École de gestion TELFER School of Management — LIÉE au LINKED with LEADERSHIP

ADM 2304
APPLIED STATISTICAL METHODS IN BUSINESS

Midterm Exam

NAME (please print): ___________________________________

Student Number: __________________

SECTION REGISTERED (Circle one): A (day) B (evening)

Instructions

Duration of Exam: Two (2) hours.
Length of Exam: 5 pages, plus a 3-page Minitab supplement (please return).
Please show all your work and explain your answers briefly where required.
You are encouraged to use the Minitab output as much as possible.
You are permitted to have a non-programmable calculator and a sheet (8.5 x 11 inch) of notes.
Statistical tables (normal, t and chi-square) are provided separately (please keep).

Marks: ________ + ________ + _______ + ________ = ___________

9	8	7	6	**28**

Statement of Academic Integrity

The School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: ____________________________________

1. [9 marks + 1 bonus]

The numbers of complaints received per hour were recorded at a service counter of a local department store. Appendix A displays and summarizes the data.

a. HR guidelines suggest an increase in the number of staff when the average number of complaints exceed 3 per hour. Test whether this threshold has been exceeded Use the 5% level of significance.

[4]

Ho: mu = 3 or mu ≤ 3 ; Ha: mu > 3
t= (3.6- 3.0)/(1.943/√(35) = 0.6 / .328 = 1.83
t(critical) = 1.69 (df = 34) or 1.645, based on normal approx.
Reject Ho if t > 1.6+.
Decision is to reject Ho and conclude the average number of complaints exceeds 3.

b. What is the p-value of the test performed in (a)?

[1] *p-value = P(t > 1.83) = P(z > 1.83) = .0336*
If only the t-table is used, then .025 < p-value < .05, based on 34 d.f.

c. Estimate the average number of complaints using the appropriate 95% 1-sided confidence interval.

[2]

At least 3.6 – 1.69 (.328) or at least 3.05 or
At least 3.6 - 1.645(.328) or at least 3.06

d. State precisely what assumption(s) you *had* to make about the distribution of the number of complaints to justify your calculations above. Explain whether this is a reasonable assumption.

[2]

Since n > 30, we only *have* to assume that complaints are not extremely skewed. This is reasonable since the boxplot is only slightly skewed.

The assumption that the complaints are normally distributed is necessary only if the sample size is small. Such an assumption might not be reasonable given the skewness in the boxplot and the Poisson nature of the distribution.

e. For extra credit, identify the probability distribution for the number of complaints.

[1]

A common distribution for modeling the number of independent events that occur in a given unit of time is the Poisson. This is clearly not a normal distribution.

2. [8 marks]

A random sample of 60 families in Ottawa-Vanier has an average income of $92,000 and a standard deviation of $31,000, while a random sample of 40 families in Ottawa-Orleans has an average income of $108,000 and a standard deviation of $23,000.

(a) Test at the .01 level of significance whether these data constitute sufficient evidence to show a difference in average income in the respective populations of Ottawa-Vanier and Ottawa-Orleans. Use the critical value approach. You may assume equal population variances, but that is not necessary. Moreover, if you do not assume equal variances, you do not have to calculate the exact degrees of freedom.

All units are in thousands of dollars.

[5]
Ho: mu1 = mu2 ; Ha: mu1 ≠ mu2
Not assuming equal variance, t = (92 – 108) / √(31^2/60 + 23^2/40) = -2.96.
Assuming equal variance, the pooled stdev is 28.09 and t = -2.79.
Reject Ho if |t| = |z| > 2.58 (2.63 for df = 100).
Decide to reject Ho since 2.96 or 2.79 are greater than either 2.58 or 2.63.
Conclude the average incomes are different in the two populations

(b) What is the p-value for the result in (a)?

[1] *p-value = 2 Prob (z > 2.79) = 2 (.0026) = .005*
p-value = 2 Prob (z > 2.96) = 2 (.0015) = .003

Using t-table with df = 100, we find t(.005) = 2.63 and t(..001) = 3.17
.002 = 2 (.001) < p-value < 2 (.005) = .01

(c) Calculate a 99% confidence interval for the difference in the average incomes between the two populations.

[2] *±16 ± 2.576 (5.41), if not assuming equal variances and assuming a normal approximation to the t.*
= ±16 ± 13.94 = (2.1, 29.9) or (-29.9, -2.1)

±16 ± 2.576 (5.71) , assuming equal variance
= ±16 ± 14.71 = (1.3, 30.7) or (-30.7, -1.3)

3. [7 marks]

A small clinical trial was conducted to test the effectiveness of a new beta-blocker in reducing the systolic blood pressure. Fifteen patients with moderately high blood pressure readings were monitored over a 4-week period. Their blood pressures were measured before and after the clinical trial. Appendix B shows the data and various analyses.

(a) Without looking at the data, are the two sets of blood pressure readings independent or paired? Explain briefly.

[1]

The two samples are paired because each pair of observations corresponds to the same patient. It is not enough to say the two measurements are taken by the same "tester".

(b) Now examine the graphs of the data and explain what would be the most appropriate test. Refer only to the specific graph(s) that is (are) relevant given your answer to (a).

[2]

The boxplot of the differences is relatively symmetric, with no outliers. It is reasonable to assume that these differences came from a normal distribution. Therefore, the most appropriate test is the matched pairs t-test. The Wilcoxon paired sample test is also possible, but it is not the most appropriate if the t-test is appropriate.

(c) Notwithstanding your answer to (b), perform a "parametric" test to determine whether the new drug is effective. Use the .05 level of significance.

[4]

```
Test of mu = 0 vs > 0


                                          95%
                                        Lower
 N      Mean     StDev  SE Mean     Bound     T      P
15  3.13333  4.27395  1.10353  1.18967  2.84  0.007
```

Ho: mu(diff) = 0 ; Ha: mu(diff) > 0
(the differences are calculated as (before - after). For the drug to be effective, the differences should be positive. If Ha is stated as mu < 0, then the differences would have to be (after –before) and the mean would be -3.13 instead of 3.13.)

t-stat = 3.13333 / 1.10353 = 2.84 (for the < 0 alternative, t-stat would be -2.84)

Reject Ho if t-stat > 1.76 (t based on alpha of .05 and 14 df) (for the < 0 alternative, the rejection region would be t < -1.76).

We reject Ho and conclude that the beta-blocker is effective.

4. [6 marks]

Test whether the poll results could be considered an accurate reflection of the actual results. Use the 5% level of significance.

The poll results are the observed counts based on a sample. The question is whether these are consistent with the actual electoral results. Since the hypotheses are always statements about population values and not sample results, we cannot specify the sample proportions in the hypotheses. However, the hypothesized population proportions are used to calculate the expected counts in a sample of 1000.

Ho: poll results reflect the electoral vote which are:

p(L) = .3811, p(PC)=.3589, p(NDP)=.2302, p(G)=.0298

Ha: poll results do not reflect the electoral breakdown, i.e., one of the probabilities is different.

	Oi	Ei	(O-E)^2 / E
Liberal	410	381.1	2.191577
PC	310	358.9	6.662608
NDP	250	230.2	1.703041
Green	30	29.8	0.001342
Total	1000	1000	10.55857

The chi-square statistic is 10.56. At the .05 level, the critical value is 7.81, based on 3 df.

We reject the Ho, since 10.56 exceeds the critical value.
Conclude that the poll results are different from the electoral vote.

1 for hypotheses
1 for calculation of observed and expected counts
1 for calculation of chi-square statistic
1 for the rejection region
1 for the decision to reject the Ho
1 for the conclusion

Four individual tests of one proportion are not equivalent to the overall goodness of fit test because the probability of the type 1 error increases with each test. However, it is possible to obtain 4 out of the 6 marks for these four tests if they were all done correctly. For example,

Ho: p(PC) = .3589, Ha: p(L) ≠ .3589
(Note that the sample proportion is .31, while the hypothesized population proportion is .3589).
z = (.31 - .3589) / √(.3589.6411/1000) = -3.22*
At the .05 level, we reject Ho if |z| > 1.96
We reject Ho, and conclude the observed proportion supporting the PCs did not reflect the actual electoral result. This implies that the poll results are not an accurate reflection of the electoral results.

Note the z-values for the other observed proportions are 1.88 (Liberal), 1.49 (NDP) and .04 (Green), leading to the conclusion that the other observed proportions from the poll are consistent with the actual electoral results.

If we had adjusted the individual significance levels using the Bonferroni method, each individual z-test would be done at the .05/4= .0125 level to ensure an overall .05 level of significance for all four tests. The critical value would have been z(.0125/2) = z(.00625) = 2.50, but this would not have changed any of the previous decisions or conclusions.