Winter 2008 Graphical Models Final Exam

April 14, 2008

Question 1

A Bayesian network is given in Figure 1. Determine whether the following statement holds true for this model: X_3 and X_7 are independent conditioned on X_2, X_4, X_5 and X_9. Justify your answer.

Question 2

A Markov random field is given in Figure 2.

1. Check whether the model is decomposable.
2. Find a junction tree of the model if it exists, otherwise first triangulate it (via the GraphElimination procedure to eliminate the vertices in the increasing order of their indices) and then obtain a junction tree for the triangulated graph.

Question 3

A small developing country consists of only 6 cities, which we will call A, B, C, D, E, and F, respectively. The six cities are geographically located, in this order, from the east end to the west end of the country. Each city has 4 towns, which we will index by 1, 2, 3, and 4 respectively.

The highway structure of the country is shown in Figure 3, where towns are indicated by nodes and highways are indicated by edges. Although in the figure the highways intersect, they in fact do not. One must pay a tax in order to travel along any of the highways, and the tax for each highway may be different.

Suppose that you are going to City F from City A and you know the tax for each highway in the map. Then you want to find the cheapest way to travel (in terms of minimizing the total amount of tax). Show that this problem can be solved by (generalized) sum-product algorithm. [You will need to formulate the problem as an optimization problem where you will optimize certain objective function; then you show that the sum-product algorithm can be a solution for solving the problem. You do not have to actually solve the problem for give the algorithm.]

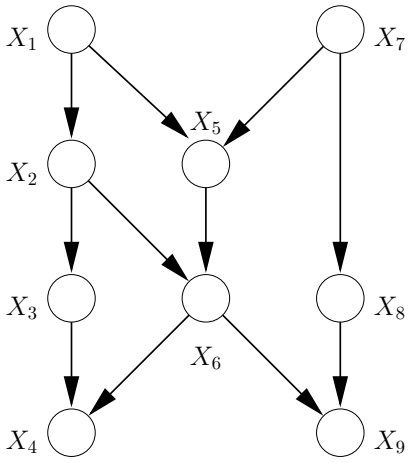

Figure 1: A Bayesian network.

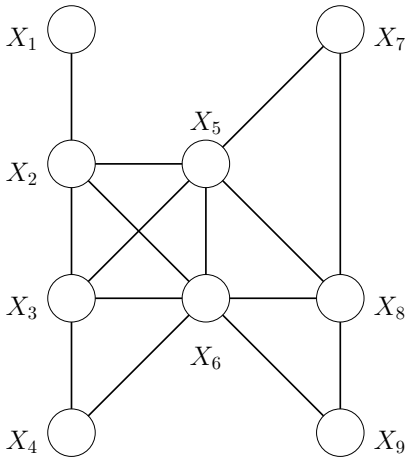

Figure 2: A Markov random field.

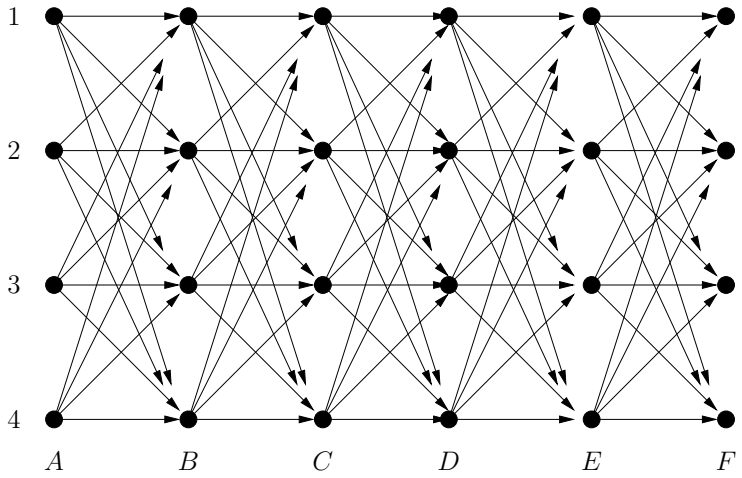

Figure 3: The highway structure