Solutions to December 2008 Final

(1) $Ax = b$

$x = A^{-1}b$

$$= \begin{bmatrix} -1 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2-3 \\ 8-1 \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$$

(d)

(2) $\left[\begin{array}{cc|c} 1 & h & 3 \\ 5 & -10 & k \end{array}\right] R_2 - 5R_1$

$$\begin{array}{rccc} R_2: & 5 & -10 & k \\ -5R_1: & -5 & -5h & -15 \\ \hline & 0 & -5h-10 & k-15 \end{array}$$

$\left[\begin{array}{cc|c} 1 & h & 3 \\ 0 & -5h-10 & k-15 \end{array}\right]$ $k \neq 15$ (If $k = 15$ then that entry is zero, and thus is consistent)

$-5h - 10 = 0.$

$h = -2$

(d)

(3) $\begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & h \end{bmatrix} R_1 + R_2$

$$\begin{array}{rccc} R_1: & 1 & 0 & 2 \\ R_2: & -1 & -1 & -2 \\ \hline & 0 & -1 & 0 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & h \end{bmatrix} \begin{array}{l} \\ \\ R_2+R_3 \\ R_4-R_1 \end{array}$$

$$\begin{array}{rrrr} R_4: & 1 & 0 & h \\ -R_1: & -1 & 0 & -2 \\ \hline & 0 & 0 & h-2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & h-2 \end{bmatrix} \begin{array}{l} \\ -R_2 \\ R_3 \leftrightarrow R_4 \\ \\ \end{array}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & h-2 \\ 0 & 0 & 0 \end{bmatrix}$$

If $h-2 \neq 0$, then we can row reduce this matrix further and get 3 distinct elementary vectors. In this case, the 3 vectors would be linearly ~~d~~independent.

So $h-2=0$ $\boxed{h=2}$ (a)

(4) $T\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $T\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $T\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$T\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{array}{l} \\ R_2-R_1 \\ \\ \end{array}$

$$\begin{array}{rrrrr} R_2: & 1 & 1 & 0 & -1 \\ -R_1: & -1 & -1 & 1 & 0 \\ \hline & 0 & 0 & 1 & -1 \end{array}$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix} R_2 \leftrightarrow R_3$$

$$\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} R_1 - R_2$$

$$\begin{array}{rcccc} R_1: & 1 & 1 & -1 & 0 \\ -R_2: & 0 & -1 & -1 & 0 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{matrix} R_1 + 2R_3 \\ R_2 - R_3 \end{matrix}$$

$$\begin{array}{rcccc} R_2: & 0 & 1 & 1 & 0 \\ -R_3: & 0 & 0 & -1 & 1 \\ \hline & 0 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{rcccc} R_1: & 1 & 0 & -2 & 0 \\ 2R_3: & 0 & 0 & 2 & -2 \\ \hline & 1 & 0 & 0 & -2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

infinitely many solutions to homogeneous system, so not one-to-one.

rank $A = 3 = \dim(\mathbb{R}^3)$, so A is onto.

(b)

(5.) Write $(2,-1,-1)$ as a linear combination of $(1,0,-1)$ and $(0,1,-1)$

$$(2,-1,-1) = 2(1,0,-1) - 1(0,+1,-1)$$

$$T(2,-1,-1) = T(2(1,0,-1) - (0,1,-1))$$
$$= T(2(1,0,-1)) - T(0,1,-1).$$

$= 2T(1,0,-1) - T(0,1,-1)$

$= 2(5,-2) - (1,2)$

$= (10,-4) - (1,2) = (9,-6)$ b

(6) $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 4 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 - 2R_1$

$R_2: \; 2 \;\; 4 \;\; 2 \;\; 6$

$-2R_1: \; -2 \;\; -4 \;\; 2 \;\; 0$

$0 \;\; 0 \;\; 4 \;\; 6$

$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $4R_1 + R_2$

$4R_1: \; 4 \;\; 8 \;\; -4 \;\; 0$

$R_2: \; 0 \;\; 0 \;\; 4 \;\; 6$

$4 \;\; 8 \;\; 0 \;\; 6$

$\begin{bmatrix} 4 & 8 & 0 & 6 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_1/4$, $R_2/4$

$\begin{bmatrix} 1 & 2 & 0 & 3/2 \\ 0 & 0 & 1 & 3/2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

not in basis　　not in basis.

(b) $\{v_2, v_4\}$.

(7) $(A^{-1} B^T (C^{-1})^T)^{-1} = ((C^{-1})^T)^{-1} (B^T)^{-1} (A^{-1})^{-1}$

$= C^T (B^T)^{-1} A.$

$= C^T (B^{-1})^T A.$ d

(8) A $\underset{m\times n.}{5\times 7}$ dim Col = 4

$n = \text{rank}(A) + \text{nullity}(A).$

$n = \dim \text{Col } A + \dim \text{Nul } A$

$7 = 4 + \dim \text{Nul} A$

$\dim \text{Nul} A = 3.$ d

(9) This is something called 'Change of Basis' which is no longer covered in the course

(10) (b) Same as Fall 2010

(11) det A = 2.

Need to figure out which elementary row operations were done to get from ~~A~~ B to ~~B~~ A ~~B~~

B ≠

$$\det B = \det \begin{bmatrix} g & h & i \\ d-a & e-b & f-c \\ -2a & -2b & -2c \end{bmatrix}$$

(b)

$$= -2 \det \begin{bmatrix} g & h & i \\ d-a & (e-b) & (f-c) \\ a & b & c \end{bmatrix}$$ $R_1 \leftrightarrow R_3$ changes det by (-1)

$$= 2 \det \begin{bmatrix} a & b & c \\ (d-a) & (e-b) & (f-c) \\ g & h & i \end{bmatrix}$$ $R_2 + R_1$ ← doesn't change anythin

$$= 2 \det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$= 2 \det(A) = 4$$

(b)

(12) $\det A = -3$. 2×2

$\det(2A^2 A^T A^{-1})$

$= 2^2 \det(A^2 A^T A^{-1})$ since $\det(A^T) = \det A$

$= 4 (\det A)^2 \det(A) (\det A)^{-1}$

$= 4 \dfrac{(-3)^2(-3)}{-3} = 36$ (a)

(13) $\det A = \det \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} = 1 + 4 = 5$

$$\det A_1 = \det\begin{bmatrix} 0 & 0 & -2 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 2$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{2}{5} \leftarrow$$ Correct (I double checked) but not listed as one of the answers

(14) No longer in the course

(15)
$$\frac{(2+4i)}{(1+i)}\frac{(1-i)}{(1-i)} = \frac{2+4i-2i-4i^2}{1-i^2}$$
$$= \frac{2+4+2i}{2}$$
$$= \frac{6+2i}{2} = 3+i$$

(d)

(16)
$$\det(A-\lambda I) = \det\begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 2-\lambda & 4 \\ 0 & 4 & 2-\lambda \end{bmatrix}$$
$$= (1-\lambda)(2-\lambda)(2-\lambda) - 16(1-\lambda)$$
$$= (1-\lambda)\left[(2-\lambda)^2 - 16\right]$$
$$= (1-\lambda)(2-\lambda-4)(2-\lambda+4)$$

$= (1-\lambda)(-2-\lambda)(6-\lambda) = 0$

$\lambda = 1 \quad \lambda = -2 \quad \lambda = 6$

e

(17) $\lambda = 1$

$$\begin{bmatrix} 1-\lambda & 0 & 0 & -2 \\ 0 & 1-\lambda & 0 & 0 \\ 0 & 0 & 1-\lambda & 0 \\ -1 & 0 & 0 & 1-\lambda \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} R_1 \leftrightarrow R_4 \\ R_4 \leftrightarrow R_2 \end{matrix}$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} x_2 = t \\ x_3 = r \end{matrix}$$

$x_1 = 0 \quad x_2 = t \quad x_3 = r \quad x_4 = 0.$

$(0, t, r, 0) = (0, t, 0, 0) + (0, 0, r, 0)$

$= t(0, 1, 0, 0) + r(0, 0, 1, 0)$

dim = 2. c

(18) Same as a question in Fall 2010.

V and H are subspaces c.

(19) $A = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$

$\det \begin{bmatrix} 1-\lambda & -3 \\ 3 & 1-\lambda \end{bmatrix} = (1-\lambda)^2 + 9.$

$= \lambda^2 - 2\lambda + 1 + 9$

$= \lambda^2 - 2\lambda + 10.$

$a = 1 \quad b = -2 \quad c = 10.$

$x = \dfrac{2 \pm \sqrt{4 - 4(10)}}{2}$

$= \dfrac{2 \pm \sqrt{-36}}{2}$

$= \dfrac{2 \pm 6i}{2} = 1 \pm 3i$ (a)

(20) $Av_1 = \lambda_1 v_1$

$A \begin{bmatrix} 4 \\ -1 \end{bmatrix} = 1 \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

Trial and error X

$\begin{bmatrix} 7 & -2 \\ -7 & 24 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 28+2 \\ -28-24 \end{bmatrix}$

$\begin{bmatrix} 4 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 16+1 \\ 12-1 \end{bmatrix}$ X

$\begin{bmatrix} 7 & -2 \\ 24 & -7 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} 28+2 \\ 96+7 \end{bmatrix}$ X

$\begin{bmatrix} -3 & 1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix} = \begin{bmatrix} -12-1 \\ 16+1 \end{bmatrix}$ X

$$\begin{bmatrix}7 & 24\\ -2 & -7\end{bmatrix}\begin{bmatrix}+4\\ -1\end{bmatrix}=\begin{bmatrix}28-24\\ -8+7\end{bmatrix}=\begin{bmatrix}+4\\ -1\end{bmatrix}$$ ✓

Check second one.

$A v_2 = \lambda_2 v_2$

$$\begin{bmatrix}7 & 24\\ -2 & -7\end{bmatrix}\begin{bmatrix}-3\\ 1\end{bmatrix}=-1\begin{bmatrix}-3\\ 1\end{bmatrix}$$

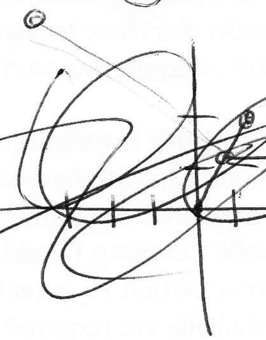
$$\begin{bmatrix}-21+24\\ 6-7\end{bmatrix}=\begin{bmatrix}3\\ -1\end{bmatrix}$$

$$\begin{bmatrix}3\\ -1\end{bmatrix}=\begin{bmatrix}3\\ -1\end{bmatrix}$$ ✓. (e)

(21) Not in this course anymore.

(22) Not in this course anymore.

(23) $a=\begin{bmatrix}-3\\ 9\end{bmatrix}$ $u=\begin{bmatrix}1\\ 2\end{bmatrix}$



midpt.

Midpoint of (0,0), (1,2)

$\left(\frac{1}{2}, 1\right)$

$x_1\ y_1 \qquad x_2\ y_2$

Distance from $(1/2, 1)$ and $(-3, 9)$

$d=\sqrt{(1/2+3)^2+(1-9)^2}=\sqrt{12.25+64}=$

(23) Not in this course anymore

(24). ~~[illegible]~~ = On Fall 2010 exam

(25) Not in this course anymore