University of Ottawa

Department of Mathematics and Statistics

MAT 1302B: Mathematical Methods II

Instructor: Alistair Savage

First Midterm Exam – February 6, 2009

Surname _____ First Name _____

Student # _____ DGD (1–4) _____

Instructions:

(1) You have 80 minutes to complete this exam.
(2) The number of points available for each question is indicated in square brackets.
(3) You must show your work and justify your answers to receive full marks. Partial marks may be awarded for making sufficient progress towards a solution.
(4) All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this **clearly**. Otherwise, the work written on the reverse side of pages will not be considered for marks.
(5) Write your student number at the top of each page in the space provided.
(6) No notes, books, calculators or scrap paper are allowed.
(7) You should write in **pen**, not pencil.
(8) The final page of the exam may be used for scrap work.
(9) Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	4	6	5	5	5	5	30
Score							

1. [**4**] Find all solutions to the vector equation

$$x_1 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}.$$

2. [**6**] Is the following linear system consistent? If so, find the general solution.

$$\begin{aligned} x_1 + 12 &= 3x_2 + 3x_3 - x_4 \\ x_1 + 2x_2 + 5x_3 + 2 &= x_2 + 3x_4 + 6 \\ -x_1 + x_2 - x_3 + x_4 - 4 &= 0 \\ -3x_2 + x_4 &= -12 - x_1 + 3x_3 \end{aligned}$$

3. Consider the following linear system:

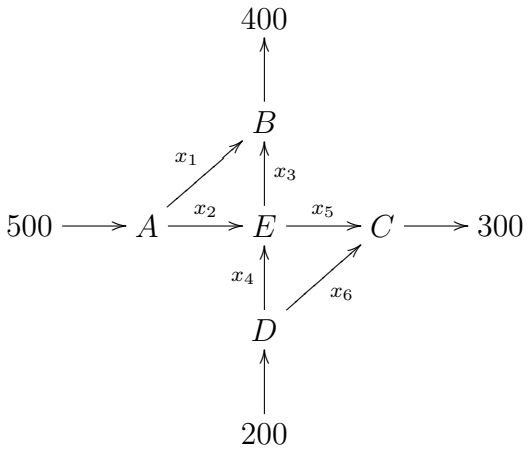
$$\begin{aligned} y + z &= -2 \\ x + 2y + z &= 1 \\ x - y + cz &= 7 \end{aligned}$$

(a) **[1]** Write down the augmented matrix $[\ A \mid \vec{b}\ ]$ of the system.

(b) **[2]** Find the reduced echelon form of $[\ A \mid \vec{b}\ ]$ when $c = -2$.

(c) **[2]** Find all possible reduced echelon forms of $[\ A \mid \vec{b}\ ]$ when c is arbitrary. Are there values of c for which the system is inconsistent?

4. Consider the traffic flow described by the following diagram. The letters A through E label intersections and numerical values indicate flow in cars per minute. The arrows indicate the direction of flow (all roads are one-way).



(a) **[2]** Write down a linear system describing the traffic flow, as well as all constraints on the variables x_i, $i = 1, \ldots, 6$. (Do not perform any calculations at this stage.)

(b) **[2]** The reduced echelon form of the augmented matrix corresponding to the linear system in part (a) is

$$\left[\begin{array}{cccccc|c} 1 & 0 & 1 & 0 & 0 & 0 & 400 \\ 0 & 1 & -1 & 0 & 0 & 0 & 100 \\ 0 & 0 & 0 & 1 & 0 & 1 & 200 \\ 0 & 0 & 0 & 0 & 1 & 1 & 300 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Find the general solution of the linear system. (Ignore the constraints for now.)

(c) **[1]** If the flow along EB is limited to a maximum of 100 cars per minute due to road work, what is the smallest possible flow along AB?

5. [**5**] Let

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}.$$

Is the vector $\mathbf{b}$ in $\text{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$?

6. Let

(a) [**4**] Describe the general solution to the equation

$$\begin{bmatrix} 2 & -4 & 8 \\ 3 & -6 & 12 \\ -1 & 2 & -4 \end{bmatrix} \vec{x} = \begin{bmatrix} 6 \\ 9 \\ -3 \end{bmatrix}$$

in *vector parametric form.*

(b) [**1**] What is the solution set to the homogeneous equation

$$\begin{bmatrix} 2 & -4 & 8 \\ 3 & -6 & 12 \\ -1 & 2 & -4 \end{bmatrix} \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad ?$$

Describe it as the span of a collection of vectors. (Note that the coefficient matrix is the same as in part (a).)