University of Ottawa

Department of Mathematics and Statistics

MAT 1302B: Mathematical Methods II

Instructor: Alistair Savage

Second Midterm Exam

Surname _____ First Name _____

Student # _____

Instructions:

(1) You have 80 minutes to complete this exam.
(2) The number of points available for each question is indicated in square brackets.
(3) You must show your work and justify your answers to receive full marks. Partial marks may be awarded for making sufficient progress towards a solution.
(4) All work to be considered for grading should be written in the space provided. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**.
(5) Write your student number at the top of each page in the space provided.
(6) No notes, books, calculators or scrap papers are allowed.
(7) The final page of the exam may be used for scrap work.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	4	6	6	7	3	4	30
Score							

1. **[4pt.]** Let $\overrightarrow{v_1} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $\overrightarrow{v_2} = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix}$, and $\overrightarrow{v_3} = \begin{bmatrix} -3 \\ 15 \\ h \end{bmatrix}$. Find the value(s) of h for which the vectors are linearly *dependent*.

2. The following matrices are given

$$A = \begin{bmatrix} 4 & -1 \\ 3 & 2 \\ 3 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{bmatrix}.$$

(a) **[4pt.]** Determine the inverse of C.

(b) **[2pt.]** If possible, solve the equation $CX = A$ for X.

3. An economy has two sectors: Electricity and Services. For each unit of output, Electricity requires 0.5 units from its own sector and 0.4 units from Services. Meanwhile, Services requires 0.5 units from Electricity and 0.2 units from its own sector to produce one unit of Services.

(a) **[1pt.]** Determine the consumption matrix C.

(b) **[1pt.]** State the Leontief input-output equation relating C to the production vector $\overrightarrow{x}$ and final demand vector $\overrightarrow{d}$.

(c) **[4pt.]** Use an inverse matrix to determine the production vector necessary to satisfy a final demand of 1000 units of Electricity and 2000 units of Services, i.e. $\overrightarrow{d} = \begin{bmatrix} 1000 \\ 2000 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 1 & 0 & 3 & 4 \\ 4 & 4 & -4 & 0 \\ 2 & 5 & -4 & -2 \end{bmatrix}$.

(a) **[3pt.]** Find a basis for $\operatorname{Col} A$.

(b) **[2pt.]** Provide a basis for $\operatorname{Nul} A$.

(c) **[2pt.]** Determine the rank of A and the dimension of the null space of A.

5. **[3pt.]** Calculate the following determinant:

$$\begin{vmatrix} 2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 2 \\ 2 & 1 & 1 & 6 \end{vmatrix}.$$

Remember to show your work.

6. Find out which of the following subsets of $\mathbb{R}^4$ are subspaces. Justify your answer.

(a) **[2pt.]** $H = \left\{ \begin{bmatrix} x \\ y \\ u \\ z \end{bmatrix} \;\middle|\; x = y \right\}$.

(b) **[2pt.]** $K = \left\{ \begin{bmatrix} x \\ y \\ u \\ z \end{bmatrix} \;\middle|\; x \geq 0 \right\}$.

This page is for scrap work and is intentionally left blank.