Root Finding:

$$x_R = x_U - \frac{f(x_U)(x_U - x_L)}{f(x_U) - f(x_L)} \quad x_{i+1} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)} \quad x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$E_{MAX} = \frac{\Delta x^0}{2^N} \quad N = \log_2 \frac{\Delta x^0}{E_{MAX}} = \frac{\log(\Delta x^0 / E_{MAX})}{\log 2}$$

Golden Section:

$$X_2 = X_U - d, \quad X_1 = X_L + d, \quad d = (\phi - 1)(X_U - X_L)$$

$$\phi = \frac{1+\sqrt{5}}{2} = 1.6180339887 \quad E_{MAX} = \frac{\Delta x^0 (\phi - 1)^N}{2} \text{ (assuming midpoint)}$$

Simultaneous Equations:

$$Ax = b \Rightarrow LUx = b \Rightarrow Ld = b \text{ and } Ux = d$$
$$Ax = b \Rightarrow P^{-1}LU = b \Rightarrow LUx = Pb \Rightarrow Ld = Pb \text{ and } Ux = d$$

Iterative methods: $x_{k+1} = Cx_k + d \quad c_{ii} = 0 \quad c_{ij} = -a_{ij} / a_{ii} \quad d_i = b_i / a_{ii}$

Lagrange Polynomial:

$$p(x) = L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 + \ldots L_N(x)y_N$$

$$L_k(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_{k-1})(x-x_{k+1})\ldots(x-x_N)}{(x_k-x_1)(x_k-x_2)\ldots(x_k-x_{k-1})(x_k-x_{k+1})\ldots(x_k-x_N)}$$

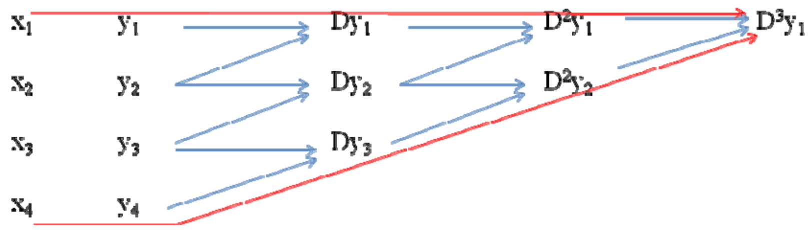
Numerator of $L_k(x)$ is product of all $(x - x_i)$ except for $(x - x_k)$
Denominator of $L_k(x)$ is product of all $(x_k - x_i)$ except for $(x_k - x_k)$

Newton's Polynomial:

$$p(x) = a_1 + a_2(x-x_1) + a_3(x-x_1)(x-x_2) + \ldots + a_N(x-x_1)\ldots(x-x_{N-1})$$

$$a_1 = y_1 \quad a_2 = Dy_1 \quad a_3 = D^2 y_1 \quad \ldots \quad a_N = D^{N-1} y_1$$

$$Dy_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \quad D^2 y_i = \frac{Dy_{i+1} - Dy_i}{x_{i+2} - x_i} \quad D^k y_i = \frac{D^{k-1}y_{i+1} - D^{k-1}y_i}{x_{i+k} - x_i}$$



Regression:

Straight line fit: $y = mx + b \quad m = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^{\,2} - \left(\sum x_i\right)^2} \quad b = \dfrac{\sum x_i^{\,2} \sum y_i - \sum x_i \sum x_i y_i}{n\sum x_i^{\,2} - \left(\sum x_i\right)^2} = \bar{y} - m\bar{x}$

$r^2 = \dfrac{S_t - S_r}{S_t} \quad S_t = \sum (y_i - \bar{y})^2 \quad S_r = \sum (y_i - f(x_i))^2$

For a straight line fit only: $r = \dfrac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^{\,2} - \left(\sum x_i\right)^2}\sqrt{n\sum y_i^{\,2} - \left(\sum y_i\right)^2}}$

For $y = \alpha e^{\beta x} \quad x' = x \quad y' = \ln(y) \quad \alpha = e^b, \beta = m$

For $y = \alpha x^{\beta} \quad x' = \log(x) \quad y' = \log(y) \quad \alpha = 10^b, \beta = m$

For $y = \alpha \dfrac{x}{\beta + x} \quad x' = \dfrac{1}{x} \quad y' = \dfrac{1}{y} \quad \alpha = 1/b, \beta = m/b$

General least squares : $y = a_0 g_0(x) + a_1 g_1(x) + a_2 g_2(x) + \cdots a_n g_n(x)$

Basic solution : $z_{ij} = g_i(x_j) \quad Z^T Z a = Z^T y$ QR decomposition : $Z = QR \quad Ra = Q^T y$

Integration:

$$I = \frac{h}{2}\left(f(x_0) + f(x_1)\right) \quad \Rightarrow \quad I = \frac{h}{2}\left(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \ldots + 2f(x_{N-1}) + f(x_N)\right)$$

$$I = \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right) \quad \Rightarrow \quad I = \frac{h}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots + 4f(x_{N-1}) + f(x_N)\right)$$

$$I = \frac{3h}{8}\left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right)$$

$I_{j,1} =$ estimate using $h_j \quad h_j = \dfrac{h_1}{2^{j-1}} \quad I_{j,k} = \dfrac{4^{k-1} I_{j+1,k-1} - I_{j,k-1}}{4^{k-1} - 1} \quad |\varepsilon| = \left|\dfrac{I_{1,k} - I_{2,k-1}}{I_{1,k}}\right|$

n	c_0	x_0	c_1	x_1	c_2	x_2	c_3	x_3
2	1	-0.57735	1	0.57735				
3	5/9	-0.77459	8/9	0	5/9	0.77459		
4	0.34785	-0.81136	0.652145	-0.339981	0.652145	0.339981	0.34785	-0.81136

Directly usable only for -1 to 1. For *a* to *b* use $x = \frac{b+a}{2} + \frac{b-a}{2} x_d \quad dx = \frac{b-a}{2} dx_d$

Differentiation:

First Derivative	**"First order"**	**"Second Order" (better)**
Forward	$\frac{f(x_{i+1}) - f(x_i)}{h}$	$\frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h}$
Backwards	$\frac{f(x_i) - f(x_{i-1})}{h}$	$\frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2})}{2h}$
Central	$\frac{f(x_{i+1}) - f(x_{i-1})}{2h}$	$\frac{-f(x_{i+2}) + 8f(x_{i+1}) - 8f(x_{i-1}) + f(x_{i-2})}{12h}$

$D_{j,1} = \text{estimate using } h_j \quad h_j = \frac{h_1}{2^{j-1}} \quad D_{j,k} = \frac{4^{k-1} D_{j+1,k-1} - D_{j,k-1}}{4^{k-1} - 1} \quad |\varepsilon| = \left| \frac{D_{1,k} - D_{2,k-1}}{D_{1,k}} \right|$

ODE's:

$\frac{dy}{dt} = f(t,y) \quad y_{i+1} = y_i + \phi h \quad \text{Euler}: \phi = f(t_i, y_i) \quad \text{Midpoint}: \phi = f(t_{i+1/2}, y_{i+1/2})$

$\text{Heun}: y^0{}_{i+1} = y_i + f(t_i, y_i) h \quad y^j{}_{i+1} = y_i + \frac{f(t_i, y_i) + f(t_{i+1}, y^{j-1}{}_{i+1})}{2} h$

$\text{no iteration } y_{i+1} = y^1{}_{i+1} \quad \text{with iteration } y_{i+1} = y^m{}_{i+1} \quad \left| \frac{y^m{}_{i+1} - y^{m-1}{}_{i+1}}{y^m{}_{i+1}} \right| \times 100\% < \varepsilon$