CARLETON UNIVERSITY
Department of Mechanical and Aerospace Engineering
Fluids I (MAAE 2300) Mid - Term Examination Solutions
Wednesday, October 31, 2007
Professor J. Gaydos

- This examination is *Open Aid Sheet* meaning that you may use exactly *One* hand-written 8.5 inch by 11 inch sheet of paper.
- Any calculator.

Question 1 (20 marks)

Briefly define or explain the following terms or concepts:

(a) Give one example of a device that uses fluid mechanical principles in its design and operation.
Any suitable answer was accepted provided it seemed reasonable ... a gift!

(b) The SI units for the dynamic viscosity μ .
Ns/m^2 or kg/ms.

(c) The gauge pressure on the dark side of the lunar surface.
Gauge pressure is only measured with respect to Earth's sea level pressure; that is, with respect to the values 101,325 N/m^2 or 14.7 $psi_{(abs)}$. On the dark side of the moon where there is no photo (light) pressure the absolute pressure would be close to zero and the gauge pressure would be negative and equal to approximately - 14.7 psi.

(d) Archimedes' principle.
States that the buoyant force has a magnitude equal to the weight of the fluid displaced by the body and that its force vector is directed vertically upward (opposite to gravity).

(e) Explain the physical basis for the no slip condition AND give one example where this condition does NOT apply.
The no slip condition states that the velocity of a fluid adjacent to a solid surface is equal to the velocity of the solid surface; that is, $\vec{\mathbf{V}}_{\text{fluid}} = \vec{\mathbf{V}}_{\text{solid}}$. On a physical basis, we can understand the reason this occurs by considering the microscopic behaviour of any typical fluid during flow when the fluid is in close proximity to the solid's surface. As is known from X-ray diffraction or electron microscopy studies, solid surfaces are rough on small length scales. Fluid molecules moving in close proximity to a solid surface will not move past the surface without occasionally colliding with the rough prominences of the surface. Collisions produce scattering (or bounce) from the surface in random directions and with arbitrary speeds. This random scattering of molecules at the surface is equivalent (after one sums-up over a sufficiently large time period and number of molecules) to the macroscopic velocity of the fluid in proximity to the wall ... which is zero if the randomness is truly random. The no slip condition may not be satisfied in rarified gas flows by streams of low density gas around a satellite. In these cases the gas moves in a kinetic flow which "slides" past the satellite in much the same manner that a baseball "slides" past a catcher who misses the catch. Spreading of fluids represents another situation where the no slip condition may not be satisfied. If a solid surface is slowly pushed into a viscous fluid

like honey, then the honey appears to roll onto the surface in much the same manner that a tire rolls on a road!

(f) Consider an iceberg floating in sea water. Find the fraction of the iceberg's volume above the sea surface if the density of the iceberg is 920 kg/m^3 and the density of the sea water is 1025 kg/m^3?

If the total volume of the iceberg is V, and the volume showing above the surface is ΔV, then the bouyance force acting vertically upward on the iceberg is given by the equation $\rho_{sw}\, g\, (V - \Delta V)$, where ρ_{sw} is the density of the sea water. For equilibrium, this must equal the weight of the iceberg, given by $\rho_{ice} g V$, where ρ_{ice} is the density of ice. Thus,

$$\rho_{sw} g\left(V - \Delta V\right) = \rho_{ice} g V$$

or

$$\frac{\Delta V}{V} = 1 - \frac{\rho_{ice}}{\rho_{sw}} \approx 0.102$$

so that only about 10% of the bulk of an iceberg is visible above the sea surface.

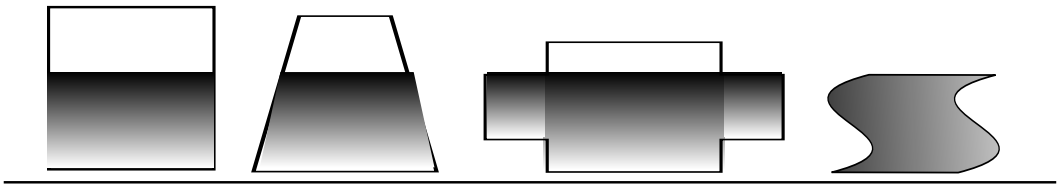
(g) Consider your typical glass that is partially filled with water. Capillary rise of the water on the glass surface occurs because of surface tension effects. If the water rise in your glass is 5 mm, then estimate the gauge pressure of the water just below the air-water surface at the glass surface.

In the middle of the glass (far away from the glass' sides) the pressure inside the liquid and just below the surface will be equal to the air's pressure just above the surface; that is, zero gauge. If you are convinced that hydrostatics applies within any liquid in close proximity to a solid (you should be convinced!), then the hydrostatic relation, given by

$$\Delta P = \rho_{liquid} g \Delta h \approx 10^3\, kg/m^3 \times 9.81 m/s^2 \times (-0.005 m) \approx -49\, Pa$$

indicates that the pressure will be reduced below zero.

(h) Four vessels, with exactly the same bottom surface area, are filled to the same level by water. According to hydrostatic theory, the water pressure at the bottom of each vessel is equal in each vessel since the water depth is the same in each vessel. Also, the hydrostatic pressure force on the bottom is the same since the bottom area is the same. If you placed each vessel on a scale would you also expect the same weight value? Explain?



The hydrostatic pressure force on the bottom surface of each vessel has the same value

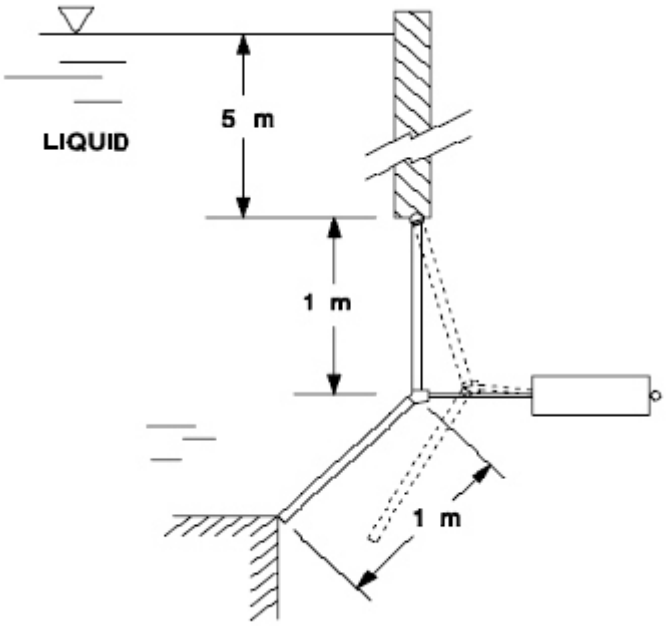
because the depth of water is the same in all vessels. This pressure force is a surface force. However, weight is a body force. Body forces can be measured by a scale as the weight product of mass (density times volume) times gravity. Thus, the third vessel from the left-hand side appears to have the most volume and thus the most weight.

Question 2 (20 marks)

The actuating mechanism of a dump door on a large storage tank is shown below. The door consists of two steel plates, dimensions one metre by two metres, welded together at an angle of 45° along the two metre side. This door is hinged at the top and is operated by a hydraulic jack pushing on the joint between the plates. The liquid in the tank has a specific weight $\gamma = 8$ kN/m^3.

(a) What force must the jack exert on the rod to hold the door shut?

(b) Evaluate the net moment on the top hinge?



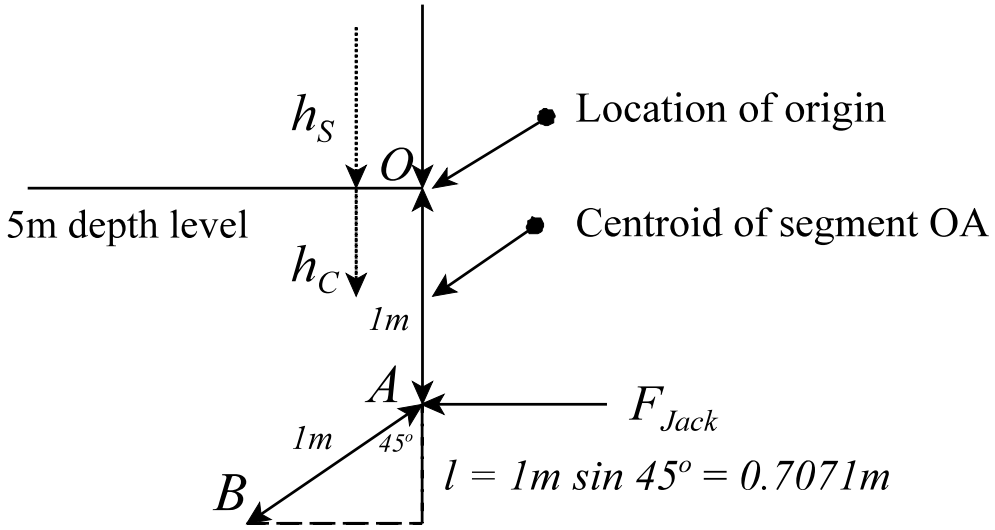
(a) There are a couple of approaches for solving this problem. We start by selecting the origin to occur at the same elevation as the hinge. We will start by evaluating horizontal forces that arise because of the free surface and from hydrostatics. Consider the door segment *OAB* as shown below. On the door segment *OA* we have two forces; F_S and F_R given by

$$F_S = \rho g h_S(@O) A_{OA} = \gamma h_S A_{OA}$$ acting at the centroid of segment *OA*.

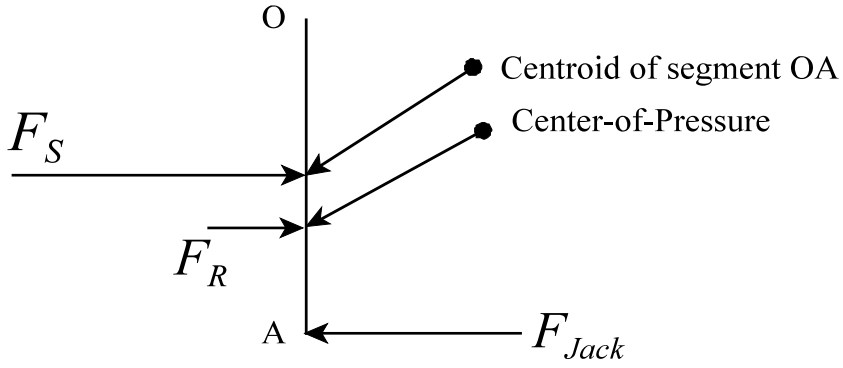
$$F_S = 8000 \times 5m \times (1m \times 2m) = 80{,}000N$$

$$F_R = \rho g h_C A_{OA} = \gamma h_C A_{OA}$$ acting at the center-of-pressure of segment *OA* which is 2/3 of the distance down the segment from the origin point *O* towards point *A*. Thus,

$$F_R = 8000 \times 0.5m \times 2m^2 = 8000N$$



On door segment *OA,* these two forces act at the positions shown below



On the door segment *AB* that is tilted with respect to the vertical by 45° we also have two similar forces given by

$$F_S = \rho g h_S(@A) A_{Vert_AB} = \gamma h_S A_{Vert_AB}$$

where the vertical area A_{Vert_AB} is the projection of the tilted area *AB* onto the vertical plane that

extends from point *O* through point *A* and directly downward (see below). Thus,

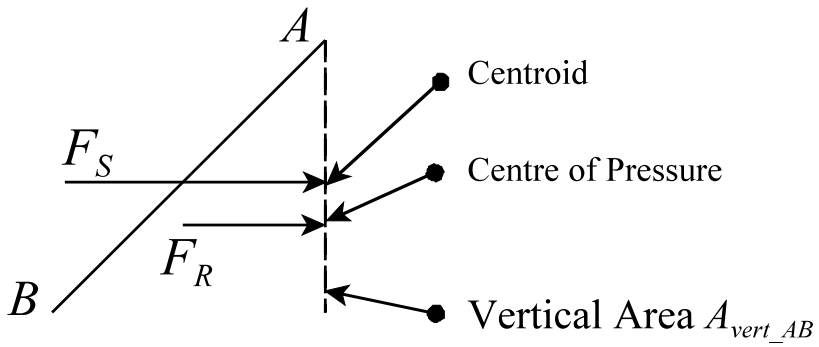
$$F_S = 8000 \times 6m \times \left(2m \times \frac{1}{\sqrt{2}} m \right) \approx 67{,}882\,N$$

acting at the centroid of the door segment *AB*. Finally, the hydrostatic force on segment *AB* is

$$F_R = \rho g h_C A_{Vert_AB} = \gamma h_C A_{Vert_AB}$$

acting at the center-of-pressure of segment *AB* which is 2/3 of the distance down the vertical segment from the point *A*. The value is

$$F_R = 8000 \times \left(\frac{1}{2} \times \frac{1}{\sqrt{2}} m \right) \times \left(2m \times \frac{1}{\sqrt{2}} m \right) = 4000N$$



Summing up all four contributions gives a net horizontal force as

$$\sum F_{Horizontal} = \left(F_S + F_R\right)_{OA} + \left(F_S + F_R\right)_{AB} \approx 159{,}900N$$

which is not sufficient to keep the dump door closed since the moment at the hinge point *O* is not zero (see below).

Alternatively, if we treat the door as one entire segment, then its vertical (projected) length will be $\left(1 + \frac{1}{\sqrt{2}}\right)m$ and the two forces become

$$F_S = \rho g h_S (@\,O) A_{Vert_OAB} = \gamma h_S A_{Vert_OAB}$$

$$F_S = 8000 \times 5m \times \left(1 + \frac{1}{\sqrt{2}} \right) m \times 2m \approx 136{,}569\,N$$

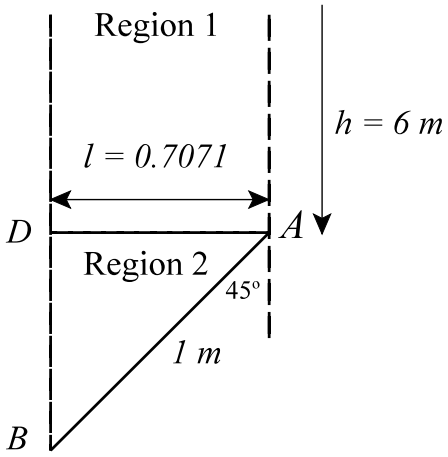
which acts at the centroid of the vertical segment for *OAB*. This centroid occurs below the level of the origin (hinge) and at a depth of $\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)m$. Finally, the hydrostatic force is

$$F_R = \rho g h_C A_{Vert_OAB} = \gamma h_C A_{Vert_OAB}$$

$$F_R = 8000 \times \left[\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)m \right] \times \left[\left(1 + \frac{1}{\sqrt{2}}\right)m \times 2m \right] \approx 23{,}314N$$

acting at the center of pressure of the entire segment *OAB*. This occurs below the level of the origin (hinge) and at a depth of $\frac{2}{3}\left(1 + \frac{1}{\sqrt{2}}\right)m$. Summing these two forces gives exactly the same result as before.

The final forces that acts on the dump door are due to the weight of the fluid above the inclined section of the door; that is, above the section *AB*. This weight force can be separated into two contributing regions (see below). The weight force acting at the centroid of region 1 is given by



$$F_{V1} = \rho g V_1 = 8000 \times \left(6m \times 2m \times \frac{1}{\sqrt{2}}m \right) \approx 67{,}882N$$

and it acts at a horizontal distance of $\left(\frac{1}{2} \times \frac{1}{\sqrt{2}}\right)m$ away from the hinge. The region 2 force is

$$F_{V2} = \rho g V_2 = 8000 \times \left(2m \times \frac{1}{2} \times \frac{1}{\sqrt{2}}m \times \frac{1}{\sqrt{2}}m \right) \approx 4000N$$

where the volume of the triangular region 2 is the area of the triangle (one-half base times height) times 2 m. This second weight force operates at a horizontal distance of $\left(\frac{2}{3} \times \frac{1}{\sqrt{2}}\right)m$ away from the hinge. All six forces contribute to the moment effect on the hinge which is

$$\sum M_o(fluid)_{CCW}$$

$$= F_{V1} \times \left(\frac{1}{2} \frac{1}{\sqrt{2}} \right) + F_{V2} \times \left(\frac{2}{3} \frac{1}{\sqrt{2}} \right) + F_{S_OA} \times \frac{1}{2} + F_{R_OA} \times \frac{2}{3}$$
$$+ F_{S_AB} \times \left(1 + \frac{1}{2} \frac{1}{\sqrt{2}} \right) + F_{R_AB} \times \left(1 + \frac{2}{3} \frac{1}{\sqrt{2}} \right)$$
$$\approx 67882 \times 0.3536 + 4000 \times 0.4714 + 80000 \times 0.5 + 8000 \times 0.6667$$
$$+ 67882 \times 1.3536 + 4000 \times 1.4714$$
$$\approx 169{,}000 Nm$$

To keep the dump door closed, the jack force, F_{Jack}, acting at a distance of *1 m* from the hinge point at *O*, must provide a force of 169,000 *N*. This force provides a clockwise moment that just balances the moment from all liquid effects to keep the dump door in mechanical equilibrium.

Alternatively, if the entire dump door approach is used, then

$F_S \approx 136{,}569\,N$ acts at a distance $\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right) m$ from the hinge at point *O,*

$F_R \approx 23{,}314\,N$ acts at a distance $\frac{2}{3}\left(1 + \frac{1}{\sqrt{2}}\right) m$ from the hinge at point *O*.

The moments which arise from these two forces in combination with the moments from the two weight forces yields the same 169 *kNm* moment on the hinge from all fluid effects. Once again, for equilibrium the jack force must equal 169 *N* to counteract these liquid moment effects.

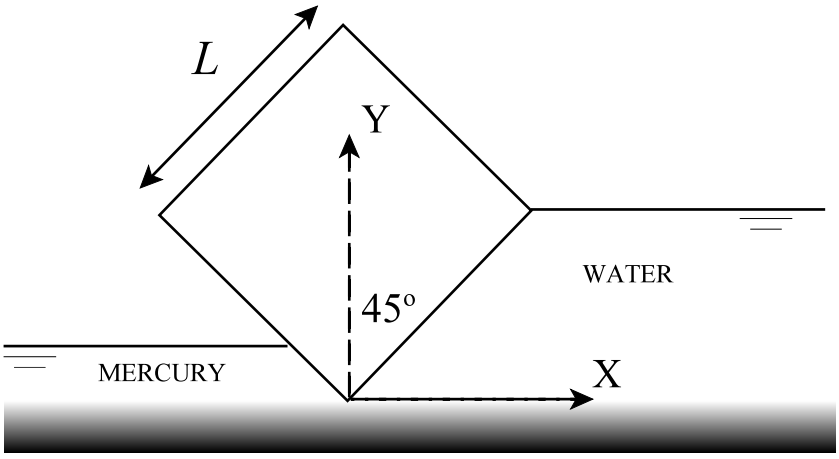
(b) If the dump door is to remain closed and in mechanical equilibrium the net moment force at the hinge must be zero. The various terms that contribute to this moment have been evaluated.

Question 3 (20 marks)

A cube of glass, with cross-section *L by L,* is shown below tilted onto its vertex. You may assume that the cube extends into this page and that it is in a container that holds the water and mercury at the levels shown. The water level is at one of the cube's vertices as shown. Mercury has a density that is 13.6 times larger than water and its level is lower than the water's level.

(a) If you wish to set the mercury depth level so that the cube is NOT under any net horizontal force, then estimate this mercury depth in terms of the cube's side length *L*.
(b) What depth of mercury is required to ensure that the cube does not rotate?
(c) Would you expect these two depths, as calculated in parts (a) and (b), to be the same or different?

(d) Is there a tilt angle (45° or any other angle) that will permit both horizontal force and net moment to balance for one depth of mercury?



(a) It is important to realize that the atmospheric pressure will induce exactly the same horizontal force on both sides of the cube. As a result, it is not necessary to explicitly evaluate this force, but it is necessary to state why you are not evaluating this force. The mercury depth, denoted by l, required to just balance the horizontal forces arises solely from hydrostatic effects. In math, this balancing is expressed as

$$F_{R_Hg} = \left(\rho g h_C A_{Vert}\right)_{Hg} = \left(\rho g h_C A_{Vert}\right)_{H_2O} = F_{R_H_2O}$$

or

$$13.6\rho_{H_2O} g \cdot \frac{l}{2} \cdot lL = \rho_{H_2O} g \cdot \frac{1}{2}\frac{L}{\sqrt{2}} \cdot \frac{L}{\sqrt{2}} L$$

which can be simplified to

$$l^2 = \frac{L^2}{2 \times 13.6} \qquad \text{or} \qquad l \approx 0.192L$$

for the required mercury depth expressed in terms of the cube's side length *L*.

(b) Two kinds of forces are involved in this part of the question; a hydrostatic horizontal force and a buoyancy force that pushes the cube directly upward (causing it to float). The net moment from all of these kinds of forces must be zero at the bottom vertex if the cube does not rotate. We will denote the mercury depth as h_M and will assume that it does not equal *0.192 L*. Next, we write the moment expressions for both the mercury side and the water side as

For the mercury side the moment is

$$\sum \left(M_{Vertex} \right)_{Hg} = F_{R_Hg} \times \frac{2}{3} h_M + SG\rho_{H_2O} g \cdot \frac{1}{2} h_M^2 L \times \frac{2}{3} h_M$$

where the first moment term arises from the hydrostatic force and the second term from the force due to buoyancy. Substituting in the hydrostatic expression

$$F_{R_Hg} = SG\rho_{H_2O} g \cdot \frac{h_M}{2} \cdot h_M L \times \frac{2}{3} h_M$$

into the moment expression above permits one to simplify to

$$\sum \left(M_{Vertex} \right)_{Hg} = SG\rho_{H_2O} g L h_M^2 \times \frac{2}{3} h_M$$

For the water side the moment is

$$\sum \left(M_{Vertex} \right)_{H_2O} = F_{R_H_2O} \times \frac{2}{3} h_W + \rho_{H_2O} g \cdot \frac{1}{2} h_W^2 L \times \frac{2}{3} h_W$$

where h_W is the water depth; equal to $\frac{L}{\sqrt{2}}$. Water's hydrostatic force expression is

$$F_{R_H_2O} = \rho_{H_2O} g \cdot \frac{h_W}{2} \cdot h_W L \times \frac{2}{3} h_W$$

Once again, if we substitute this expression into the moment expression for water and simplify we obtain the result

$$\sum \left(M_{Vertex} \right)_{H_2O} = \rho_{H_2O} g L^3 \cdot \frac{1}{3} \frac{L}{\sqrt{2}}$$

Equating this moment expression for water to the analogous moment for mercury gives, after cancellations, the equality

$$h_M^3 = \frac{L^3}{2\sqrt{2}(SG)} \qquad \text{or} \qquad h_M \approx 0.296 L$$

(c) We discover that the two depths are not the same and that the depth of mercury required to keep the cube from rotating at its lowest vertex is larger than the depth required to just maintain a horizontal force balance. Specifically, $l < h_M$. This result could have been anticipated since the line-of-action of the water force $F_{R_H_2O}$ is further away from the horizontal line of support that passes through the cube's vertex. As a consequence, if the two F_R forces are balanced (as required in part (a)) it should follow that the CW moment from the mercury force will be smaller than the CCW moment from the water force. Thus, with just horizontal forces balanced the cube would have rotated towards the mercury side and away from the water side. To prevent this, the depth of mercury is increased so that the moments balance. What happens to the horizontal force balance?

(d) We use the following variables in this part of the solution:

h_M, h_W Mercury and water vertical depths measured from the horizontal line that contains the cube's lowest vertex.

ρ_M, ρ Mercury and water densities

ϑ Theta angle measured through the mercury liquid from the horizontal line that contains the cube's lowest vertex to the cube's side (see below).

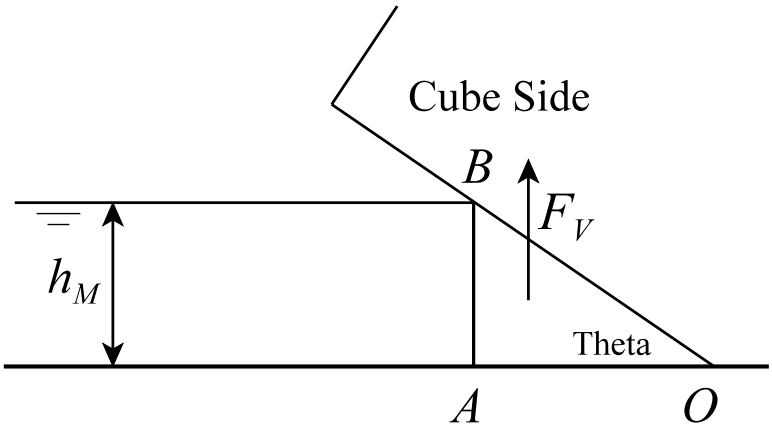
A net horizontal force balance requires that

$$F_{R_Hg} = \rho_M g \cdot \frac{h_M}{2} \cdot h_M L = \rho g \cdot \frac{h_W}{2} \cdot h_W L = F_{R_H_2O}$$

and if these are to be equal, then with $h_W = \frac{L}{\sqrt{2}}$ (unchanged) we must require that

$$\rho_M h_M^2 = \rho h_W^2$$

Now, let's consider the geometry on the mercury side of the cube (see sketch below).



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g with the hydrostatic force there will be a weight (or buoyancy) force contributing to the moment about the cube's vertex at point *O*. The moment expression on the mercury side is

$$\sum \left(M_O\right)_{Hg} = \rho_M g \cdot \frac{h_M}{2} \cdot h_M L \times \frac{2}{3} h_M + \rho_M g \cdot \frac{1}{2} h_M d_{OA} L \times \frac{2}{3} d_{OA}$$

where d_{OA} is the distance from the origin point *O* to the point *A* as shown in the sketch. Simple geometry of the triangle *OAB* gives

$$\tan \vartheta = \frac{h_M}{d_{OA}} \qquad \text{with} \qquad d_{OA} = h_M \tan \vartheta$$

so that the volume of the triangular region becomes

$$V_{OAB} = \frac{1}{2} base \times height \times length = \frac{1}{2}\frac{h_M^2}{\tan\vartheta} L$$

On the water side of the cube, there is an analogous length to d_{OA}, denoted by d_W, given by

$$\tan\left(\frac{\pi}{2} - \vartheta\right) = \frac{h_W}{d_W} \qquad \text{or} \qquad d_W = h_W \tan\left(\frac{\pi}{2} - \vartheta\right)$$

The water volume in the triangular section is

$$V_W = \frac{1}{2} d_W h_W L$$

and the corresponding moment on the water side is

$$\sum \left(M_O\right)_{H_2O} = F_{R_H_2O} \times \frac{2}{3} h_W + \rho g \cdot \frac{1}{2} h_W \frac{h_W}{\tan\left(\frac{\pi}{2} - \vartheta\right)} L \times \frac{2}{3} d_W$$

Equating moment expressions and dividing through by common factors gives

$$\frac{2}{3} h_M + \frac{2}{3}\frac{h_M}{\tan^2\vartheta} = \frac{2}{3} h_W + \frac{2}{3}\frac{h_W}{\tan^2\left(\frac{\pi}{2} - \vartheta\right)}$$

or

$$h_M = h_W \left(\frac{1 + \tan^2\vartheta}{1 + \frac{1}{\tan^2\vartheta}}\right) = h_W \tan^2\vartheta$$

Using our force equality expression in the relation above gives

$$\tan^2\vartheta = \frac{h_M}{h_W} = \left(\frac{\rho}{\rho_M}\right)^{1/2}$$

whereupon

$$\tan\vartheta = \left(\frac{\rho}{\rho_M}\right)^{1/4} = \left(\frac{1}{13.6}\right)^{1/4} \approx 0.5207$$

and the required angle is about *27.5°*. At this tilt angle the hydrostatic forces will be balanced and the moments will sum to zero at the cube's lowest vertex so the cube will be in equilibrium ... a meta-stable equilibrium. If the cube is pushed onto its side so that an entire side-length rests on the "floor" rather than just a vertex, then the cube will be in stable equilibrium with either unequal horizontal forces or unequal moments.