CARLETON UNIVERSITY

FINAL EXAMINATION
MATH 1005 A, B, C, D
Winter 2011

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH 1005 A, B, C, D.
Course Instructor(s): Dr. A.B. Mingarelli (Sect. A), Mr. A. Khanchi (Sect. B), Dr. J. Abdulrahman (Sect. C), Dr. S. Melkonian (Sect. D).

AUTHORIZED MEMORANDA
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1. **[5 marks] Solve the differential equation** $\frac{dy}{dx} = e^{3x+2y}$.

   (a) $-3e^{-2y} = 2e^{-3x} + C$ (b) $-3e^{2y} = 2e^{-3x} + C$ (c) $-3e^{-2y} = 2e^{3x} + C$ (d) $-e^{-2y} = 2e^{-3x} + C$

2. **[5 marks] Let** y **be the solution of the initial value problem** $x\frac{dy}{dx} - y = x^2 \sin x$, $y(\frac{\pi}{2}) = \pi$. **Determine** $y(0)$.

   $xy' - y = x^2 \sin x$

   (a) 0 (b) 1 (c) 2 (d) π

3. **[5 marks] Solve the exact equation** $(3x^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$.

   (a) $x^3y - xe^y - y^2 = C$ (b) $x^3y + xe^y + y^2 = C$ (c) $x^3y + xe^y - y^2 = C$ (d) $x^3y - xe^y + y^2 = C$

4. **[5 marks] Solve the differential equation** $(y^2 + yx)dx - x^2dy = 0$ **by an appropriate substitution.**

   (a) $y \ln|x| + x = cy$ (b) $y \ln|x| - x^2 = cy$ (c) $x \ln|x| + x = cy$ (d) $y \ln|x| + x = cx$

5. **[5 marks] Let** y **be the solution of the initial value problem** $xy^2\frac{dy}{dx} = y^3 - x^3$, $y(1) = 2$. **Determine** $y(-1)$.

   (a) 0 (b) -1 (c) 2 (d) -2

6. **[5 marks] The sum of the series** $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$ **is**

   ~~(a)~~ $\cancel{\frac{10}{3}}$ (b) $\frac{1}{2}$ (c) $\frac{2}{5}$ (d) $\cancel{\frac{5}{3}}$.

7. **[5 marks] Which of the following three series below converge(s)?**

$$(1)\ \sum_{n=1}^{\infty} \frac{1}{n^5+1}, \qquad (2)\ \sum_{n=1}^{\infty} \frac{2n-1}{3n+1}, \qquad (3)\ \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$$

   (a) (1) only ~~(b) (3) only~~ (c) (1), (2) and (3) ~~(d) (2) and (3)~~

$\frac{1}{n^2} > \frac{1}{n^5+1}$

8. **[5 marks] Which of the following three series below converge(s) conditionally?**

$$(1)\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{5^n} \qquad (2)\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n+1} \qquad (3)\quad \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$$

(a) (1) and (2) (b) (2) only (c) (3) only (d) (1) and (3).

9. **[5 marks] The radius of convergence, R, of the series $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$ is**

(a) R= 2 (b) R=3 (c) R=1 (d) R=4

10. **[5 marks] Two linearly independent solutions of the system $x' = x + 2y$, $y' = x + 2y$ are given by**

(a) $\begin{bmatrix} 2 \\ -1 \end{bmatrix}, e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, e^{2t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, e^{2t}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}, e^{3t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

11. **[5 marks] The coefficient of $(x-1)^3$ in the Taylor series of $f(x) = \ln(x)$ about $a = 1$ (centred at 1) is given by**

(a) 2 (b) -2 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

12. **[5 marks] The coefficient of x^2 in the MacLaurin series of $f(x) = \sqrt[3]{1+x}$ is given by**

(a) $\frac{2}{9}$ (b) $-\frac{2}{9}$ (c) $\frac{1}{9}$ (d) $-\frac{1}{9}$

13. **[5 marks] Let $f(x) = x$, $g(x) = x^2$ and $h(x) = x^3$ on the interval $[-1, 1]$. Among the pairs (f, g), (f, h) and (g, h), the pairs which are orthogonal on $[-1, 1]$ are**

(a) (f, g) but not (f, h) or (g, h) (b) (f, g) and (f, h) but not (g, h) (c) (f, g) and (g, h) but not (f, h) (d) (f, h) but not (f, g) or (g, h)

14. **[5 marks] Let $f(x) = x$ for $0 \le x \le 1$. The half-range sine series of f is $\sum_{n=1}^{\infty} b_n \sin(n\pi x)$, where**

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$b_n =$

(a) $\frac{-2(-1)^n}{n\pi}$ (b) $\cancel{\frac{(-1)^{n+1}}{n\pi}}$ (c) $\cancel{\frac{(-1)^n}{n\pi}}$ (d) $\frac{2(-1)^n}{n\pi}$

27.5

15. **[5 marks] Let $f(x) = \begin{Bmatrix} 0, & -1 \le x \le 0 \\ x, & 0 < x \le 1 \end{Bmatrix}$. The Fourier series of f is of the form $\frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\pi x) + b_n \sin(n\pi x)]$, where $n \ge 1$ and**

(a) $a_0 = 1$, $a_n = \frac{2}{n^2\pi^2}[(-1)^n - 1]$, $b_n = \frac{-2(-1)^n}{n\pi}$; (b) $a_0 = \frac{1}{2}$, $a_n = \frac{1}{n^2\pi^2}[(-1)^n - 1]$, $b_n = \frac{-(-1)^n}{n\pi}$;

(c) $a_0 = 1$, $a_n = \frac{2}{n^2\pi^2}[1 - (-1)^n]$, $b_n = \frac{(-1)^n}{n\pi}$; (d) $a_0 = \frac{1}{2}$, $a_n = \frac{1}{n^2\pi^2}[1 - (-1)^n]$, $b_n = \frac{(-1)^n}{n\pi}$

16. **[5 marks] Determine the general solution of the differential equation $y'' + 5y = 0$:**

(a) $y(x) = c_1 \sin\sqrt{5}x + c_2 \cos\sqrt{5}x$ (b) $y(x) = c_1 \sin 5x + c_2 \cos 5x$ ~~(c)~~ $\cancel{y(x) = c_1 e^{5x} + c_2 e^{-5x}}$ ~~(d)~~ $\cancel{y(x) = c_1 x + c_2 e^{5x}}$.

17. **[5 marks] Find the general solution of the non-homogeneous differential equation $y'' - 6y' + 9y = e^x$:**

~~(a)~~ $\cancel{y(x) = c_1 e^{3x} + c_2 e^{-3x} + e^x}$ ~~(b)~~ $\cancel{y(x) = c_1 \sin 6x + c_2 \cos 9x - e^x}$ ~~(c)~~ $\cancel{y(x) = c_1 e^{3x} + c_2 \sin 3x + e^x/2}$. (d) $y(x) = c_1 x e^{3x} + c_2 e^{3x} + e^x/4$.

18. **[5 marks] The solution, $y(x)$, of the initial value problem for the homogeneous differential equation $y'' + y' - 6y = 0$ satisfying the initial conditions $y(0) = 1$, $y'(0) = -1$ is given by :**

~~(a)~~ $\cancel{y(x) = e^{3x} - xe^{-2x}}$ ~~(b)~~ $\cancel{y(x) = e^{-x}}$ (c) $y(x) = 5e^{-3x} - 4e^{2x}$ (d) $y(x) = 2e^{2x}/5 + 3e^{-3x}/5$.

19. **[5 marks] The number of linearly independent solutions of a second order linear ordinary differential equation is:**

(a) 1 (b) 2 (c) 3 (d) 4.

20. **[5 marks] The general solution of the differential equation $x^2y'' + xy' - 9y = 1$ is:**

(a) $y = c_1x^2 + c_2x^3$ (b) $y = c_1x^2 + c_2x^3 + 1/9$ (c) $y = -1/9 + c_1x^3$ (d) $y = c_1x^3 + c_2/x^3 - 1/9$

y'' y'