2 A_2$$

$$\bar{v}_2 = \bar{v}_1 A_1 / A_2$$

$$= \frac{\pi (0.15 \text{ m})^2 (2 \text{ m/s})}{\pi (0.08 \text{ m})^2}$$

$$= 7.03 \text{ m/s}$$

$$\Sigma F_y = F_y - P_1 A_1 + P_{\text{atm}} A_1$$

$$= (\dot{m}\bar{v})_{\text{out}} - (\dot{m}\bar{v})_{\text{in}} + A_1 (P_{\text{atm}} - P_1)$$

$$= -\rho A_1 \bar{v}_1^2 + A_1 (P_{\text{atm}} - P_1)$$

$$= -(1000 \text{ kg/m}^3) [\pi (0.15 \text{ m})^2] (2 \text{ m/s})^2 + [\pi (0.15 \text{ m})^2] (101000 \text{ Pa} - 100000 \text{ Pa})$$

$$= -282.7 \text{ kg m/s} - 7068.6 \text{ kg m/s}$$

$$= -7351 \text{ N}$$

$$\boxed{F_y = -7351 \text{ N}}$$

3.6) Known

$$P_1 = 100 \text{ Psia}$$

$$T_1 = 80^\circ\text{F} = 540^\circ\text{R}$$

$$\bar{v}_1 = 50 \text{ ft/s}$$

$$r_1 = 3 \text{ in}$$

$$P_2 = 200 \text{ Psig} = 214.7 \text{ Psia}$$

$$\bar{v}_2 = 100 \text{ ft/s}$$

$$\dot{m}_2 = 0.181 \text{ lbm/s}$$

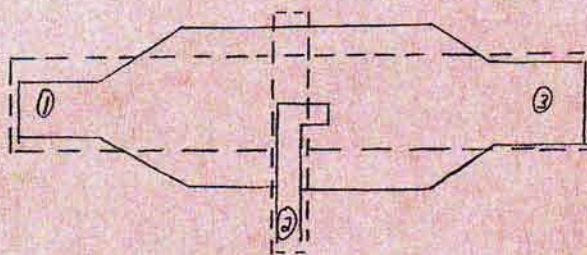
$$r_2 = 0.5 \text{ in}$$

$$P_3 = 70 \text{ Psia}$$

$$T_3 = 1500^\circ\text{F} = 1960^\circ\text{R}$$

$$r_3 = 6 \text{ in}$$

$$R = 53.3 \text{ ft}\cdot\text{lb}/\text{lbm}\cdot^\circ\text{R}$$



$$\rho = P/RT \quad \dot{m} = \rho \bar{v} A = P \bar{v} A / RT$$

$$a) \quad \dot{m}_1 = \frac{P_1 \bar{v}_1 A_1}{RT_1} = \frac{(100 \text{ lb}/\text{in}^2)(50 \text{ ft/s})[\pi(3 \text{ in})^2]}{(53.3 \text{ ft}\cdot\text{lb}/\text{lbm}\cdot^\circ\text{R})(540^\circ\text{R})} = 4.91 \text{ lbm/s}$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

$$= 4.91 \text{ lbm/s} + 0.181 \text{ lbm/s}$$

$$= 5.09 \text{ lbm/s}$$

$$\boxed{\dot{m}_3 = 5.09 \text{ lbm/s}}$$

$$\dot{m}_3 = \frac{P_3 \bar{v}_3 A_3}{RT_3} \quad \bar{v}_3 = \frac{\dot{m}_3 RT_3}{P_3 A_3} = \frac{(5.09 \text{ lbm/s})(53.3 \text{ ft}\cdot\text{lb}/\text{lbm}\cdot^\circ\text{R})(1960^\circ\text{R})}{(70 \text{ lb}/\text{in}^2)[\pi(6 \text{ in})^2]} = 67.2 \text{ ft/s}$$

$$\boxed{\bar{v}_3 = 67.2 \text{ ft/s}}$$

$$b) \quad \sum F_y = -P_2 A_2 + P_{atm} A_2 = -(P_2 - P_{atm}) A_2 = -(214.7 - 14.7) \text{ lb}/\text{in}^2 \cdot \pi(0.5 \text{ in})^2 = -157 \text{ lbf}$$

$$\boxed{F_y = 157 \text{ lbf}}$$

$$\begin{aligned} \sum F_x &= -P_1 A_1 - P_{atm} (A_3 - A_1) + P_3 A_3 \\ &= -(100 \text{ lb}/\text{in}^2)[\pi(3 \text{ in})^2] - (14.7 \text{ lb}/\text{in}^2)[\pi(6 \text{ in})^2 - \pi(3 \text{ in})^2] + (70 \text{ lb}/\text{in}^2)[\pi(6 \text{ in})^2] \\ &= 3842 \text{ lbf} \end{aligned}$$

$$\boxed{F_x = 3842 \text{ lbf}}$$

3.7) Known

$$Q_1 = 25 \text{ m}^3/\text{s}$$

$$r_1 = 0.375 \text{ m}$$

$$\rho = 1.3 \text{ kg}/\text{m}^3$$

$$P_{atm} = 101 \text{ kPa(a)}$$

$$r_2 = 0.25 \text{ m}$$

$$P_2 = 5 \text{ kPa(g)}$$

$$\bar{v}_1 = 0$$

$$\dot{m}_1 = \rho Q = (1.3 \text{ kg}/\text{m}^3)(25 \text{ m}^3/\text{s}) = 32.5 \text{ kg/s}$$

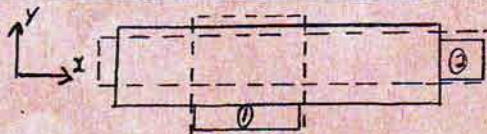
$$\dot{m}_1 = \dot{m}_2$$

$$\dot{m}_2 = \rho \bar{v}_2 A_2$$

$$\bar{v}_2 = \dot{m}_2 / \rho A_2$$

$$= (32.5 \text{ kg/s}) / (1.3 \text{ kg}/\text{m}^3)[\pi(0.25 \text{ m})^2]$$

$$= 127.3 \text{ m/s}$$



$$3.7...) \Sigma F_y = F_y + P_{atm} A_1 - P_{atm} A_2 = (\dot{m}v)_{out} - (\dot{m}v)_{in} = 0 \quad \boxed{F_y = 0}$$

Note: use of (g) instead of both sides in absolute

$$\begin{aligned} \Sigma F_x &= (\dot{m}v)_{out} - (\dot{m}v)_{in} + P_2 A_2 \\ &= (32.5 \text{ kg/s})(127.3 \text{ m/s}) + (5000 \text{ Pa})(g) [\pi (0.25 \text{ m})^2] \\ &= 5119 \text{ N} \end{aligned}$$

$$\boxed{F_x = 5119 \text{ N}}$$

3.8) Known

$$r_1 = 0.15 \text{ m}$$

$$r_2 = 0.10 \text{ m}$$

$$\bar{v}_1 = 1000 \text{ km/h} = 277.8 \text{ m/s}$$

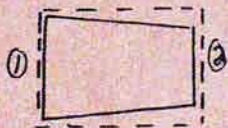
$$\bar{v}_2 = 800 \text{ m/s}$$

$$P_2 = 15 \text{ kPa}(g)$$

$$\dot{m}_1 = 20 \text{ kg/s}$$

$$\dot{m}_2 = (20 \text{ kg/s})(1/50) = 0.4 \text{ kg/s}$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 20.4 \text{ kg/s}$$



$$\begin{aligned} a) F_x &= (\dot{m}v)_{out} - (\dot{m}v)_{in} \\ &= (20.4 \text{ kg/s})(800 \text{ m/s}) - (20 \text{ kg/s})(277.8 \text{ m/s}) \\ &= 10764 \text{ kg m/s}^2 \end{aligned}$$

$$\boxed{F_x = 10764 \text{ N}}$$

$$\times F_x = 10764 \text{ N}$$

$$\begin{aligned} b) \Sigma F_x &= F_x + P_2 A_2 \\ &= 10764 \text{ N} + (15000 \text{ Pa}) [\pi (0.10 \text{ m})^2] \\ &= 11235 \text{ N} \end{aligned}$$

$$\boxed{F_x = 11235 \text{ N}}$$

3.9) Known

$$P_1 = P_2$$

$$\dot{m} = \rho \bar{v} A$$

$$\bar{v}_1 = 100 \text{ ft/s}$$

$$\rho = 64.4 \text{ lbm/ft}^3$$

$$A_1 = 0.1 \text{ ft}^2$$

$$A_2 = 0.9 \text{ ft}^2$$

$$A_3 = 1.0 \text{ ft}^2$$

$$\begin{aligned} F_x &= (\dot{m}v)_{out} - (\dot{m}v)_{in} \\ &= \dot{m}_3 \bar{v}_3 - \dot{m}_1 \bar{v}_1 - \dot{m}_2 \bar{v}_2 \\ &= \rho (\bar{v}_3^2 A_3 - \bar{v}_1^2 A_1 - \bar{v}_2^2 A_2) \\ &= (64.4 \text{ lbm/ft}^3) [(19 \text{ ft/s})^2 (1.0 \text{ ft}^2) - (100 \text{ ft/s})^2 (0.1 \text{ ft}^2) \\ &\quad - (10 \text{ ft/s})^2 (0.9 \text{ ft}^2)] \\ &= -46947.6 \text{ lbm ft/s}^2 \cdot (1 \text{ lbf} \cdot \text{s}^2 / 32.2 \text{ lbm ft}) \\ &= -1458 \text{ lbf} \end{aligned}$$

$$\Sigma F_x = F_x + P_3 A_3 - P_2 A_2 - P_1 A_1 = F_x + P_3 A_3 - P_2 (A_2 + A_1)$$

$$0 = -1458 \text{ lbf} + P_3 (1.0 \text{ ft}^2) - P_2 (0.9 \text{ ft}^2 + 0.1 \text{ ft}^2)$$

$$P_3 - P_2 = 1458 \text{ lbf} / 1.0 \text{ ft}^2$$

$$= 1458 \text{ lbf/ft}^2 \cdot (5 \text{ ft}^2 / 144 \text{ in}^2)$$

$$= 10.1 \text{ lbf/in}^2$$

$$\boxed{P_3 - P_2 = 10.1 \text{ psi}}$$

Problem Set 4

4.1) Known

$$P = P_{atm} = 101200 \text{ Pa (a)}$$

$$T_{atm} = 18^\circ\text{C} = 291 \text{ K}$$

$$\Delta h = 0.15 \text{ m}$$

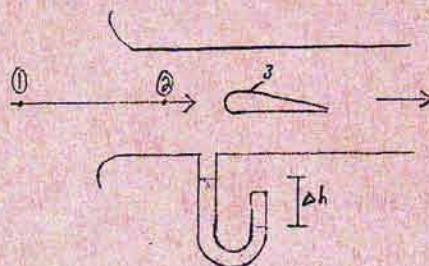
$$P_f = 95000 \text{ Pa (a)}$$

Assume: steady state

streamline

incompressible

$$\bar{v}_1 = 0$$



$$a) P_1 + \cancel{f g z_1} + \frac{1}{2} \rho \bar{v}_1^2 = P_2 + \cancel{f g z_2} + \frac{1}{2} \rho \bar{v}_2^2$$

$$P_T = P_1 = P_2 + \frac{1}{2} \rho \bar{v}_2^2 = \boxed{101200 \text{ Pa}}$$

$$b) P_2 = P_1 - \rho g \Delta h \quad P_T = P_2 + \frac{1}{2} \rho \bar{v}_2^2 \quad f_a = P/RT$$

$$P_2 = P_T - \rho g \Delta h \quad P_T - P_2 = \frac{1}{2} \rho \bar{v}_2^2$$

$$\textcircled{1} P_T - P_2 = \rho g \Delta h \quad \textcircled{2} P_T - P_2 = \frac{1}{2} \rho \bar{v}_2^2 / RT$$

$$\textcircled{1} = \textcircled{2} \quad \rho g \Delta h = \frac{1}{2} \rho \bar{v}_2^2 / RT \rightarrow \bar{v}_2 = \sqrt{\frac{2 \rho g \Delta h R T}{P}}$$

$$\bar{v}_2 = \sqrt{\frac{2(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.15 \text{ m})(287 \text{ J/kgK})(291 \text{ K})}{101200 \text{ Pa}} \cdot \left(\frac{\text{Pa m}^2}{\text{N}}\right) \cdot \left(\frac{\text{Nm}}{\text{J}}\right)}$$

$$= \sqrt{2428.76 \text{ m}^2/\text{s}^2} = 49.3 \text{ m/s} \cdot (3.6 \text{ km/h/m} \cdot \text{s})$$

$$= 177.4 \text{ km/h}$$

$$\boxed{\bar{v}_2 = 177.4 \text{ km/h}}$$

$$c) \underbrace{P_2 + \frac{1}{2} \rho \bar{v}_2^2}_{P_{atm}} + \cancel{f g z_2} = P_f + \frac{1}{2} \rho \bar{v}_f^2 + \cancel{f g z_f}$$

$$\rightarrow \bar{v}_f = \sqrt{\frac{2(P_{atm} - P_f)}{\rho}} = \sqrt{\frac{2(101200 - 95000) \text{ N/m}^2 \cdot (\text{kg m})}{(1.212 \text{ kg/m}^3) (\text{N s}^2)}} = \sqrt{10231.0 \text{ m}^2/\text{s}^2} = \boxed{101.1 \text{ m/s}}$$

$$d) \text{Altitude} = 5000 \text{ m}$$

$$\text{Table } \rho_a = 0.7364 \text{ kg/m}^3$$

$$\text{info: } P_{atm} = 54048 \text{ Pa}$$

$$i) P_{stag} = P_{atm} + \frac{1}{2} \rho_a \bar{v}_2^2$$

$$= 54048 \text{ Pa} + \frac{1}{2} (0.73643 \text{ kg/m}^3) (49.3 \text{ m/s})^2$$

$$= \boxed{54942 \text{ Pa}}$$

$$ii) P_{stag} = P_a - \frac{1}{2} \rho_a \bar{v}_f^2 \rightarrow P_f = P_{stag} - \frac{1}{2} \rho_a \bar{v}_f^2$$

$$\rightarrow P_f = 54942 - \frac{1}{2} (0.73643 \text{ kg/m}^3) (101.1 \text{ m/s})^2$$

$$= \boxed{51178 \text{ Pa}}$$

Problem Set 5

5.1) Known

$$P_{atm} = 14.7 \text{ Psia}$$

$$T_{atm} = 60^\circ\text{F} = 520^\circ\text{R}$$

$$\rho_a = 0.076 \text{ lbm/ft}^3$$

$$r = 3.5 \text{ ft}$$

$$\dot{m}_i = 1200 \text{ lbm/s}$$

$$\dot{m}_c = 0.83(1200) = 996 \text{ lbm/s}$$

$$\dot{m}_H = 0.17(1200) = 204 \text{ lbm/s}$$

$$\bar{V}_c = 900 \text{ ft/s}$$

$$\bar{V}_H = 1450 \text{ ft/s}$$

$$\dot{m}_F = 22000 \text{ lbm/h} = 6.1 \text{ lbm/s}$$

Assume

$$V_o = 0 \text{ negligible}$$

$$V_F = 0 \text{ negligible}$$

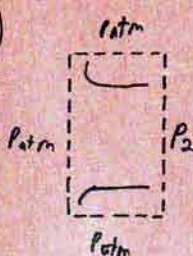
Pressure equal on all side

$$\dot{m}_H + \dot{m}_F = \dot{m}_H = 210.1 \text{ lbm/s}$$

$$\begin{aligned} a) \sum F_x &= (\dot{m}v)_{out} - (\dot{m}v)_{in} \\ &= \dot{m}_c \bar{V}_c + \dot{m}_H \bar{V}_H \\ &= (996 \text{ lbm/s})(900 \text{ ft/s}) + (210.1 \text{ lbm/s})(1450 \text{ ft/s}) \\ &= 1201045 \text{ lbm} \cdot \text{ft/s}^2 \cdot \left(\frac{\text{lb} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \right) \\ &= 37300 \text{ lbf} \end{aligned}$$

$$F_{stand} = 37300 \text{ lbf}$$

b)



Known

$$r = 3.5 \text{ ft}$$

$$\dot{m}_2 = \rho_a \bar{V}_2 A_2 \quad \bar{V}_2 = \frac{\dot{m}}{\rho A_2} = \frac{1200 \text{ lbm/s}}{(0.076 \text{ lbm/ft}^3)(\pi (3.5 \text{ ft})^2)} = 410.3 \text{ ft/s}$$

$$\begin{aligned} \sum F_x &= (\dot{m}v)_{out} - (\dot{m}v)_{in} \\ &= (1200 \text{ lbm/s})(410.3 \text{ ft/s}) \cdot \left(\frac{\text{lb} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \right) \\ &= 15290.7 \text{ lbf} \end{aligned}$$

$$P_1 + \cancel{\rho_a g z_1} + \frac{1}{2} \cancel{\rho_a \bar{V}_1^2} = P_2 + \cancel{\rho_a g z_2} + \frac{1}{2} \rho_a \bar{V}_2^2$$

$$P_2 = P_1 - \frac{1}{2} \rho_a \bar{V}_2^2$$

$$\begin{aligned} &= 14.7 \text{ lbf/in}^2 - \frac{1}{2} (0.076 \text{ lbm/ft}^3) (410.3 \text{ ft/s})^2 \left(\frac{\text{lb} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \right) \left(\frac{1 \text{ ft}^2}{144 \text{ in}^2} \right) \\ &= 14.7 \text{ lbf/in}^2 - 1.4 \text{ lbf/in}^2 = 13.3 \text{ lbf/in}^2 \end{aligned}$$

$$P_{2g} = -1.4 \text{ psig}$$

$$b) \sum F_x = F_x - P_o A$$

$$F_x = \sum F_x / 260 \text{ lbf}$$

$$= 15290 \text{ lbf} / 260 \text{ lbf} = 7645 \text{ lbf/601 ft}$$

5.2) Known

$$h_1 = 20 \text{ m}$$

$$h_2 = 3 \text{ m}$$

$$w = 10 \text{ m}$$

$$Q = 554 \text{ m}^3/\text{s}$$

$$F_x = \int P dA$$

$$= \int \rho g h dA$$

$$= \int \rho g w h dh$$

$$\begin{aligned} a) F_{x_1} &= \int_0^{20} \rho g w h dh = \frac{1}{2} \rho g w h^2 \Big|_0^{20} \\ &= \frac{1}{2} (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) (10 \text{ m}) (20 \text{ m})^2 \\ &= 1.962 \times 10^7 \text{ kg} \cdot \text{m/s}^2 \end{aligned}$$

$$F_{x_1} = 1.96 \times 10^7 \text{ N}$$

$$\begin{aligned} b) F_{x_2} &= \int_3^{20} \rho g w h dh = \frac{1}{2} (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) (10 \text{ m}) [(20 \text{ m})^2 - (3 \text{ m})^2] \\ &= 19178550 \text{ N} \end{aligned}$$

$$5.2b...) \sum F_x = F_{x_b} - [(mv)_{out} - (mv)_{in}]$$

$$= F_{x_b} - [\rho Q^2/A_2 - \rho Q^2/A_1]$$

$$= F_{x_b} - \rho Q^2 \left[\frac{1}{A_2} - \frac{1}{A_1} \right]$$

$$\dot{m} = \rho Q$$

$$\bar{V} = Q/A$$

$$= 19178550 \text{ N} - (1000 \text{ kg/m}^3)(554 \text{ m}^3/\text{s})^2 \left[1/(2\text{m})(10\text{m}) - 1/(20\text{m})(10\text{m}) \right]$$

$$= 19178550 \text{ N} - 8695953 \text{ N}$$

$$= 10482597 \text{ N}$$

$$\boxed{\sum F_x = 1.05 \times 10^7 \text{ N}}$$

5.3) known

$$Q_1 = 12000 \text{ cfm} = 200 \text{ ft}^3/\text{s}$$

$$T_1 = 70^\circ \text{F} = 530^\circ \text{R}$$

$$\rho_{\text{air}} = 20 \text{ in } \#20 \text{ (g)} = 1.67$$

$$r_1 = 8 \text{ in}$$

$$A_2 = (18 \text{ in}) \times (18 \text{ in}) = 324 \text{ in}^2 = 2.25 \text{ ft}^2$$

$$\bar{V}_1 = 0$$

$$\therefore F_y = \bar{V}A = 0$$

$$h = 5 \text{ ft}$$

$$\dot{m} = \rho Q$$

$$\bar{V} = Q/A$$

$$F_x = (m\bar{V})_{out} - (m\bar{V})_{in}$$

$$= \rho Q \bar{V}_2 = \rho Q^2/A_2$$

$$= \frac{(0.075 \text{ lbm/ft}^3)(200 \text{ ft}^3/\text{s})^2}{(2.25 \text{ ft}^2)} \cdot \left(\frac{\text{lbft} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \right)$$

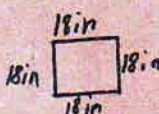
$$= 41.67 \text{ lbf}$$

$$P_{2g} = \rho g h$$

$$= (62.2 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(1.67 \text{ ft}) \cdot \left(\frac{\text{lbft} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \right)$$

$$= 103.9 \text{ lbf/ft}^2 \text{ (g)}$$

Assume: air density to be 0.075 lbm/ft^3



Note: If using absolute
↓

$$\sum F_x = F_x + P_{2g}A_2$$

$$= 41.67 \text{ lbf} + (103.9 \text{ lbf/ft}^2 \text{ (g)})(2.25 \text{ ft}^2)$$

$$= 275 \text{ lbf}$$

$$\boxed{\sum F_x = 275 \text{ lbf}}$$

$$\sum M_{y \text{ Floor}} = F_x h = (275 \text{ lbf})(5 \text{ ft}) = 1375 \text{ lbf} \cdot \text{ft}$$

5.4) known

$$SG = 0.86$$

$$P_g = 200 \text{ kPa (g)}$$

$$\dot{m} = 20 \text{ kg/s}$$

$$r_o = 0.075 \text{ m}$$

$$r_1 = 0.05 \text{ m}$$

$$r_2 = 0.025 \text{ m}$$

$$L_1 = 0.30 \text{ m}$$

$$L_2 = 0.25 \text{ m}$$

$$\bar{V} = \dot{m}/\rho A$$

$$P_1 + \frac{1}{2} \rho \bar{V}_1^2 + \rho g z_1 = P_2 + \frac{1}{2} \rho \bar{V}_2^2 + \rho g z_2$$

nozzle lies in horizontal

$$a) P_1 = P_2 + \frac{1}{2} \rho (\bar{V}_2^2 - \bar{V}_1^2)$$

$$= P_2 + \frac{1}{2} \rho \left[\frac{\dot{m}^2}{\rho^2} \left(\frac{1}{A_2^2} - \frac{1}{A_1^2} \right) \right]$$

$$= 200000 \text{ Pa} + \frac{(20 \text{ kg/s})^2}{2(0.86)(1000 \text{ kg/m}^3)} \cdot \left[\left(\frac{1}{\pi(0.025 \text{ m})^2} \right)^2 - \left(\frac{1}{\pi(0.05 \text{ m})^2} \right)^2 \right]$$

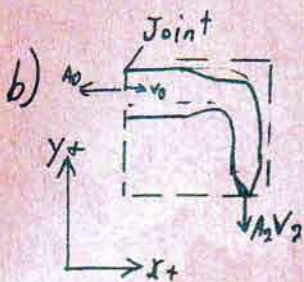
$$= 200000 \text{ Pa} + 60291 \text{ kg/m} \cdot \text{s}^2$$

$$= 260291 \text{ Pa (g)} + 101300 \text{ Pa (g)}$$

$$= 361591 \text{ Pa (g)}$$

$$\boxed{P_1 = 361.6 \text{ kPa (g)}}$$

$$\boxed{P_1 = 361.6 \text{ kPa (g)}}$$



$$\begin{aligned}
 F_x &= (\dot{m}v)_{out} - (\dot{m}v)_{in} - \Delta P A \\
 &= -\dot{m}^2 / \rho A_0 = (260.291 - 200000) \text{ Pa} / (9) \cdot [\pi (0.025 \text{ m})^2] \\
 &= -(20 \text{ kg/s})^2 / (0.86 \times 1000 \text{ kg/m}^3) [\pi (0.025 \text{ m})^2] - 1065.4 \text{ N} \\
 &= -26.3 \text{ N} - 1065.4 \text{ N} \\
 &= -1091.7
 \end{aligned}$$

$$F_x = -1091.7 \text{ N}$$

$$\begin{aligned}
 F_y &= (\dot{m}v)_{out} - (\dot{m}v)_{in} \quad \Delta P \text{ not included in a nozzle} \\
 &= \dot{m}^2 / \rho A_2 \\
 &= (20 \text{ kg/s})^2 / (0.86)(1000 \text{ kg/m}^3) [\pi (0.025 \text{ m})^2] \\
 &= 236.9 \text{ N}
 \end{aligned}$$

$$F_y = 237 \text{ N}$$

$$\sum M_{z \text{ flange}} = -F_y \cdot l_1 + F_x \cdot 0 = (236.9 \text{ N})(0.3 \text{ m}) = -71 \text{ N}\cdot\text{m}$$

$$M_z = -71 \text{ N}\cdot\text{m}$$

5.5) Known

$$P_i = P_F = 300 \text{ kPa g}$$

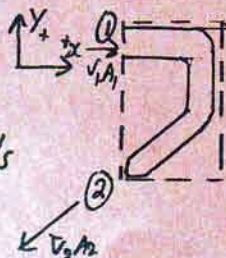
$$Q = 250 \text{ l/min} = 4.17 \times 10^{-3} \text{ m}^3/\text{s}$$

$$r_1 = 0.015 \text{ m}$$

$$r_2 = 0.0075 \text{ m}$$

$$L = 2 \text{ m}$$

note: dirⁿ of flow on diagram



$$\begin{aligned}
 \dot{m} &= \rho Q = (1000 \text{ kg/m}^3)(4.17 \times 10^{-3} \text{ m}^3/\text{s}) \\
 &= 4.17 \text{ kg/s}
 \end{aligned}$$

$$\begin{aligned}
 \bar{v}_1 &= Q/A_1 = Q/[\pi r_1^2] \\
 &= (4.17 \times 10^{-3} \text{ m}^3/\text{s}) / [\pi (0.015 \text{ m})^2] \\
 &= 5.9 \text{ m/s}
 \end{aligned}$$

$$\begin{aligned}
 \bar{v}_2 &= Q/A_2 = (4.17 \times 10^{-3} \text{ m}^3/\text{s}) / [\pi (0.0075 \text{ m})^2] \\
 &= 23.6 \text{ m/s}
 \end{aligned}$$

$$\sum F_x = (\dot{m}v)_{out} + (\dot{m}v)_{in} = F_x - P_1 A_1$$

$$F_x = \dot{m}_2 \bar{v}_2 \sin 45^\circ + \dot{m}_1 \bar{v}_1 + P_1 A_1$$

$$\begin{aligned}
 &= (4.17 \text{ kg/s}) [(23.6 \text{ m/s}) \sin 45^\circ + 5.9 \text{ m/s}] + (300000 \text{ Pa}) [\pi (0.015 \text{ m})^2] \\
 &= 306.2 \text{ N}
 \end{aligned}$$

$$F_x = 306 \text{ N}$$

$$\begin{aligned}
 \sum F_y &= (\dot{m}v)_{out} = (4.17 \text{ kg/s}) [(23.6 \text{ m/s}) \sin 45^\circ] \\
 &= 69.6 \text{ N}
 \end{aligned}$$

$$F_y = 69.6 \text{ N}$$

$$\sum M_z = -F_y \cdot L = -(69.6 \text{ N})(2 \text{ m}) = -138.4 \text{ N}\cdot\text{m}$$

$$M_z = 138.4 \text{ N}\cdot\text{m}$$

5.6) Known

$$\dot{m} = 5 \text{ kg/s}$$

$$\bar{v}_1 = 100 \text{ m/s}$$

$$\bar{v}_2 = 300 \text{ m/s}$$

$$r = 0.075 \text{ m}$$

$$L_x = 2 \text{ m}$$

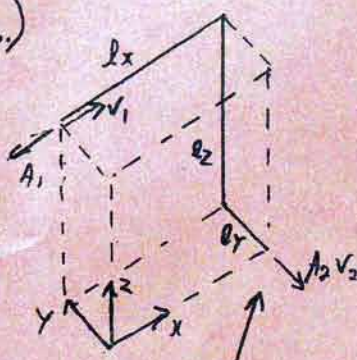
$$L_z = 5 \text{ m}$$

$$L_y = 1 \text{ m}$$

$$P_1 = 200 \text{ kPa(g)} = 301 \text{ kPa(a)}$$

$$P_2 = P_{atm} = 101 \text{ kPa(a)}$$

5.6...)



$$\Sigma F_x = (\dot{m}v)_{out} - (\dot{m}v)_{in} = F_x + P_1 A_1$$

$$F_x = -\dot{m}_1 v_1 - P_1 A_1$$

$$= (5 \text{ kg/s})(100 \text{ m/s}) - (200000 \text{ Pa})(\pi(0.075 \text{ m})^2)$$

$$= -4034 \text{ N}$$

$$F_x = -4034 \text{ N}$$

$$\Sigma F_y = (\dot{m}v)_{out} - (\dot{m}v)_{in}$$

Note: atmospheric on both sides

$$= \dot{m}_2 v_2 = (5 \text{ kg/s})(300 \text{ m/s})$$

$$= -1500 \text{ N}$$

$$F_y = -1500 \text{ N}$$

Note: negative dirⁿ

$\Sigma F_z = 0$ By inspection - atmospheric both sides
- no flow enter/exit

$$\Sigma M_y = F_x \cdot 0 = 0$$

$$\Sigma M_z = F_y \cdot l_x = (-1500 \text{ N})(2 \text{ m}) = -3000 \text{ N}\cdot\text{m}$$

$$\Sigma M_x = F_y \cdot l_z = (-1500 \text{ N})(5 \text{ m}) = -7500 \text{ N}\cdot\text{m}$$

$$M_y = 0$$

$$M_z = -3000 \text{ N}\cdot\text{m}$$

$$M_x = -7500 \text{ N}\cdot\text{m}$$

5.7) known

$$r_1 = 0.3 \text{ m (in)}$$

$$r_2 = 0.4 \text{ m (out)}$$

$$Q = 1000 \text{ m}^3/\text{min}$$

$$= 16.7 \text{ m}^3/\text{s}$$

$$\Delta h_1 = 0.1 \text{ m, H}_2\text{O}(\text{g})$$

$$\dot{m}_s = 6000 \text{ kg/h}$$

$$= 1.67 \text{ kg/s}$$

$$T_a = 20^\circ\text{C} = 293 \text{ K}$$

$$r_{cyc} = 1.5 \text{ m}$$

$$P_1 = \rho g \Delta h = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.1 \text{ m})$$

$$= 981 \text{ Pa}$$

$$\dot{m}_a = \rho Q = (1.225 \text{ kg/m}^3)(16.7 \text{ m}^3/\text{s})$$

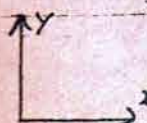
$$= 20.46 \text{ kg/s}$$

From Tables

$$\dot{m}_1 = \dot{m}_a + \dot{m}_s = (20.46 + 1.67) \text{ kg/s} = 22.13 \text{ kg/s}$$

$$\bar{v}_1 = Q/A_1 = (16.7 \text{ m}^3/\text{s})/[\pi(0.3 \text{ m})^2] = 59.06 \text{ m/s}$$

$$\bar{v}_2 = Q/A_2 = (16.7 \text{ m}^3/\text{s})/[\pi(0.4 \text{ m})^2] = 33.22 \text{ m/s}$$



$$\Sigma F_x = (\dot{m}v)_{out} - (\dot{m}v)_{in} = F_x - P_1 A_1$$

$$F_x = -\dot{m}_1 v_1 + P_1 A_1 = -(22.13 \text{ kg/s})(59.06 \text{ m/s}) + (981 \text{ Pa})(\pi(0.3 \text{ m})^2)$$

$$F_x = -1584 \text{ N}$$

$$\Sigma F_y = (\dot{m}v)_{out} - (\dot{m}v)_{in}$$

Note: atmospheric on both sides

$$= \dot{m}_a v_2 = (20.46 \text{ kg/s})(33.22 \text{ m/s})$$

$$F_y = 679.7 \text{ N}$$

$$\Sigma M_z = F_x \cdot (r_{cyc} - r_1) = (-1584 \text{ N})[(1.5 \text{ m}) - (0.3 \text{ m})] = 1900.8 \text{ N}\cdot\text{m}$$

$$M_z = 1900.8 \text{ N}\cdot\text{m}$$

5.8) Known

$$Q = 1 \text{ L/s} = 0.001 \text{ m}^3/\text{s}$$

$$A = 30 \text{ mm}^2 = 3 \times 10^{-5} \text{ m}^2$$

$$r = 0.2 \text{ m}$$

$$\dot{m} = \rho Q = (1000 \text{ kg/m}^3)(0.001 \text{ m}^3/\text{s}) = 1 \text{ kg/s}$$

$$\bar{v} = Q/A = (0.001 \text{ m}^3/\text{s}) / (3 \times 10^{-5} \text{ m}^2) = 16.67 \text{ m/s}$$

$$\bar{v}_{\text{nozzle}} = \bar{v} \cos 30 = 14.4 \text{ m/s}$$

$$\dot{m}_{\text{nozzle}} = \dot{m}/2 = 0.5 \text{ kg/s}$$

$$\begin{aligned} \sum F_y &= (m\bar{v})_{\text{out}} - (m\bar{v})_{\text{in}} = 2\dot{m}_{\text{nozzle}} \bar{v}_{\text{nozzle}} \\ &= 2(1 \text{ kg/s})(14.4 \text{ m/s}) = 2(0.5 \text{ kg/s})(14.4 \text{ m/s}) = 14.4 \text{ N} \end{aligned}$$

a) $\sum M_2 = F_y \cdot r = (14.4 \text{ N})(0.2 \text{ m}) = 2.88 \text{ N}\cdot\text{m}$ $M_2 = 2.88 \text{ N}\cdot\text{m}$

b) $\omega = 500 \text{ RPM} = 52.36 \text{ rad/s}$

$$\bar{v} = r\omega = 0.2 \text{ m}(52.36 \text{ rad/s})$$

$$= 10.47 \text{ m/s}$$

$$\sum M_2 = \dot{m} \bar{v} r = (1 \text{ kg/s})(14.4 - 10.47) \text{ m/s}(0.2 \text{ m}) = 0.786 \text{ N}\cdot\text{m}$$

$M_2 = 0.786 \text{ N}\cdot\text{m}$

c) $\omega = \bar{v}/r = (14.4 \text{ m/s})/(0.2 \text{ m}) = 72.2 \text{ rad/s} \cdot (60 \text{ min/s} / 2\pi) = 689 \text{ RPM}$ $\omega = 689 \text{ RPM}$

5.9) Known

$$r_h = 1.3 \text{ ft}$$

$$r_f = 1.7 \text{ ft}$$

$$r_m = 1.5 \text{ ft}$$

$$Q = 250 \text{ ft}^3/\text{s}$$

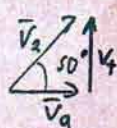
$$\omega = 1000 \text{ RPM} = 104.72 \text{ rad/s}$$

$$\rho = 0.077 \text{ lbm/ft}^3$$

$$\dot{m} = \rho Q = (0.077 \text{ lbm/ft}^3)(250 \text{ ft}^3/\text{s}) = 19.25 \text{ lbm/s}$$

$$\bar{v}_1 = Q/A_1 = (250 \text{ ft}^3/\text{s}) / (\pi(1.7^2 - 1.3^2) \text{ ft}^2) = 66.3 \text{ ft/s}$$

$$\bar{v}_a = \bar{v}_1$$



$$\bar{v}_2 = \bar{v}_1 \tan 50$$

$$= 66.3 \tan 50 = 79.0 \text{ ft/s}$$

a) $\sum F_r = \dot{m} \bar{v}_2 = (19.25 \text{ lbm/s})(79.0 \text{ ft/s}) \cdot (2 \text{ ft} / 32.2 \text{ lbm}\cdot\text{ft})$
 $= 47.22 \text{ lbf}$

$$\sum M = r_m F_r = (1.5 \text{ ft})(47.22 \text{ lbf}) = 70.8 \text{ ft}\cdot\text{lbf}$$

$$\begin{aligned} W &= M\omega = (70.8 \text{ ft}\cdot\text{lbf})(104.72 \text{ rad/s}) \cdot (\text{hp}\cdot\text{s} / 550 \text{ ft}\cdot\text{lbf}) \\ &= 13.5 \text{ hp} \end{aligned}$$

$W = 13.5 \text{ hp}$

b) If the swirl doesn't change $M = 70.8 \text{ ft}\cdot\text{lbf}$

Problem Set 6

6.1) known

a) $v_2 = 150 \text{ ft/s}$

$\eta = 0.70$

$\Delta z = 25 \text{ ft}$

$d_2 = 1 \text{ in } r_2 = 0.5 \text{ in}$

$$A_2 = \pi r^2 = \pi (0.5/12)^2 = 0.005454 \text{ ft}^2$$

$P_1 = P_2$

Assume: $\Delta u = 0$ negligible
adiabatic $Q_H = 0$

$$\cancel{Q_H} - W_p = \sum \left(\frac{P}{\rho} + \frac{V^2}{2} + gz + u \right) \rho V A = \left(\frac{V_2^2}{2} - \frac{V_1^2}{2} + g\Delta z \right) \rho V_2 A_2$$

$$-W_p = \left[\frac{(150 \text{ ft/s})^2}{2} + (32.2 \text{ ft/s}^2)(25 \text{ ft}) \right] (1.94 \text{ slug/ft}^3) (150 \text{ ft/s}) (0.005454 \text{ ft}^2)$$

$$= (12055 \text{ ft}^2/\text{s}^2) (1.58595 \text{ slug/s}) \cdot \frac{2.6 \text{ ft}^2}{\text{slug} \cdot \text{ft}} \cdot \frac{1 \text{ hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lb}}$$

$$W_p = -34.76 \text{ hp} \rightarrow W_p/0.7 \rightarrow \boxed{W_{p_{\text{new}}} = -50 \text{ hp}}$$

-ive means input

b) $v_3^2 = v_2^2 + 2gh \rightarrow h = \frac{v_2^2}{2g} = \frac{(150 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 350 \text{ ft}$

$\boxed{h = 350 \text{ ft}}$

6.2) known

a) $d_1 = 6 \text{ in} \rightarrow r_1 = 0.25 \text{ ft}$

$d_2 = 4 \text{ in} \rightarrow r_2 = 0.16 \text{ ft}$

$Q = 8 \text{ ft}^3/\text{s}$

$\eta = 0.70$

$\Delta z = 0$

$A_1 = \pi r_1^2$

$= 0.196 \text{ ft}^2$

$A_2 = 0.0873 \text{ ft}^2$

$V_1 = Q/A_1 = (8 \text{ ft}^3/\text{s}) / (0.196 \text{ ft}^2) = 40.82 \text{ ft/s}$

$V_2 = 91.6 \text{ ft/s}$

Assume: $\Delta u = 0$ negligible

$P_1 = -10 \text{ psig} = -1440 \text{ psf g}$

adiabatic $Q_H = 0$

$P_2 = 25 \text{ psig} = 3600 \text{ psf g}$

$$\cancel{Q_H} - W_p = \sum \left(\frac{P}{\rho} + \frac{V^2}{2} + gz + u \right) \rho Q \quad -W_p = \left(\frac{P_2 - P_1}{\rho} + \frac{V_2^2 - V_1^2}{2} \right) \rho Q$$

$$-W_p = \left[\frac{(3600 - 1440) \frac{\text{lb}}{\text{ft}^2}}{1.94 \frac{\text{slug}}{\text{ft}^3}} + \frac{(91.6 \text{ ft/s})^2 - (40.82 \text{ ft/s})^2}{2} \right] (1.94 \text{ slug/ft}^3) (8 \text{ ft}^3/\text{s})$$

$$= \left[2597.9 \frac{\text{lb}}{\text{ft}^2} \cdot \frac{\text{slug} \cdot \text{ft}}{\text{lb} \cdot \text{s}^2} + 3362.1 \text{ ft}^2/\text{s}^2 \right] (15.52 \text{ slug/s})$$

$$= 92494.2 \text{ slug} \cdot \text{ft}^2/\text{s}^2 \cdot \frac{1 \text{ hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lb}} = 168.2 \text{ hp} \rightarrow \boxed{W_p = 168.2 \text{ hp}}$$

b) $W_p/0.7 \rightarrow \boxed{W_{p_{\text{new}}} = 240.3 \text{ hp}}$

6.3) Known

$$h_1 = 100 \text{ m}$$

$$h_2 = -5 \text{ m}$$

$$Q = 200 \text{ m}^3/\text{s}$$

$$d_2 = 7 \text{ m} \quad r_2 = 3.5 \text{ m}$$

$$\eta = 0.94$$

$$A_2 = \pi r^2 = 38.48 \text{ m}^2$$

$$v_2 = Q/A_2 = (200 \text{ m}^3/\text{s}) / (38.48 \text{ m}^2) = 5.20 \text{ m/s}$$

Take water level as datum

Assume: $\Delta u = 0$ negligible

$$P_1 = P_{\text{atm}}$$

Adiabatic $Q_H = 0$

$$v_1 = 0$$

$$Q_H - W_T = \sum \left(P_p + \frac{v^2}{2} + gz + u \right) \rho V A \rightarrow -W_T = \left(\frac{P_2 - P_1}{\rho} + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1) \right) \rho Q$$

$$W_T = -\rho Q \left(\frac{P_{\text{atm}} + \rho g(h_2) - P_{\text{atm}}}{\rho} + \frac{v_2^2}{2} + g(z_2 - z_1) \right)$$

$$= (1000 \text{ kg/m}^3)(200 \text{ m}^3/\text{s}) \left[\frac{(9.81 \text{ m/s}^2)(5 \text{ m})}{1} + \frac{(5.20 \text{ m/s})^2}{2} + (9.81 \text{ m/s}^2)(-5 \text{ m} - 100 \text{ m}) \right]$$

$$= 193.5 \text{ kg m}^2/\text{s}^2 \cdot \frac{\text{N s}^2}{\text{kg m}} \cdot \frac{\text{J}}{\text{N s}} \rightarrow W_T = 193.5 \text{ W}$$

$$W_T \cdot 0.94 \rightarrow \boxed{W_{\text{new}} = 182 \text{ W}}$$

6.4) Known

$$Q = 12000 \text{ ft}^3/\text{min} = 200 \text{ ft}^3/\text{s}$$

$$T_1 = 70^\circ\text{F} = 530^\circ\text{R}$$

$$T_2 = 315^\circ\text{F} = 775^\circ\text{R}$$

$$P_1 = P_2 = 14.7 \text{ psia} = 2116.8 \text{ psfa}$$

$$P_2 = 30 \text{ psig} = 6436.8 \text{ psfa}$$

$$v_2 = 1600 \text{ ft/s}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}, Q = \frac{P_1 Q}{RT_1} = \frac{(2116.8 \text{ psfa})(200 \text{ ft}^3/\text{s})}{(1716.3 \text{ ft}^2/\text{R s}^2)(530^\circ\text{R})} \cdot \frac{1 \text{ slug/ft}^3}{1 \text{ psfa}} = 0.465 \text{ slug/s}$$

Assume: $Q_H = 0$ Adiabatic

$$P = P/R T$$

$\Delta z = 0$ negligible

$$v_1 = v_2 = 0$$

$$R = 53.3 \frac{\text{ft} \cdot \text{lb}_f}{\text{lbm} \cdot ^\circ\text{R}} \cdot 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2} = 1716.3 \text{ ft}^2/\text{R s}^2$$

$$c_v = 0.17 \frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{R}} \cdot 778 \frac{\text{ft} \cdot \text{lb}_f}{\text{Btu}} \cdot 32.2 \frac{\text{lbm} \cdot \text{ft}}{\text{lb}_f \cdot \text{s}^2} = 4258.8 \text{ ft}^2/\text{R s}^2$$

$$a) Q_H - W = \sum \left[\frac{P}{\rho} + \frac{v^2}{2} + gz + u \right] \dot{m} \rightarrow -W = \left[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} + u_2 - u_1 \right] \dot{m}$$

$$W = -\dot{m} \left[\frac{P_2}{\rho_2} - \frac{P_1}{\rho_1} + c_p(T_2 - T_1) \right] = -\dot{m} \left[R(T_2 - T_1) + c_p(T_2 - T_1) \right] = -\dot{m} (R + c_p)(T_2 - T_1)$$

$$W = -(0.465 \text{ slug/s}) (1716.3 + 4258.8) \text{ ft}^2/\text{R s}^2 (775 - 530)^\circ\text{R} \cdot \frac{1 \text{ slug/ft}^3}{1 \text{ psfa}} \cdot \frac{\text{hp s}}{550 \text{ ft} \cdot \text{lb}_f/\text{s}}$$

$$\boxed{W = 7238 \text{ hp}}$$

6.4b) similar to part a but no work and add V_2

$$0 = -\dot{m} \left[(R + c_v)(T_3 - T_2) + \frac{V_2^2}{2} \right] \rightarrow V_2^2 = 2(R + c_v)(T_3 - T_2)$$

$$T_3 = \frac{-V_2^2}{2(R + c_v)} + T_2 = \frac{-(1600 \text{ ft/s})^2}{2(1716 + 4258.8) \text{ ft}^2/\text{R} \cdot \text{s}^2} + 775^\circ\text{R} = 561^\circ\text{R} = 101^\circ\text{F} \rightarrow \boxed{T_3 = 101^\circ\text{F}}$$

6.5) Known

$$W = -100 \text{ W}$$

$$Q_H = 1000 \text{ W}$$

$$P_1 = P_2 = 101325 \text{ Pa}$$

$$T_1 = 25^\circ\text{C} = 298 \text{ K}$$

$$Q = 60 \text{ m}^3/\text{h}$$

$$d_1 = 0.04 \text{ m} \quad r_1 = 0.02 \text{ m}$$

$$d_2 = 0.03 \quad r_2 = 0.015 \text{ m}$$

$$A_1 = \pi r_1^2 = 1.26 \times 10^{-3} \text{ m}^2$$

$$A_2 = 7.07 \times 10^{-4}$$

$$c_p = 1 \text{ kJ/kg} \cdot \text{K} \quad f = P/RT$$

$$R = 287 \text{ J/kg} \cdot \text{K}$$

$$\dot{m} = \rho Q = \frac{P_1 Q}{RT_1} = \frac{(101325 \text{ Pa})(60 \text{ m}^3/\text{h})}{(287 \text{ J/kg} \cdot \text{K})(298 \text{ K})} = 0.0197 \text{ kg/s}$$

$$V_2 = Q/A = 60 \text{ m}^3/\text{h} / 7.07 \times 10^{-4} \text{ m}^2 = 23.6 \text{ m/s}$$

Assume: Δz negligible, $v_1 = 0$, $P_1 = P_2$, $P_1 = P_2$

$$Q_H - W = \sum \left(\frac{P}{\rho} + \frac{V^2}{2} + gz + u \right) \dot{m} = \dot{m} \left(\frac{V_2^2 - V_1^2}{2} + c_p(T_2 - T_1) \right) \rightarrow Q_H - W = \dot{m} \left(\frac{V_2^2}{2} + c_p(T_2 - T_1) \right)$$

$$\rightarrow T_2 = \left[\frac{(Q_H - W)/\dot{m} - V_2^2/2}{c_p} \right] + T_1 = \left[\frac{(1000 + 100) \text{ W} / 0.0197 \text{ kg/s} - (23.6 \text{ m/s})^2/2}{(1000 \text{ J/kg} \cdot \text{K})} \right] + 298$$

$$\boxed{T_2 = 353.6 \text{ K} = 80.6^\circ\text{C}}$$

Iterations not necessary

6.6) Known

$$h = 450 \text{ ft}$$

$$Q = 10000 \text{ ft}^3/\text{s}$$

$$\eta = 0.94$$

$$H_L = 0.016 H_{DYN} L/D$$

$$L = 585 \text{ ft}$$

$$D = 26 \text{ ft}$$

$$P_1 = P_2 = P_{atm}$$

Assume: $v_1 = 0$

$$A_2 = \pi (13)^2 = 531 \text{ ft}^2$$

$$V_2 = Q/A_2 = 10000 \text{ ft}^3/\text{s} / 531 \text{ ft}^2 = 18.8 \text{ ft/s}$$

$$H_{DYN} = V^2/2g$$

$$H_L = 0.016 H_{DYN} L/D = 0.016 \left(\frac{V^2}{2g} \right) \frac{L}{D} = \frac{0.016 (18.8 \text{ ft/s})^2 (585 \text{ ft})}{2 (32.2 \text{ ft/s}^2) (26 \text{ ft})} = 1.98 \text{ ft}$$

This is taken into account in H_L

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + H_L + h_f$$

$$h_f = z_1 - h_L = 448.02 \text{ ft}$$

$$P_f = \rho g Q h_f = (1.94 \text{ slug/ft}^3) (32.2 \text{ ft/s}^2) (10000 \text{ ft}^3/\text{s}) (448.02 \text{ ft}) \cdot \frac{1 \text{ hp} \cdot \text{s}}{550 \text{ ft} \cdot \text{lb}} = 508853 \text{ hp}$$

$$P_f (0.94) = \boxed{P_{f_{new}} = 478321 \text{ hp}}$$

6.7) Known

$$SG = 1.26$$

$$\mu = 0.4 \text{ Pa}\cdot\text{s}$$

$$Q = 20 \text{ l/s} = 0.02 \text{ m}^3/\text{s} = 2.55 \text{ m/s}$$

$$d = 0.1 \text{ m} \quad r = 0.05 \text{ m} \quad v_1 = v_2$$

$$L = 45 \text{ m}$$

$$\theta = 15^\circ$$

$$P_1 = 590 \text{ kPa(g)}$$

Smooth Pipe

$$z_2 = 45 \sin 15^\circ$$

$$= 11.65 \text{ m}$$

$$z_1 = 0$$

$$A_1 = \pi r^2 = 7.85 \times 10^{-3} \text{ m}^2$$

$$Re = \frac{\rho v D}{\mu} = \frac{1.26 (1000 \text{ kg/m}^3) (0.1 \text{ m}) (2.55 \text{ m/s})}{0.4 \text{ Pa}\cdot\text{s}}$$

$$Re < 2000 \quad \therefore \text{Laminar Flow}$$

$$f = 64/Re = 0.1793$$

$$h_f = \frac{f v^2 L}{2 g D} = \frac{0.1793 (2.55 \text{ m/s})^2 (45 \text{ m})}{2 (9.81 \text{ m/s}^2) (0.1 \text{ m})} = 26.7 \text{ m}$$

$$P_1/\rho g + v_1^2/2 + z_1 = P_2/\rho g + v_2^2/2 + z_2 + h_f$$

$$z_1 = 0 \rightarrow P_2 = P_1 - \rho g (z_2 + h_f) = 590000 \text{ Pa} - (1.26) (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) (11.65 \text{ m} + 26.7 \text{ m})$$

$$P_2 = 116 \text{ kPa}$$

$$h_f = 4 \tau_w L / \rho g D \rightarrow \tau_w = h_f \rho g D / 4 L = 26.7 \text{ m} (1.26) (1000 \text{ kg/m}^3) (9.81 \text{ m/s}^2) (0.1 \text{ m}) / 4 (45 \text{ m})$$

$$\tau_w = 183 \text{ Pa}$$

6.8) Known

Assume: $\Delta z = 0$ negligible

$$v = Q/A = 4Q/\pi D^2 = \frac{4 (8.0 \text{ ft}^3/\text{s})}{\pi D^2}$$

$$\nu = 0.0002 \text{ ft}^2/\text{s}$$

$$gh_f = 0.4 \text{ ft lbf/lbm} \cdot \frac{32.2 \text{ lbm ft/s}^2}{32.2 \text{ ft}\cdot\text{lbf/lbm}} = 12.88 \text{ ft}^2/\text{s}^2$$

$$= 10.2 \text{ ft}^2/\text{s}^2 \quad v = \sqrt{gh_f}$$

$$Q = 8.0 \text{ ft}^3/\text{s}$$

$$\epsilon = 0.00015$$

$$\epsilon/D = 0.00015/D$$

$$Re = \frac{\rho v D}{\mu} = \frac{4QD}{\nu \pi D^2} = \frac{4 (8.0 \text{ ft}^3/\text{s})}{(0.0002 \text{ ft}^2/\text{s}) \pi D} = 5.0 \times 10^4 \text{ ft}/D$$

$$gh_f = \frac{f v^2 L}{2D} = \frac{f v^2 (100 \text{ ft})}{2D} = \frac{f v^2 (50 \text{ ft})}{D}$$

D (ft)	Re	ϵ/D	f	v (ft/s)	gh _f	
0.5	1.0×10^5	0.0003	0.028	40.80	4161.6	high
2.0	2.5×10^4	0.000075	0.026	2.55	4.23	low
1.5	3.4×10^4	0.00010	0.024	4.53	16.40	low
1.6	3.2×10^4	0.00009	0.024	4.00	12.00	closest

$$D_{\min} = 1.6 \text{ ft}$$

for square pipes



6.9) Known

$$h = 0.4 \text{ m}$$

$$w = 0.6 \text{ m}$$

$$Q = 200 \text{ m}^3/\text{h}$$

$$P_1 = 101 \text{ kPa}$$

$$T_H = 35^\circ\text{C} = 308 \text{ K}$$

$$T_C = 12^\circ\text{C} = 285 \text{ K}$$

$$E = 0.00015 \text{ m}$$

$$L = 100 \text{ m}$$

Use only for Re $\rightarrow D_H = \frac{4A}{P} = \frac{4hw}{2(h+w)} = \frac{4(0.4)(0.6)}{2(0.4+0.6)} \text{ m} = 0.48 \text{ m}$
 $r_H = 0.24 \text{ m}$

$$V = \frac{Q}{A} = \frac{200 \text{ m}^3/\text{h}}{(0.4 \text{ m})(0.6 \text{ m})} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = 0.231 \text{ m/s}$$

$$\mu = 1.88 \times 10^{-5} \text{ kg/ms}$$

$$\mu = 1.77 \times 10^{-5} \text{ kg/ms}$$

Heating

$$\rho = \frac{P}{RT_H} = \frac{101000 \text{ Pa}}{(287 \text{ J/kg}\cdot\text{K})(308 \text{ K})} = 1.14 \text{ kg/m}^3$$

$$Re = \frac{\rho V D_H}{\mu} = \frac{(1.14 \text{ kg/m}^3)(0.231 \text{ m/s})(0.48 \text{ m})}{(1.88 \times 10^{-5} \text{ kg/ms})} = 6.7 \times 10^2$$

$$h_f = \frac{f V^2 L}{2g D_H} = \frac{0.035 (0.231 \text{ m/s})^2 (100 \text{ m})}{2(9.81 \text{ m/s}^2)(0.48 \text{ m})} = 0.0198 \text{ m}$$

$$h_f = z + \frac{\Delta P}{\rho g}$$

$$\Delta P = h_f \rho g = (0.0198 \text{ m})(1.14 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 0.221 \text{ kg/ms}^2$$

$$\Delta P = 0.221 \text{ Pa}$$

Cooling

$$\rho = \frac{P}{RT_C} = \frac{101000 \text{ Pa}}{(287 \text{ J/kg}\cdot\text{K})(285 \text{ K})} = 1.23 \text{ kg/m}^3$$

$$Re = \frac{\rho V D_H}{\mu} = \frac{(1.23 \text{ kg/m}^3)(0.231 \text{ m/s})(0.48 \text{ m})}{(1.77 \times 10^{-5} \text{ kg/ms})} = 7.7 \times 10^2$$

$$h_f = \frac{f V^2 L}{2g D_H} = \frac{0.0335 (0.231 \text{ m/s})^2 (100 \text{ m})}{2(9.81 \text{ m/s}^2)(0.48 \text{ m})} = 0.0190 \text{ m}$$

$$h_f = z + \frac{\Delta P}{\rho g}$$

$$\Delta P = h_f \rho g = (0.0190 \text{ m})(1.23 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 0.0229 \text{ kg/ms}^2$$

$$\Delta P = 0.0229 \text{ Pa}$$

6.10) Known

$$D_1 = 8 \text{ in} = 0.6 \text{ ft}$$

$$r_1 = 0.3 \text{ ft}$$

$$D_2 = 24 \text{ in} = 2 \text{ ft}$$

$$r_2 = 1 \text{ ft}$$

$$L_1 = 50 \text{ ft}$$

$$L_2 = 300 \text{ ft}$$

$$T = 60^\circ\text{F}$$

$$V_1 = 18 \text{ ft/s}$$

$$V_2 =$$

$$\rho = 62.4 \text{ lbm/ft}^3 = 1.94 \text{ slug/ft}^3$$

$$\mu = 23.7 \times 10^{-6} \text{ lb}\cdot\text{s/ft}^2$$

$$Q = V_1 A_1 = (18 \text{ ft/s})(\pi (0.3 \text{ ft})^2) = 6.28 \text{ ft}^3/\text{s}$$

Assume: Smooth Pipes

z negligible

$$V_2 = Q/A_2 = (6.28 \text{ ft}^3/\text{s})/(\pi (1 \text{ ft})^2) = 2 \text{ ft/s}$$

$$K = \left(1 - \frac{A_1^2}{A_2^2}\right)^2 = \left(1 - \frac{D_1^2}{D_2^2}\right)^2 = \left(1 - \frac{(0.6)^2}{(2)^2}\right)^2 = 0.79$$

$$h_f = \frac{K V_1^2}{2g} = \frac{0.79 (18 \text{ ft/s})^2}{2(32.2 \text{ ft/s}^2)} = 3.97 \text{ ft}$$

$$Re_1 = \frac{\rho V_1 D_1}{\mu} = \frac{(1.94 \text{ slug/ft}^3)(18 \text{ ft/s})(0.5 \text{ ft})}{(23.7 \times 10^{-6} \text{ lb} \cdot \text{s/ft}^2)} = 9.8 \times 10^5$$

$$h_{f1} = \frac{f V_1^2 L_1}{2g D_1} = \frac{0.0125 (18 \text{ ft/s})^2 (50 \text{ ft})}{2(32.2 \text{ ft/s}^2)(0.5 \text{ ft})} = 4.72 \text{ ft}$$

$$Re_2 = \frac{(1.94 \text{ slug/ft}^3)(2 \text{ ft/s})(2 \text{ ft})}{(23.7 \times 10^{-6} \text{ lb} \cdot \text{s/ft}^2)} = 3.3 \times 10^5$$

$$h_{f2} = \frac{0.014 (2 \text{ ft/s})^2 (300 \text{ ft})}{2(32.2 \text{ ft/s}^2)(2 \text{ ft})} = 0.13 \text{ ft}$$

$$h_{f_{total}} = h_{f1} + h_{f2} + h_e = 4.72 \text{ ft} + 0.13 \text{ ft} + 3.97 \text{ ft} = \boxed{8.8 \text{ ft}}$$

6.11) Known

$$Q = 1 \text{ ft}^3/\text{s}$$

$$\mu = 2.11 \times 10^{-5} \text{ lb} \cdot \text{s/ft}^2$$

$$D = 3 \text{ in} = 0.25 \text{ ft} \quad r = 0.125 \text{ ft}$$

$$L = 200 \text{ ft}$$

$$P_1 = 14.7 \text{ psia} = 2116.8 \text{ lb/ft}^2$$

$$P_2 = 20 \text{ psia} = 2880 \text{ lb/ft}^2$$

$$\Delta z = 0$$

$$\epsilon = 0.006 \text{ in} = 0.0005 \text{ ft}$$

$$\epsilon/D = 0.002$$

$$V = Q/A = (1 \text{ ft}^3/\text{s}) / \pi (0.125 \text{ ft})^2 = 20.37 \text{ ft/s}$$

$$Re = \frac{\rho V D}{\mu} = \frac{(1.94 \text{ slug/ft}^3)(20.37 \text{ ft/s})(0.25 \text{ ft})}{2.11 \times 10^{-5} \text{ lb} \cdot \text{s/ft}^2} = 4.68 \times 10^5$$

$$h_f = \frac{f V^2 L}{2g D} = \frac{0.024 (20.37 \text{ ft/s})^2 (200 \text{ ft})}{2(32.2 \text{ ft/s}^2)(0.25 \text{ ft})} = 123.7 \text{ ft}$$

$$h_L = \frac{P_2 - P_1}{\rho g} = \frac{(2880 - 2116.8) \text{ lb/ft}^2}{(1.94 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)} = 12.2 \text{ ft}$$

$$h_{pump} = h_L + h_f = 135.9 \text{ ft}$$

$$P_p = \rho Q g h_{pump} = (1.94 \text{ slug/ft}^3)(1 \text{ ft}^3/\text{s})(32.2 \text{ ft/s}^2)(135.9 \text{ ft})(\text{lb} \cdot \text{s}^2/\text{slug} \cdot \text{ft}) (h_p / 550 \text{ lb} \cdot \text{ft/s})$$

$$= 15.4 \text{ hp}$$

$$\boxed{P_{pump} = 15.4 \text{ hp}}$$

6.12) Known $\epsilon = 0.0005 \text{ ft}$

$$K = 0.54 \quad \epsilon_b = 0.002$$

$$n = 0.70$$

$$D_b = 2 \text{ in} = 0.167 \text{ ft}$$

$$C_c = 0.6$$

$$D_v = 0.6(0.167 \text{ ft})$$

$$= 0.1 \text{ ft} \quad r_v = 0.05 \text{ ft}$$

$$Q = 1 \text{ ft}^3/\text{s}$$

$$D_i = 3 \text{ in} \quad r_i = 0.125 \text{ ft}$$

$$\mu = 2.11 \times 10^{-5} \text{ lb} \cdot \text{s/ft}^2$$

$$V_i = Q/A = (1 \text{ ft}^3/\text{s}) / [\pi (0.125 \text{ ft})^2] = 20.37 \text{ ft/s}$$

$$V_v = Q/A_v = (1 \text{ ft}^3/\text{s}) / [\pi (0.05 \text{ ft})^2] = 127.32 \text{ ft/s}$$

$$Re_{D_i} = \frac{\rho V_i D_i}{\mu} = \frac{(1.94 \text{ slug/ft}^3)(20.37 \text{ ft/s})(0.25 \text{ ft})}{2.11 \times 10^{-5} \text{ lb} \cdot \text{s/ft}^2} = 468220.4$$

$$f = 0.023 \text{ from chart}$$

$$6.12...) \quad h_{f1} = \frac{f \bar{v}^2 L}{2gD} = \frac{0.023 (20.37 \text{ ft/s})^2 (200 \text{ ft})}{2 (32.2 \text{ ft/s}^2) (0.25 \text{ ft})} = 118.55 \text{ ft}$$

$$h_{f2} = \frac{[2 \times (0.54) + 0.6] (20.37 \text{ ft/s})^2}{2 (32.2 \text{ ft/s}^2)} = 10.8 \text{ ft} \quad h_f = h_{f1} + h_{f2} = 129.37 \text{ ft}$$

$$h_p = h_f + h_L = 12.2 \text{ ft} + 129.37 \text{ ft} = 141.57 \text{ ft}$$

$$P_p' = \rho Q g h_p = (1.94 \text{ slug/ft}^3) (1 \text{ ft}^3/\text{s}) (32.2 \text{ ft/s}^2) (141.57 \text{ ft}) (1 \text{ ft}^2/\text{slug} \cdot \text{ft}) (h_p \cdot \text{s} / 550 \text{ ft} \cdot \text{ft})$$

$$= 16.1 h_p$$

$$P_p = P_p' / 0.7 = \boxed{23.0 h_p} \leftarrow \text{I am uncertain of the accuracy of this}$$

6.13) Known

$$SG = 0.9$$

$$\mu = 0.0008 \text{ lb} \cdot \text{s} / \text{ft}^2$$

$$Q = 1 \text{ ft}^3/\text{s}$$

$$L = 3000 \text{ ft}$$

$$\Delta z = 40 \text{ ft}$$

$$\Delta P = 0$$

$$P = ?$$

$$\epsilon = 0.0008 \text{ s}$$

$$K = 0.5$$

$$h_{fi} = \frac{K \bar{v}^2}{2g} = \frac{0.5 \bar{v}^2}{2 (32.2 \text{ ft/s}^2)} = 0.00776 \bar{v}^2$$

$$\bar{v} = Q/A = 40 / \pi D^2$$

$$= 4 (1 \text{ ft}^3/\text{s}) / \pi D^2$$

$$= 1.27 \text{ ft}^2 / D^2$$

$$Re = \frac{\rho \bar{v} D}{\mu} = \frac{(1.27 \text{ ft}^2) \rho}{(0.0008 \text{ lb} \cdot \text{s} / \text{ft}^2) D} = \frac{(1.94 \text{ slug/ft}^3) (0.9) (1.27 \text{ ft}^2)}{\text{slug} \cdot \text{ft}} \cdot \frac{\text{lb} \cdot \text{s}^2}{\text{slug} \cdot \text{ft}}$$

$$Re = 2771.78 / D$$

$$h_f = \frac{f \bar{v}^2 L}{2gD} = \frac{f (1.27 \text{ ft}^2 / D^2)^2 L}{2gD} = \frac{f (1.27 \text{ ft}^2)^2 (3000 \text{ ft})}{2 (32.2 \text{ ft/s}^2) D^5} \quad h_f = 75.1 / D^5$$

D	Re	ϵ/D	f	$h_{fi} (\text{ft})$	$h_f (\text{ft})$	$h_{\Sigma} (\text{ft})$	
0.5	5.5×10^3	0.0003	0.038	0.2	91.4	91.6	high
0.7	4×10^3	0.00021	0.040	0.1	17.9	18.0	low
→ 0.6	4.6×10^3	0.00025	0.040	0.2	38.5	38.7	≈ Δz = 40 ft

$$\boxed{D = 0.6 \text{ ft}}$$

6.14) known

$$V = 100 \text{ m}^3$$

$$L = 50 \text{ m}$$

$$D = 0.02 \text{ m}$$

$$r = 0.01 \text{ m}$$

$$P_1 = 400 \text{ kPa (g)}$$

$$= 501 \text{ kPa (a)}$$

$$P_2 = 101 \text{ kPa (a)}$$

$$T = 15^\circ\text{C} = 288 \text{ K}$$

$$\mu = 1.136 \times 10^{-3} \text{ kg/ms}$$

$$\Delta z = 0$$

$$\rho \approx 1000 \text{ kg/m}^3$$

$$A = \pi(0.01 \text{ m})^2 = 0.000314 \text{ m}^2$$

$$Re = \frac{\rho \bar{v} D}{\mu} = \frac{(1000 \text{ kg/m}^3) \bar{v} (0.02 \text{ m})}{(1.136 \times 10^{-3} \text{ kg/ms})}$$

$$\rightarrow \bar{v} = \frac{Re}{17605.6} \text{ m/s}$$

$$h_f = \frac{f \bar{v}^2 L}{2gD} = \frac{f \bar{v}^2 (50 \text{ m})}{2(9.8 \text{ m/s}^2)(0.02 \text{ m})} \rightarrow h_f = 127.4 f \bar{v}^2$$

$$\text{Rubber hose } \varepsilon = 0.0015 \text{ mm}$$

$$= 1.5 \times 10^{-6} \text{ m}$$

$$\varepsilon/D = 0.000075$$

$$\frac{P_2}{\rho g} + z_2 + h_f = \frac{P_1}{\rho g} + z_1 \quad h_f = \frac{P_1 - P_2}{\rho g} = \frac{(501000 - 101000) \text{ Pa}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} = 40.8 \text{ m}$$

f	Re	$\bar{v} \text{ (m/s)}$	$h_f \text{ ft}$	Result
0.025	2.5×10^4	1.42	6.42	low
0.015	3.5×10^5	19.88	755.3	high
0.020	6.5×10^4	3.69	26.02	low
0.019	8.0×10^4	4.54	49.89	high
0.0195	7.0×10^4	3.98	39.27	Close enough

$$\begin{aligned} t &= V/Q \\ &= V/\bar{v}A \\ &= (100 \text{ m}^3)/\bar{v}(0.000314 \text{ m}^2) \\ &= 318471 \text{ m}/\bar{v} (3600 \text{ s/h}) \\ &= 88.46/\bar{v} \text{ h} \\ &= 88.46/(3.98) \\ &= 22.2 \text{ h} \end{aligned}$$

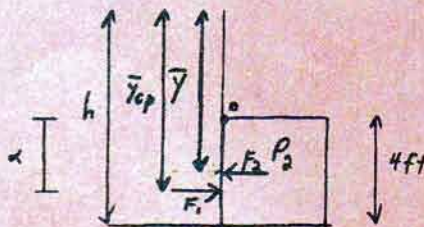
$$t = 22.2 \text{ h}$$

Problem Set 7

7.1) known

$$P_2 = 4 \text{ psig} = 576 \text{ psfg}$$

a) Square $A_s = 4 \cdot 4 = 16 \text{ ft}^2$ $\bar{y} = h - 2$



$$I_{xx} = bh^3/12 = (4)(4)^3/12 = 21.3 \text{ ft}^4$$

$$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y} A_s} + \bar{y} = \frac{21.3 \text{ ft}^4}{(h-2)(16 \text{ ft}^2)} + (h-2)$$

$$P_1 = \rho g \bar{y} F_1 = P_1 A_s \quad F_2 = P_2 A_s$$

$$= \frac{1.3}{(h-2)} + (h-2)$$

$$\sum M = 0 \quad F_1 \alpha = F_2 2$$

$$\alpha = 4 - (h - \bar{y}_{cp})$$

$$\rho g \bar{y} A_s (4 - h + \bar{y}_{cp}) = 2 P_2 A_s$$

$$\rho g (h-2) \left(4 - h + \frac{1.3}{h-2} + h-2 \right) = 2 P_2 \Rightarrow (1.94 \text{ slug/ft}^3) (32.2 \text{ ft/s}^2) \left(\frac{1.3}{h-2} + 2 \right) (h-2) = 2 (576 \text{ psfg})$$

$$\Rightarrow \left(\frac{1.3}{h-2} + 2 \right) (h-2) = 18.44 \Rightarrow \text{Calculator Equ}^n$$

$$\text{Solver} \longrightarrow \boxed{h = 10.6 \text{ ft}}$$

b) Triangle ∇ $A_T = \frac{1}{2} (4 \cdot 4) = 8 \text{ ft}^2$ $\bar{y} = h - \frac{4}{3} = h - \frac{4}{3}$

$$I_{xx} = bh^3/36 = 4 \cdot 4^3/36 = 7.1 \text{ ft}^4$$

$$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y} A_T} + \bar{y} = \frac{7.1 \text{ ft}^4}{(h - \frac{4}{3}) 8 \text{ ft}^2} + (h - \frac{4}{3})$$

$$\sum M = 0 \quad F_1 \alpha = \frac{4}{3} F_2$$

$$\rho g \bar{y} A_T \alpha = \frac{4}{3} P_2 A_T$$

$$\rho g (h - \frac{4}{3}) \left(\frac{0.8}{h - \frac{4}{3}} + \frac{4}{3} \right) = \frac{4}{3} (576)$$

$$(1.94 \text{ slug/ft}^3) (32.2 \text{ ft/s}^2) (h - \frac{4}{3}) \left(\frac{0.8}{h - \frac{4}{3}} + \frac{4}{3} \right) = 768$$

$$(h - \frac{4}{3}) \left(\frac{0.8}{h - \frac{4}{3}} + \frac{4}{3} \right) = 12.29 \Rightarrow \text{Calculator Equ}^n$$

$$\text{Solver} \longrightarrow \boxed{h = 11.2 \text{ ft}}$$

7.2) known

$$P_1 = 100 \text{ kPa(g)}$$

Water

$$\rho = 958 \text{ kg/m}^3$$

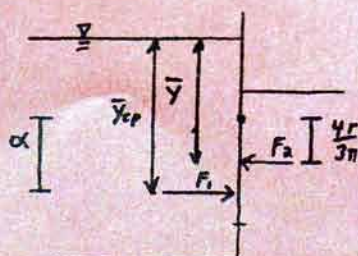
$\mu = 1$

$$A_c = \frac{1}{2} \pi r^2$$

$$= \pi/2 (1)^2$$

$$= 1.5708 \text{ m}^2$$

$$\bar{y} = 2 + \frac{4(1)}{3\pi} = 2.424$$



$$7.2...) I_{xx} = 0.1098 r^4 = 0.1098(1)^4 = 0.1098 m^4$$

$$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y}A} + \bar{y} = \frac{0.1098 m^4}{(2.424)(1.5708) m^2} + 2.424 m = 2.453 m$$

$$F_2 = P_2 A_c$$

$$P_1 = \rho g \bar{y}$$

$$F_1 = \rho g \bar{y} A_c$$

$$\sum M = 0 \quad F_1 \bar{y}_{cp} = F_2 \bar{y}_1$$

$$(P_1 \rho g \bar{y}) A_c \bar{y}_{cp} = P_2 A_c (2.424 m)$$

$$[(100000 Pa + (958 kg/m^3)(9.81 m/s^2)(2.424 m))(2.453 m) = P_2 (2.424 m) \rightarrow \boxed{P_2 = 124.2 kPa(g)}$$

7.3) Known
S.G. = 1.5

Displaced Volume $\rightarrow V = l \cdot w \cdot h - \frac{1}{2}(\pi r^2)l$
 $= (1m)(2m)(2m) - \frac{1}{2}(\pi(1m)^2)(1m)$
 $= 4m^3 - \frac{\pi}{2}m^3 = 2.429 m^3$

$$F = \rho g V = 1.5(1000 kg/m^3)(9.81 m/s^2)(2.429 m^3)$$

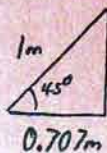
$$\rightarrow \boxed{F = 35.7 kN} \quad \boxed{F_{atch} = F/2 = 17.9 kN}$$

7.4) Known
 $\gamma = 8 kN/m^3$

$$\gamma = \rho g$$

$$P_y = \gamma \bar{y}$$

$$F_y = \gamma \bar{y} A_y$$



$$\bar{y} = 5 + (1 + 0.707)/2 = 5.854 m$$

$$A_y = (1.5)(1.707) = 2.561 m^2$$

$$I_{xx} = bh^3/12 = 1.5(1.707)^3/12 = 0.622 m^4$$

$$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y}A_y} + \bar{y} = \frac{0.622 m^4}{(5.854 m)(2.561 m^2)} + 5.854 m = 5.896 m$$

$$\bar{x} = 0.707/2$$

$$A_x = (0.707 m)(1.5 m)$$

$$P_x = \gamma h$$

$$F_x = \gamma h A_x$$

$$= 0.3535 m$$

$$= 1.061 m^2$$

$$h = (5 + 1 + 0.3535)$$

$$= 6.3535$$

$$\sum M = 0 \quad F_y(\bar{y}_{cp} - 5m) + F_x(\bar{x}) = F_j(1.0m)$$

$$F_j \cdot 1m = \gamma \bar{y} A_y(\bar{y}_{cp} - 5m) + \gamma h A_x \bar{x}$$

$$= \gamma [\bar{y} A_y(\bar{y}_{cp} - 5m) + h A_x \bar{x}]$$

$$= 8 kN/m^3 [(5.854 m)(2.561 m^2)(5.896 - 5)m + (6.3535 m)(1.061 m^2)(0.3535 m)]$$

$$F_j = 8 kN/m^3 (15.8 m^3) / 1m$$

$$= 126.4 kN$$

$$\boxed{F_j = 126.4 kN}$$

7.5) Note: Force acting on a circle are normal to the surface and pass through the hinge, thus creating no moment about the hinge.

7.5.) Known

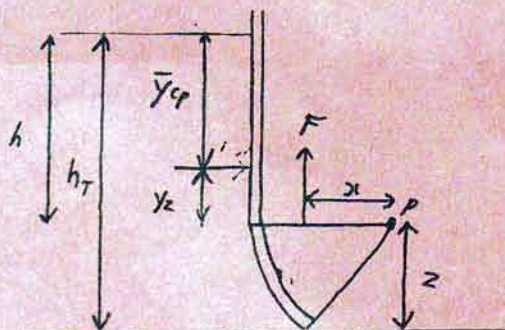
$$z = 6 \sin 45 = 4.24 \text{ ft}$$

$$x = 6 - 2 = 4 \text{ ft}$$

$$\rho = 62.4 \text{ lbm/ft}^3$$

$$F = 20000 \text{ lbf}$$

$$\bar{y}_{cp} = \frac{2}{3}h \quad \bar{y}_2 = h - \bar{y}_{cp} = \frac{1}{3}h$$



$$\sum M_p = 0 \quad (30000 \text{ lbf})(4 \text{ ft}) - F A y_2 = 0$$

$$120000 \text{ lbf}\cdot\text{ft} = \left(\frac{1}{2}\rho g h\right)(w h)\left(\frac{1}{3}h\right)$$

$$h^3 = \frac{6(120000 \text{ lbf}\cdot\text{ft})}{\rho g w} = \frac{720000 \text{ lbf}\cdot\text{ft}}{(62.4 \text{ lbm/ft}^3)(32.2 \text{ ft/s}^2)(30 \text{ ft})}$$

$$h^3 = 384.6 \text{ ft}^3 \rightarrow h = 7.27 \text{ ft}$$

$$h_T = h + z = (4.24 + 7.27) \text{ ft} = 11.5 \text{ ft} \quad \boxed{h = 11.5 \text{ ft}}$$

7.6) Known

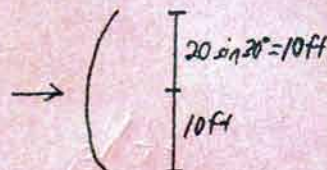
$$w = 1 \text{ ft}$$

$$r = 20 \text{ ft}$$

$$\rho = 1.94 \text{ slug/ft}^3$$

$$I_{xx} = \frac{b h^3}{12} = \frac{(1 \text{ ft})(20 \text{ ft})^3}{12} = 666.7 \text{ ft}^4$$

$$\bar{y}_{cp} = \frac{I_{xx}}{\bar{y} A} + \bar{y} = \frac{666.7 \text{ ft}^4}{(10 \text{ ft})(20 \text{ ft})} + 10 \text{ ft} = 13.3 \text{ ft}$$



$$a) F_x = \rho g \bar{y}_{cp} A = (1.94 \text{ slug/ft}^3)(32.2 \text{ ft/s}^2)(10 \text{ ft})(20 \text{ ft}) = 12493.6 \text{ slug ft/s} = 12493.6 \text{ lbf}$$

$$\boxed{F_x = 12493.6 \text{ lbf}}$$

b) Buoyancy

$$V = \left[\frac{1}{6} \pi r^2 - 2 \left(\frac{1}{2} b h \right) \right] w$$

$$= \left[\frac{1}{6} \pi (20 \text{ ft})^2 - (20 \cos 30^\circ)(20 \sin 30^\circ) \right] (1 \text{ ft})$$

$$= 36.2 \text{ ft}^3$$

$$F_{up} = \rho V g = (1.94 \text{ slug/ft}^3)(36.2 \text{ ft}^3)(32.2 \text{ ft/s}^2) = 2261 \text{ lbf} \quad \boxed{F_{up} = 2261 \text{ lbf}}$$

c, d) 0, due to all forces acting through the center

7.7) Known

$$\sigma_{max} = 100 \text{ MPa}$$

$$SG = 1.5$$

$$h = 20$$

$$D = 6 \text{ m}$$

$$h_{ring} = 2 \text{ m}$$

$$\sigma = \frac{PD}{2t} \rightarrow t = \frac{PD}{2\sigma}$$

$$t = \frac{\rho g h D}{2\sigma} = \frac{1.5(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(h)(6 \text{ m})}{2(100000000 \text{ N/m}^2)} \cdot \left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}} \right) \cdot \left(\frac{1000 \text{ mm}}{\text{m}} \right)$$

$$= 0.44145 h \cdot \text{mm/m}$$

$$P = \rho g h$$

top

$$h = 2 \text{ m}$$

$$\rightarrow t = 0.89$$

middle

$$h = 10 \text{ m}$$

$$\rightarrow t = 4.42$$

bottom

$$h = 20 \text{ m}$$

$$\rightarrow t = 8.84$$

Note: I believe the manual adds a safety factor

$$\boxed{t_{min} = 8.84}$$

7.8) Known

$$V = 4000 \text{ m}^3$$

$$h = 2000 \text{ m}$$

$$T = 275.16 \text{ K} \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{ tables}$$

$$P = 79510 \text{ Pa}$$

$$\rho_a = 1.0066 \text{ kg/m}^3$$

$$R_{H_2} = 4125 \text{ J/kgK}$$

$$R_{He} = 2077 \text{ J/kgK}$$

$$R_a = 287 \text{ J/kgK}$$

$$T_{Ha} = 40^\circ\text{C} = 313 \text{ K}$$

$$\rho_{H_2} = P/RT = 79510 \text{ Pa} / (4125 \text{ J/kgK})(275.16 \text{ K})$$

$$= 0.070 \text{ kg/m}^3$$

$$\rho_{He} = 79510 \text{ Pa} / (2077 \text{ J/kgK})(275.16 \text{ K})$$

$$= 0.139 \text{ kg/m}^3$$

$$\rho_{Ha} = 79510 \text{ Pa} / (287 \text{ J/kgK})(313 \text{ K})$$

$$= 0.885 \text{ kg/m}^3$$

$$i) F = \rho_a g V - \rho_{H_2} g V = g V (\rho_a - \rho_{H_2}) = (9.81 \text{ m/s}^2)(4000 \text{ m}^3)(1.0066 - 0.070) \text{ kg/m}^3 = 36752 \text{ kg m/s}^2$$

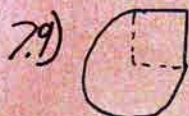
$$= 36.8 \text{ kN}$$

$$ii) F = g V (\rho_a - \rho_{He}) = (9.81 \text{ m/s}^2)(4000 \text{ m}^3)(1.0066 - 0.139) \text{ kg/m}^3 = 34044 \text{ kg m/s}^2$$

$$= 34.0 \text{ kN}$$

$$iii) F = g V (\rho_a - \rho_{Ha}) = (9.81 \text{ m/s}^2)(4000 \text{ m}^3)(1.0066 - 0.885) \text{ kg/m}^3 = 4771.6 \text{ kg m/s}^2$$

$$= 4.8 \text{ kN}$$



$$r = 0.5 \text{ m}$$

$$l = 1.0 \text{ m}$$

$$A = r \cdot r + \frac{3}{4} \pi r^2$$

$$= (0.5 \text{ m})(0.5 \text{ m}) + \frac{3}{4} (\pi) (0.5)^2$$

$$= 0.839 \text{ m}^2$$

$$V = A \cdot l = (0.839 \text{ m}^2)(1.0 \text{ m}) = 0.839 \text{ m}^3$$

$$V_L = \pi r^2 l = \pi (0.5 \text{ m})^2 (1.0 \text{ m}) = 0.785 \text{ m}^3$$

$$F_B = \rho g V = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.839 \text{ m}^3)$$

$$= 8230.59 \text{ kg m/s}^2$$

$$W_L = \rho g V_L \quad W_L = F_B$$

$$\rho_L = \frac{W_L}{V_L} = \frac{8230.59 \text{ kg m/s}^2}{(9.81 \text{ m/s}^2)(0.785 \text{ m}^3)}$$

$$= 1069 \text{ kg/m}^3$$

$$\rho_L = 1069 \text{ kg/m}^3$$

$$F_x = \rho g h A = \rho g h r l = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.25 \text{ m})(0.5 \text{ m})(1.0 \text{ m})$$

$$= 1226.25 \text{ kg m/s}^2$$

$$F_x = 1226 \text{ N}$$

7.10) known

$$h_{oil} = 4 \text{ m}$$

$$S_{G_{oil}} = 0.75$$

$$S_{G_w} = 1.03$$

$$w = 10 \text{ m}$$

$$h = 5 \text{ m}$$

$$\rho_w = 1000 \text{ kg/m}^3$$

$$V_b = h b \cdot l$$

$$= (5 \text{ m})(10 \text{ m})l = 50 \text{ m}^2 \cdot l$$

$$V_{oil} = b \cdot h_{oil} \cdot l$$

$$= (10 \text{ m})(4 \text{ m}) \cdot l = 40 \text{ m}^2 \cdot l$$

$$W = W_b + W_{oil} = W_b + \rho_{oil} V_{oil} g$$

$$\sum F_y = 0 \quad \rho_w V_b g - (W_b + \rho_{oil} V_{oil} g) = \rho_w S_{G_w} W_b - W_b - \rho_w S_{G_{oil}} V_{oil} g$$

$$W_b = \rho_w g (V_b S_{G_w} - V_b S_{G_{oil}})$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.03 \cdot 50 \text{ m}^2 \cdot l - 0.75 \cdot 40 \text{ m}^2 \cdot l)$$

$$= 210915 \text{ kg/s}^2 \cdot l$$

7.10...) $\Sigma F_y = 0$ $\rho_w S h_2 g - W_b = 0$ Without oil

$$V_{b2} = \frac{W_b}{\rho_w S g} \rightarrow \rho_w h_2 = \frac{210915 \text{ kg/s}^2 \cdot \text{m}}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(1.03)}$$

$$h_2 = 20.87 \text{ m}^2 / (10 \text{ m}) \quad \boxed{h_2 = 2.09 \text{ m}}$$

$$= 2.09 \text{ m}$$

$\Sigma M = 0$ $F_w d_w - F_o d_o - F_d d_d = 0$ $d_d = 6.0 \text{ m}$ $h_w = 5$ $\bar{y}_w = 2.5 \text{ m}$ $\bar{y}_{cpw} = \frac{1}{3} h_w = 1.67 \text{ m}$

$$F_d = (F_w d_w - F_o d_o) / d_d$$

$$= [\rho_w g S h_w \bar{y}_w \bar{y}_{cpw} A_w - \rho_w g S h_o \bar{y}_o \bar{y}_{cpo} A_o] / d_d$$

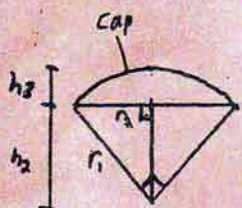
$h_o = 4$ $\bar{y}_o = 2 \text{ m}$ $\bar{y}_{cpo} = \frac{1}{3} h_o = 1.33 \text{ m}$

$$\rightarrow = (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) [(1.03)(2.5 \text{ m})(1.67 \text{ m})(5 \text{ m}) - (0.75)(2 \text{ m})(1.33 \text{ m})(4 \text{ m})] / 6 \text{ m}$$

$$= 22107 \text{ kg/s} = 22107 \text{ kg/s}^2 \cdot \text{m/m} = \boxed{22.11 \text{ kN/m}}$$

2.11) known

$h_1 = 0.3 \text{ m}$
 $r_1 = 0.05 \text{ m}$



$r_2 = r_1 \cos 45^\circ$
 $= 0.03535 \text{ m}$

$h_2 = 0.03535$
 $h_3 = r_1 - h_2$
 $= 0.01465$

$V_{cap} = V_{dome}$
 $= \frac{1}{3} \pi h_2^2 (3r_1 - h_2)$
 $= \frac{\pi}{3} (0.01465 \text{ m})^2 (3(0.05) - 0.01465)$
 $= 3.042 \times 10^{-5} \text{ m}^3$



$V_1 = (h_1 + h_3) \pi r_1^2 - V_{cap}$
 $= (0.3 + 0.01465) \text{ m} \cdot \pi (0.05 \text{ m})^2 - 3.042 \times 10^{-5} \text{ m}^3 = 1.205 \times 10^{-3} \text{ m}^3$



$V_2 = V_{sphere} - V_{center} = \frac{4}{3} \pi r_1^3 - 2V_{cap} - \pi r_2^2 (2h_1)$
 $= \frac{4}{3} \pi (0.05 \text{ m})^3 - 2(3.042 \times 10^{-5} \text{ m}^3) - \pi (0.03535 \text{ m})^2 (2)(0.03535 \text{ m})$
 $= 1.85 \times 10^{-4} \text{ m}^3$

$F = \rho_g (V_1 - V_2)$
 $= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2) (1.205 \times 10^{-3} \text{ m}^3 - 1.85 \times 10^{-4} \text{ m}^3)$
 $= 10.01 \text{ N}$

$\boxed{F = 10.0 \text{ N}}$