CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)	
Mathematics	208/4	All	
Examination	**Date**	**Time**	**Pages**
Final	April 2010	3 Hours	3

Instructors	Course Examiner
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FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT\frac{(1+i)^n - 1}{i}, \quad PV = PMT\frac{1-(1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
- ▷ **Only approved calculators are allowed.**

MARKS

[10] **1.** Given the quadratic function $f(x) = -0.12x^2 + 0.96x + 1.2$

(A) Find x and y intercepts algebraically.

(B) Find the vertex form of f.

(C) Find the vertex and the maximum or minimum.

(D) Find the range of f.

[10] **2.** Solve for x in the following equations:

(A) $e^{2x} = e^{x^2-3}$

(B) $2^{3x} = \frac{1}{8}$

(C) $\log_{10}(2x-5) - \log_{10}(x-3) = 1$

(D) $\log_b(x+2) + \log_b(x) = \log_b 24$

(E) $\log_3(9x) - \log_3(x-8) = 4$

[10] **3.** For $f(x) = -20x + 15$ and $g(x) = 3(1.8)^x$ find the following:

(A) $\sum_{k=0}^{29} f(k) = f(0) + f(1) + f(2) + \cdots + f(29)$.

(B) $\sum_{h=0}^{19} g(h) = g(0) + g(1) + g(2) + \cdots + g(19)$.

[10] **4.** Earl and Larry each begin full-time jobs in January 2009 and plan to retire in January 2057 after working for 48 years. Assume that any money they deposit into IRAs earns 6% interest compounded annually.

(A) Suppose Earl opens a traditional IRA account immediately and deposits \$5000 into his account at the end of each year for twelve years. After that he makes no further deposits and just lets the money earn interest. How much money will Earl have in his account when he retires in January 2057?

(B) Suppose Larry waits 12 years before opening his traditional IRA and then deposits \$5000 into the account at the end of each year untill he retires. How much money will Larry have in his account when he retires in January 2057?

(C) Who had the most money in his account upon retirement?

[10] **5.** A person wants to establish an annuity for retirement purposes. He wants to make quarterly deposits for 20 years so that he can then make quarterly withdrawals of \$5,000 for 10 years. The annuity earns 7.32% interest compounded quarterly.

(A) How much will have to be in the account at the time he retires?

(B) How much should be deposited each quarter for 20 years in order to accumulate the required amount?

(C) What is the total amount of interest earned during the 30 year period?

[10] **6.** A chemical manufacturer wants to lease a fleet of 24 railroad tank cars with a combined carrying capacity of 520,000 gallons. Tank cars with three different carrying capacities are available: 8,000 gallons, 16,000 gallons, and 24,000 gallons.

(A) Write the linear system of equations in terms of x, y and z; x, y and z being the number of tank-cars with carrying capacities of 8,000, 16,000 and 24,000 gallons respectively.

(B) Solve this system of equations.

(C) If the cost of leasing an 8,000 gallon tank car is \$450 per month, a 16,000 gallon tank car is \$650 per month, and a 24,000 gallon tank car is \$1,150 per month, then which of the solutions would minimize the monthly leasing cost?

[10] **7.** A large energy company produces electricity, natural gas, and oil. The production of a dollar's worth of electricity requires an input of \$0.30 from electricity, \$0.10 from natural gas, and \$0.20 from oil. Production of a dollar's worth of natural gas requires an input of \$0.30 from electricity, \$0.10 from natural gas, and \$0.20 from oil. Production of a dollar's worth of oil requires an input of \$0.10 from each sector.

(A) Write the technology matrix M for this economy.

(B) If a final demand of \$25 million for electricity, \$15 million for gas, and \$20 million for oil is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.

(C) Solve the respective inputs satisfying these demands.

[10] **8.** Extremize $P(x, y) = 30x + 10y$ subject to

$$2x + y \leq 20,\ 10x + y \geq 36,\ 2x + 5y \geq 36,\ x \geq 0,\ y \geq 0\,.$$

[10] **9.** A basketball team has 5 distinct positions. Out of 8 players, how many starting teams are possible if

(A) The distinct positions are not taken into consideration?

(B) The distinct positions are taken into consideration?

(C) The distinct positions are not taken into consideration, but either Mike or Ken (but not both) must start?

[10] **10.** From a survey involving 1,000 people in a certain city, it was found that 500 people had tried a certain brand of diet cola, 600 had tried a certain brand of regular cola, and 200 had tried both types of cola. If a resident of the city is selected at random, what is the probability that

(A) The resident has not tried either cola?

(B) The resident has tried the diet cola or has not tried the regular cola?

(C) The resident has tried one of the colas but not both?

Math 208 Final April 2010 Answers

1. (A) x intercepts: $x_{1,2} = 4 \pm \sqrt{26} = \begin{cases} -1.099 \\ 9.099 \end{cases}$

y-intercept: $y = 1.2$

(B) $f(x) = -0.12(x-4)^2 + 3.12$

(C) vertex: $(4, 3.12)$, max. value is 3.12

(D) Range $= (-\infty, 3.12]$

2. (A) $x = -1$ or $x = 3$ (B) $x = -1$

(C) $x = 3.125$ (D) $x = 4$ (E) $x = 9$

3. (A) $S_{30} = -8,250$ (B) $S_{20} = 478,055.1081$

4. (A) $(1.06)^{36} \cdot 5,000 \dfrac{(1.06)^{12} - 1}{0.06} = \$687,218.31$

(B) $5,000 \dfrac{(1.06)^{36} - 1}{0.06} = \$595,604.33$

5. (A) $\$140,945.57$ (B) $D = \$789.65$

(C) $40 \cdot 5000 - 80 \cdot 789.65 = \$136,828$

6. (A) $\begin{cases} x + y + z = 24 \\ 8,000x + 16,000y + 24,000z = 520,000 \end{cases}$

x, y, z integers ≥ 0

(B) $\begin{cases} x = z - 17 \\ y = 41 - 2z \\ z = z \end{cases}$

Possible answers:

z	17	18	19	20
x	0	1	2	3
y	7	5	3	1
	24,100	24,400	24,700	25,000

(C) Cost min

7. (A) $M = \begin{array}{c} \\ E \\ N.G \\ O \end{array} \begin{array}{c} \begin{array}{ccc} E & NG & O \end{array} \\ \begin{bmatrix} 0.3 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.1 \end{bmatrix} \end{array}$

(B) $X = MX + D$ or $(I - M)X = D$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

x = total production of E
y = " " of N.G & $D = \begin{bmatrix} 25 \\ 15 \\ 20 \end{bmatrix}$ in millions
z = " " of O

(C) $[I - M \mid D] = \left[\begin{array}{ccc|c} 0.7 & -0.3 & -0.1 & 25 \\ -0.1 & 0.9 & -0.1 & 15 \\ -0.2 & -0.2 & 0.9 & 20 \end{array}\right]$ by row operations $\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 53 \\ 0 & 1 & 0 & 27 \\ 0 & 0 & 1 & 40 \end{array}\right]$

So we need $x = \$53$ million of E, \$27 million of N.G & \$40 m. of O

8. Feasible region is bounded (a triangle) with corner points: (2,16), (8,4) & (3,6). P has the max. value of 280 at (8,4) & the min. value of 150 at (3,6)

9. (A) $C_{8,5} = \binom{8}{5} = 56$ (B) $P_{8,5} = \frac{8!}{3!} = 6,720$

(C) $\binom{7}{4} + \binom{7}{4} - \binom{6}{3} = 2 \cdot \frac{7!}{4!3!} - \frac{6!}{3!3!} = \underline{\underline{50}}$

Mike shirts, Ken shirts, both shirt

Or: $n(S) = \binom{8}{5} = 56$ E - event (either M or K shirts)

E' = none of them shirts $n(E') = \binom{6}{5} = 6$

$n(E) = n(S) - n(E') = 56 - 6 = 50$

10. (A) $1,000 - n(D \cup L) = 100 \Rightarrow$ Prob. $= \frac{100}{1,000} = 0.1$

(B) $\frac{500+100}{1,000} = \frac{600}{1,000} = 0.6$ (C) $\frac{700}{1,000} = 0.7$

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