

## ASSIGNMENT 2 PART II

**Note: Your submission will consist of two steps. First, use the answer-area to provide a statement to the TA alerting them that a PDF document containing your answers to this assignment's Part II questions has been submitted (via document sharing discussed next). Second, use the shared-documents tool within MyStatLab to upload your PDF document. Do not forget to include a statement of academic integrity within the statement that you provide in the answer area to part II. Finally, note that you are required to show your work for full credit --- correct numeric answers may earn you little credit unless you show your work.**

### QUESTION 1 [20 marks]

Your sister owns and operates a restaurant called Quiet Waters (QW) and seeks your help on Human Resource issues.

Depending on a number of factors the wait-staff requirements (number of waitresses and/or waiters) at a QW restaurant can be either *low* (4 people), or *moderate* (7 people), or *peak* (12 people).

QW has two wage levels for wait-staff: \$20.5/hr for experienced staff; and \$10.5/hr for inexperienced staff (less than two years experience).

Suppose that the flagship QW restaurant has a large pool of staff (1/5th of which are experienced), and that each person in this pool has an equal chance of being assigned a shift (regardless of wage level). Based on this information answer the following (**make sure to report units where appropriate**):

- (a) What is the probability model for the number of experienced staff assuming a *low* staffing level (i.e., 4 wait staff used)? Specify the model and tabulate the possibilities and probabilities associated with that model [2 mark].
- ❖ **Binomial model because the number of trials is given and the question asks for the probability of success in each given number of trials from 0-4.**
  - ❖ **1/5 of staffs (experienced) earn 20.5/hr.**
  - ❖ **4/5 of staffs (non-experienced) earn 10.5/hr**

$$P(x) = \frac{n!}{x!(n-x)!} * p^x * ((1-p)^{(n-x)})$$

Staff level (x)	Pr[x]
0	.410
1	.410
2	.154
3	0.026
4	0.002

- (b) What is the probability model for the total per hour wage assuming a *low* staffing level (i.e., 4 wait staff used) [2 marks];
- ❖ **Poisson probability model due to the number of arrivals in a set**

**period of time.**

- (c) Compute the following summaries for the probability model for total wage described in part (b) above (*low* case, only) [3 marks]

i. **Expected value**

$$1/5 * 20.5hr + 3/5 * 10.5hr = 12.5/hr.$$

$$E(x) = n * p = 4(12.5/hr) = 50/hr$$

ii. **Variance, and**

$$\text{Var}[x] = n * p (1-p) = (10)^2 * (0.64) = (100) * (0.64) = 64$$

$$\text{SD}[x] = \text{sqrt}(\text{Var}[x]) = 8$$

iii. **Coefficient of variation;**  $\text{SD}[x] / \text{Var}[x] = 8 / 64 = 0.125$

- (d) Compute the following (total wage) summaries for *moderate* and *peak* cases as well [3 marks].

i. **Expected value**

(moderate)

$$\text{TW}/h = 10x + 73.5$$

$$E(x) = n * p = 7 * 1/5 = 1.4$$

$$\text{Var}[x] = (10)^2 * (1.12) = (100) * (1.12) = 112$$

$$\text{Expected value: } E[\text{TW}/h] = 10(1.4) + 73.5 = 14 + 73.5 = 87.5$$

(Peak)

$$\text{TW}/h = 10x + 126$$

$$E[x] = n * p = 12 * 1/5 = 2.4$$

$$\text{Var}[x] = n * p (1-p) = 12 * 0.8 (1 - 0.8) = 1.92$$

$$E[\text{TW}/h] = 10(2.4) + 126 = 24 + 126 = 150$$

ii. **Variance, and**

**(moderate)**

$$\text{Var}[x] = \text{Var}[x] = (10)^2 * (1.12) = (100) * (1.12) = 112$$

$$\text{SD}[x] = \text{sqrt}(\text{Var}[x]) = 10.58$$

**(Peak)**

$$\text{Var}[x] = (10)^2 * (1.92) = (100) * (1.92) = 192$$

$$\text{SD}[x] = \text{sqrt}(\text{Var}[x]) = 13.86$$

iii. **Coefficient of variation;**

(moderate)

$$\text{SD}[x] / \text{Var}[x] = 10.58 / 112 = .0945$$

**(Peak)**

$$\text{SD}[x] / \text{Var}[x] = 13.86 / 192 = .072$$

- (e) Your sister examined staffing patterns across a large, and equal, number of **low** (4 staff) versus **peak** (12 staff) shifts and noticed something interesting. She has been particularly interested in shifts that have no experienced staff on the floor; referred to as “fragile shifts” because the staff (inexperienced) is less able to react to challenges--- for a low-shift, a fragile-shift refers to the event of having 0

experienced workers out of 4, whereas for a peak-shift a fragile shift refers to the

event of having 0 experienced workers out of 12. Your sister notes that fragile-shifts happen roughly 6 times (see her graph below) more often on low-shifts than on peak-shifts. Noting that the junior manager is always in charge on “low-shifts” your sister suspects that the junior-manager may be to blame for the excess “fragile-shifts” --- she speculates that experienced staff avoid working when they know the junior manager will be in charge. Provide an alternative explanation, possibly exploiting some calculations and/or information from above [4 marks]

$$X=6$$

$$n=12$$

$$P=25/400=0.0625 \text{ for peak shift}$$

$$P=175/400=0.44 \text{ for low shift}$$

$$\text{(Peak shift)} P(X=K=6) = [12] 0.0625^6(1-0.0625)^6 = 924 \cdot 0.0625^6(1-0.0625)^6 = 0.000037$$

$$\text{(Low shift)} P(X=K=6) = [12] 0.44^6(1-0.44)^6 = 924 \cdot 0.44^6(1-0.44)^6 = 0.207$$

Therefore,  $P(\text{peak shift}) < P(\text{low shift})$  that aligns with the case of low-shift excess stated in question. The chances for experienced staff in comparison to inexperienced staff since they compose only 1/5 of the overall staff.

- (f) How would your answers to question (d) change if the wage rates for wait-staff were increased by 25 percent (multiplied by 1.25). Provide numeric answer for the **moderate** case. *Be sure to address expected value, variance and coefficient of variation.* [3 marks].

$$a = 1.25$$

$$E[ax] = aE[x]$$

$$SD[ax] = aSD[x]$$

$$E[TW/h] = (87.5) \cdot (a) = (87.5) \cdot (1.25) = 109.38$$

$$SD[TW/h] = (10.58) \cdot (a) = (10.58) \cdot (1.25) = 13.56$$

$$SD[x] / \text{Var}[x] = 13.56 / 13.56^2 = 0.0737$$

- (g) How would your answers to question (d) change if you were instructed to include the manager’s hourly wage (manager has a fixed hourly rate of \$32/hour).

Provide numeric answer for the **moderate** case. *Be sure to address expected value, variance and coefficient of variation.* [3 marks].

Let  $y$  be the fixed hourly rate of 32

$$E[x + c] = E[x] + y$$

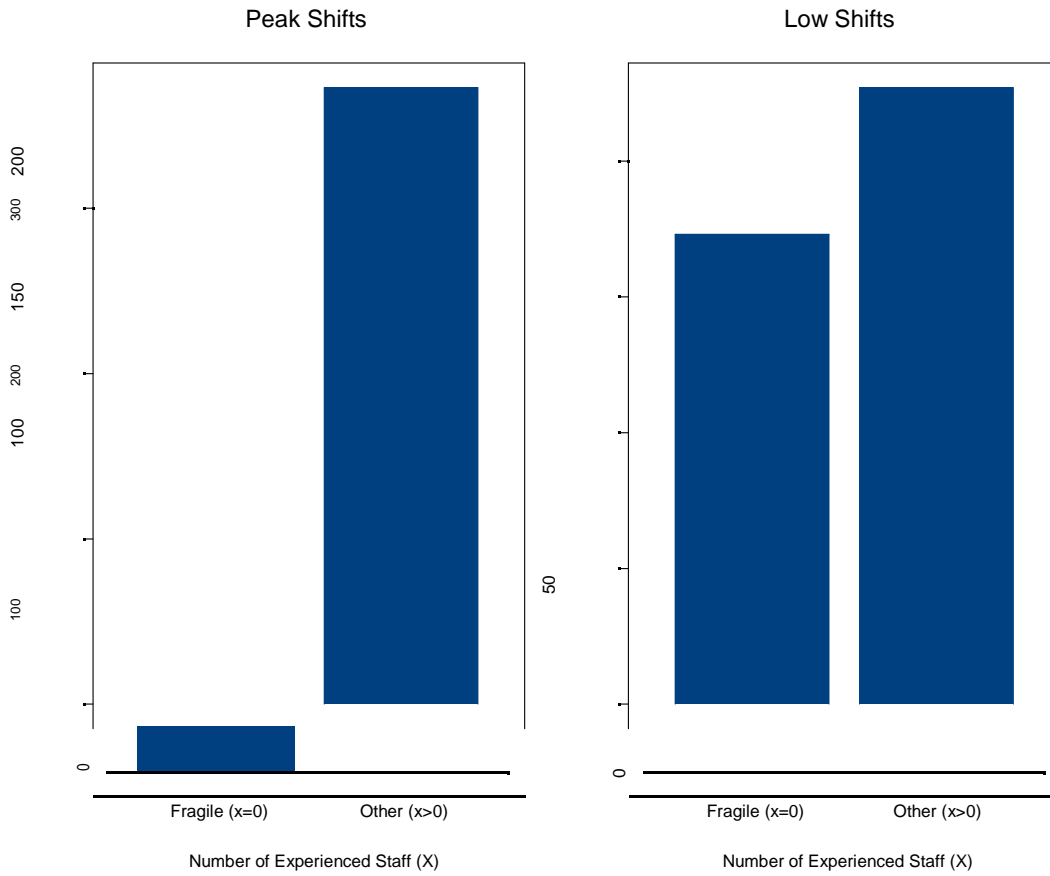
$$SD[x + c] = SD[x]$$

$$E[TW/h] = + y = 87.5 + 32 = 119.5$$

$$SD[TW/h] = 10.58$$

$$\text{Coefficient of Variation: } SD[x] / \text{Var}[x] = 10.58 / 10.58^2 = .0945$$

[THERE ARE NO OTHER QUESTIONS]



**Figure 1: Your sister had data from 400 peak shifts and 400 low shifts. She computed the number of "fragile" shifts ( $x=0$ ) among the peak and then among the low shifts, producing the barcharts in this figure. Note that peak shifts (total staff,  $n=12$ ) and low shifts (total staff,  $n=4$ )**

[PDF to Word](#)