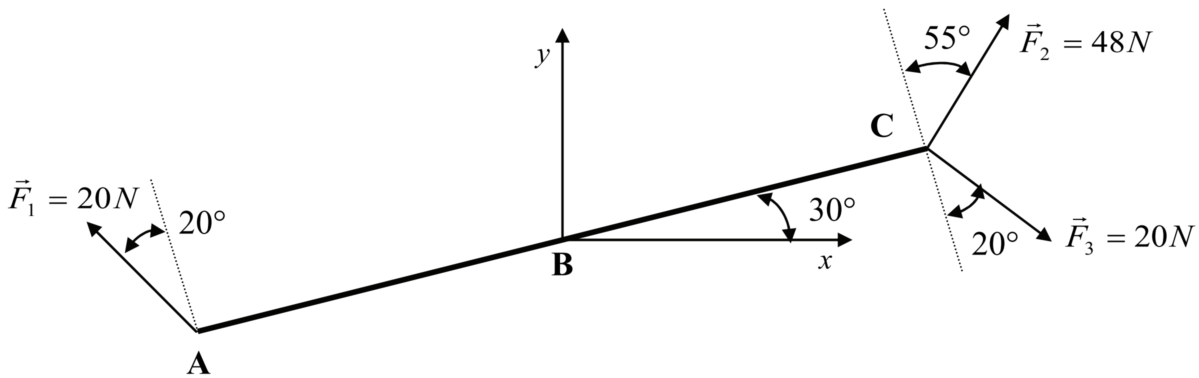
GNG1100E Mid-term Exam Winter 2006

Question 1



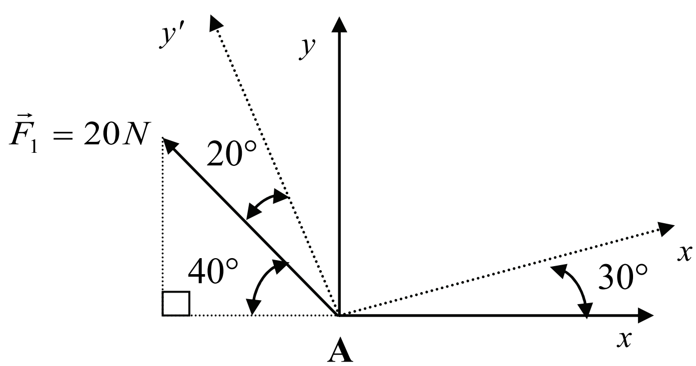
(a)

Moment about *B*:

$$\sum \vec{M}_B = -F_1 \cdot \cos 20^\circ \cdot 40 + F_2 \cdot \cos 55^\circ \cdot 30 - F_3 \cdot \cos 20^\circ \cdot 30$$
$$= -20 \cdot \cos 20^\circ \cdot 40 + 48 \cdot \cos 55^\circ \cdot 30 - 20 \cdot \cos 20^\circ \cdot 30$$
$$= -489.6 \hat{k} \ \ Nmm$$

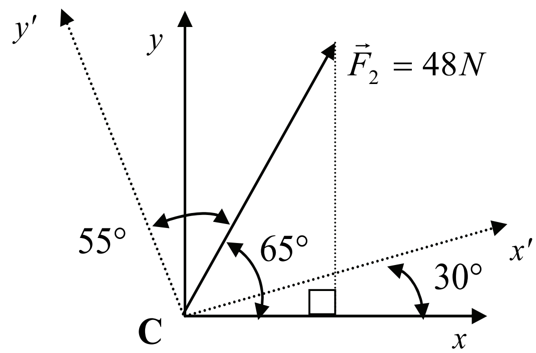
Resultant force, $\vec{R}$:

i) for F_1,



$$\left|F_1\right|_x = F_1 \cdot \cos 40^\circ = 20 \cdot \cos 40^\circ = 15.32$$
$$\left|F_1\right|_y = F_1 \cdot \sin 40^\circ = 20 \cdot \sin 40^\circ = 12.86$$
$$\therefore \vec{F}_1 = -15.32\hat{i} + 12.86\hat{j} \ N$$

ii) for F_2,

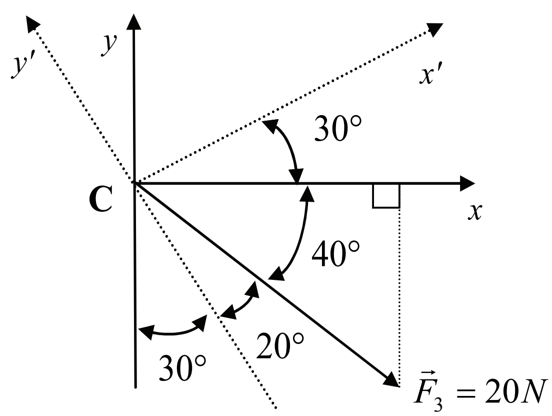


$$\left|F_2\right|_x = F_2 \cdot \cos 65^\circ = 48 \cdot \cos 65^\circ = 20.29$$

$$\left|F_2\right|_y = F_2 \cdot \sin 65^\circ = 48 \cdot \sin 65^\circ = 43.50$$

$$\therefore \vec{F}_2 = 20.29\hat{i} + 43.50\hat{j}\ N$$

iii) for F_3,



$$\left|F_3\right|_x = F_3 \cdot \cos 40^\circ = 20 \cdot \cos 40^\circ = 15.32$$

$$\left|F_3\right|_y = F_3 \cdot \sin 40^\circ = 20 \cdot \sin 40^\circ = 12.86$$

$$\therefore \vec{F}_3 = 15.32\hat{i} - 12.86\hat{j}\ N$$

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

$$= \left(-15.32 + 20.29 + 15.32\right)\hat{i} + \left(12.86 + 43.50 - 12.86\right)\hat{j}$$

$$= 20.29\hat{i} + 43.50\hat{j}\ N$$

$$\left|\vec{R}\right| = \sqrt{20.29^2 + 43.50^2} = 48\,N$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{43.50}{20.29}\right) = 65^\circ$$

(b)

$$\vec{M}_B = \vec{R} \times \vec{d}$$
$$= \left(20.29\hat{i} + 43.50\hat{j}\right) \times \left(d_x\hat{i} + d_y\hat{j}\right)$$
$$= \left(20.29\hat{i} + 43.50\hat{j}\right) \times \left(d \cdot \cos 30^\circ \hat{i} + d \cdot \sin 30^\circ \hat{j}\right)$$
$$= d \cdot \left(20.29 \cdot \sin 30^\circ - 43.50 \cdot \cos 30^\circ\right)\hat{k}$$
$$= -27.53 d\,\hat{k}$$

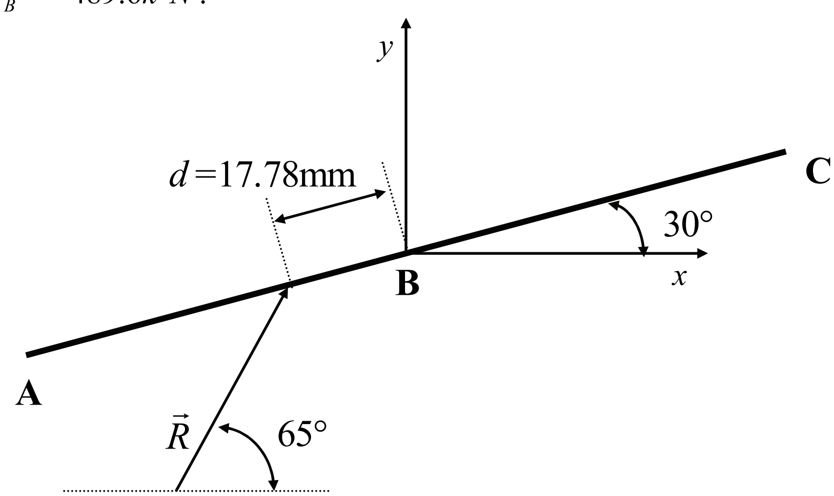
Since $\vec{M}_B = -489.6\hat{k}$ from (a)

$$-27.53d = -489.6$$

or

$$d = 17.78\,mm$$

Single force $\vec{R} = 20.29\hat{i} + 43.50\hat{j}\ N$ at $d = 17.78\,mm$ to the left of point B is required to produce $\vec{M}_B = -489.6\hat{k}\ N$.



(c)

The equivalent force-couple system at C is

$$\vec{M}_C = \vec{M}_b + \vec{r} \times \vec{R}$$
$$= \left(-489.6\hat{k}\right) + \left(26\hat{i} + 15\hat{j}\right) \times \left(20.29\hat{i} + 43.50\hat{j}\right)$$
$$= -489.6\hat{k} + \left(26 \times 43.50\right)\hat{k} - \left(15 \times 20.29\right)\hat{k}$$
$$= 337.05\hat{k}\ Nmm$$

where, $\vec{r} = \overrightarrow{BC} = 30 \cdot \cos 30^\circ \hat{i} + 30 \cdot \sin 30^\circ \hat{j} = 26\hat{i} + 15\hat{j}$

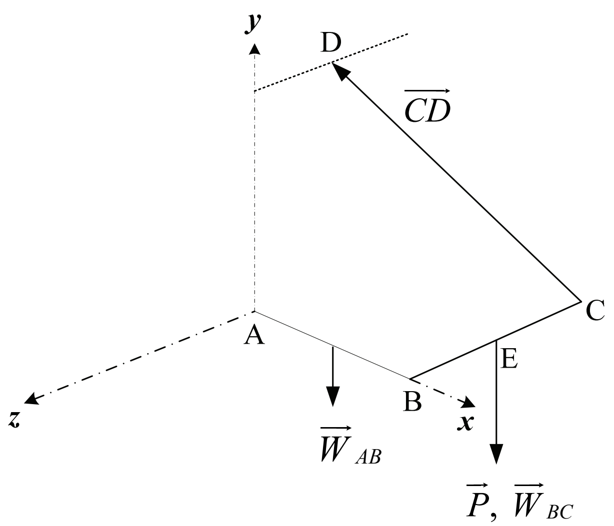
Therefore,

$$\vec{F} = -\vec{R} = -\left(20.29\hat{i} + 43.50\hat{j}\right)N$$
$$\vec{M}_{CC} = -\vec{M}_C = -337.05\hat{k}\ Nmm$$

is required to keep the lever at rest.

<u>Question 2</u>

Free-body Diagram:



AB and BC are uniform rods with a mass of 10 kg. Therefore,

Weight of bars AB and BC, $\overrightarrow{W}_{AB} = \overrightarrow{W}_{BC} = mg = 10*9.8 = 98N$

(a) We know, $C = (1.2,\ 0,\ -1.2)$ and $D = (0,\ 1.2,\ -0.6)$

Therefore,

$$\overrightarrow{CD} = -1.2\ \hat{i} + 1.2\ \hat{j} + 0.6\ \hat{k}$$

$$\left|\overrightarrow{CD}\right| = \sqrt{(-1.2)^2 + (1.2)^2 + (0.6)^2} = 1.8$$

Force in cable CD,

$$\overrightarrow{F}_{CD} = \left|\overrightarrow{F}\right| \bullet \frac{\overrightarrow{CD}}{\left|\overrightarrow{CD}\right|} = \frac{20}{1.8}\left(-1.2\ \hat{i} + 1.2\ \hat{j} + 0.6\ \hat{k}\right) = \left(-13.33\ \hat{i} + 13.33\ \hat{j} + 6.67\ \hat{k}\right) KN$$

Other forces acting on the system,

$$\overrightarrow{P} = -P\hat{j} = -5\hat{j}\ KN$$

$$\overrightarrow{W}_{AB} = -W\hat{j} = -0.098\hat{j}\ KN$$

$$\overrightarrow{W}_{BC} = -W\hat{j} = -0.098\hat{j}\ KN$$

(b) Resultant of the system of forces acting on the bar is given by:

$$\vec{R} = \vec{F}_{CD} + \vec{P} + \vec{W}_{AB} + \vec{W}_{BC} = \left(-13.33\ \hat{i} - 8.496\ \hat{j} + 6.67\ \hat{k}\right) KN$$

$$\left|\vec{R}\right| = \sqrt{(-13.33)^2 + (-8.496)^2 + (6.67)^2} = 17.16\ KN$$

(c) Taking Moment about A

$$M_{A-\vec{F}_{CD}} = r_{AC} \bullet \vec{F}_{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.2 & 0 & -1.2 \\ -13.33 & 13.33 & 6.67 \end{vmatrix} = \left(-15.996\ \hat{i} + 7.992\ \hat{j} + 15.996\ \hat{k}\right) KNm$$

$$M_{A-\vec{P}} = r_{AE} \bullet \vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1.2 & 0 & -0.6 \\ 0 & -5 & 0 \end{vmatrix} = \left(-3\ \hat{i} - 6\ \hat{k}\right) KNm$$

$$M_{A-\vec{W}_{AB}} = r_1 \bullet \vec{W}_{AB} = \left(-0.0588\ \hat{k}\right) KNm$$

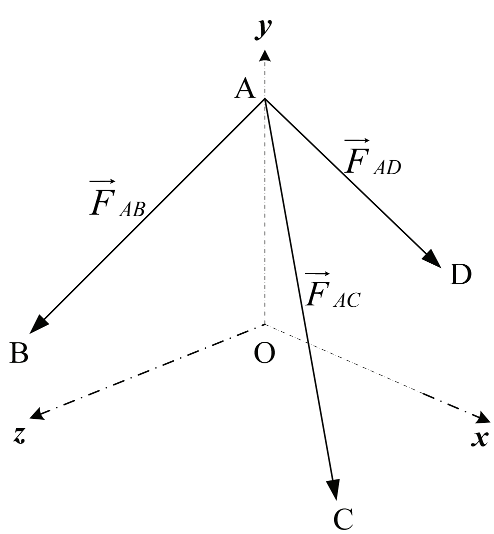
$$M_{A-\vec{W}_{BC}} = r_{AE} \bullet \vec{W}_{BC} = \left(-0.0588\ \hat{i} - 0.1176\ \hat{k}\right) KNm$$

(d) Moment Resultant,

$$\sum M_A = M_{A-\vec{F}_{CD}} + M_{A-\vec{P}} + M_{A-\vec{W}_{AB}} + M_{A-\vec{W}_{BC}} = \left(12.9372\ \hat{i} + 7.992\ \hat{j} + 9.8196\ \hat{k}\right) KNm$$

Question 3

Free-body Diagram:



(a)

$$A = (0, 480, 0)$$
$$B = (-320, 0, 360)$$
$$C = (450, 0, 360)$$
$$D = (250, 0, -360)$$

$$\overrightarrow{AB} = -320\ \hat{i} - 480\ \hat{j} + 360\ \hat{k} \qquad \left|\overrightarrow{AB}\right| = 680$$
$$\overrightarrow{AC} = 450\ \hat{i} - 480\ \hat{j} + 360\ \hat{k} \qquad \left|\overrightarrow{AC}\right| = 750$$
$$\overrightarrow{AD} = 250\ \hat{i} - 480\ \hat{j} - 360\ \hat{k} \qquad \left|\overrightarrow{AD}\right| = 650$$

$$\vec{F}_{AB} = \left|\vec{F}\right| \bullet \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{100}{680}\left(-320\ \hat{i} - 480\ \hat{j} + 360\ \hat{k}\right) = \left(-47.06\ \hat{i} - 70.59\ \hat{j} + 52.94\ \hat{k}\right) N$$
$$\vec{F}_{AC} = \left|\vec{F}\right| \bullet \frac{\overrightarrow{AC}}{\left|\overrightarrow{AC}\right|} = \frac{100}{750}\left(450\ \hat{i} - 480\ \hat{j} + 360\ \hat{k}\right) = \left(60\ \hat{i} - 64\ \hat{j} + 48\ \hat{k}\right) N$$
$$\vec{F}_{AD} = \left|\vec{F}\right| \bullet \frac{\overrightarrow{AD}}{\left|\overrightarrow{AD}\right|} = \frac{100}{650}\left(250\ \hat{i} - 480\ \hat{j} - 360\ \hat{k}\right) = \left(38.46\ \hat{i} - 73.85\ \hat{j} - 55.38\ \hat{k}\right) N$$

(b) $\vec{R}_A = \vec{F}_{AB} + \vec{F}_{AC} + \vec{F}_{AD} = \left(51.4\ \hat{i} - 208.44\ \hat{j} + 45.58\ \hat{k}\right) N$

(c) $M_O = r_{OA} \bullet \vec{R}_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 480 & 0 \\ 51.4 & 208.44 & 45.56 \end{vmatrix} = \left(21.87\ \hat{i} - 24.67\ \hat{k}\right) N.mm$

$$\vec{R} = \left(51.4\ \hat{i} - 208.44\ \hat{j} + 45.58\ \hat{k}\right) N \quad [= \vec{R}_A]$$