91.266B Mid-term Examination
February 27, 2002 16:40 – 17:50
Time: 1 hr. 10 min

Lab Section: ________________

Time: ________________

Instructions

- Write you name, student number, lab section and time on ***this question paper***, and also write your name and student number at the top of every page.
- Write your answer in the space provided below the question. If additional writing space is required, you may write on the back of (blank) adjacent pages.
- The weight of each question is provided at the beginning of the question.
- If you feel that any data is missing or in error, make an assumption, state it clearly, and proceed. ***DO NOT ASK QUESTIONS.***
- All work done for marks must be neat and well organized; marks may be deducted for very untidy or disorganized work. Show all important work; wrong answers with no work shown will be awarded zero marks.
- No formula sheet, notes or textbooks are allowed during the examination.

Marks

1	2	3	4	5	6	7	8	9	10	B1	Total

1. (5%) The computer represents a floating-point in three parts: the sign, fraction part and exponent part. Convert the floating point number below from **Base 2 to Base 10. Round your answer to four digits.** Note the significand and exponent for the number below are in the binary format.

$+.101011 * 2^{-1011} =$

$$=\left[1\left(\tfrac{1}{2}\right)+0\left(\tfrac{1}{4}\right)+1\left(\tfrac{1}{8}\right)+0\left(\tfrac{1}{16}\right)+1\left(\tfrac{1}{32}\right)+1\left(\tfrac{1}{64}\right)\right] \times 2^{-(1\times2^0+1\times2^1+0\times2^2+1\times2^3)}$$

$$= 0.671875 \times 2^{(-11)} = 3.28064\times10^{-4}$$

$$= 0.3281 \times 10^{-3}.$$

2. (15%) Find the root of the following equation using **Newton's Method. Only complete 2 iterations.**

$y(x) = x^2 + x - 2$ Use the starting value of $x = 2$

$y' = 2x+1$

$i=0$ $x_0 = 2.$

$f(x_0) = 2^2+2-2 = 4.$

$f'(x_0) = 2\times2+1 = 5$

$x_1 = 2 - \frac{4}{5} = 1.2$

$i=1$ $x_1 = 1.2$

$f(x_1) = 1.2^2 + 1.2 - 2 = 0.64.$

$f'(x_1) = 2\times1.2+1 = 3.4$

$x_2 = 1.2 - \frac{0.64}{3.4} = 1.0118.$

$f(x_2) = (1.0118)^2 + (1.0118) - 2 = 0.0355$

3. (15%) Find the root of the following equation using the **Secant Method**. **Only complete 2 iterations.**

$y(x) = x^2 + \ln(x) - 2$ Use the starting values (1.0, 1.5)

$i=0$. $x_0 = 1.0$ $y(1.0) = 1.0^2 + \ln(1.0) - 2 = -1$

$x_1 = 1.5$ $y(1.5) = 1.5^2 + \ln(1.5) - 2 = 0.6555$

$$x_2 = 1.5 - 0.6555 \times \frac{(1.5 - 1.0)}{(0.6555 + 1)} = 1.3020$$

$f(x_2) = 1.302^2 + \ln(1.302) - 2 = -0.0409$

$i=1$ $x_0 = 1.5$ $y(1.5) = 0.6555$

$x_1 = 1.302$ $y(1.302) = -0.0409$

$$x_2 = 1.302 - (-0.0409) \times \frac{(1.302 - 1.5)}{(-0.0409 - 0.6555)}$$

$= 1.3136$

$f(x_2) = 1.3136^2 + \ln(1.3136) - 2 = -0.0017$

OR.

$i=2$ $x_0 = 1.0$ $y(1.0) = -1$

$x_1 = 1.302$ $y(1.302) = -0.0409$

$$x_2 = 1.302 - (-0.0409) \times \frac{(1.302 - 1.0)}{(-0.0409 + 1)} = 1.3149$$

$f(x_2) = 1.3149^2 + \ln(1.3149) - 2 = 0.0027$

4. (12%) The augmented matrix below is the representation of a set of 3x3 linear equations with the unknown variables (x_1, x_2, x_3). Find the solution using **Gauss-Jordan Method** with partial-pivoting.

$$\begin{bmatrix} 1.000 & -0.250 & 0.500 & 3.000 \\ 0.000 & 2.500 & 6.000 & 14.000 \\ 0.000 & -5.500 & 0.000 & 5.000 \end{bmatrix}$$

Switch Row 2 & 3.

$$\begin{bmatrix} 1 & -0.250 & 0.5 & 3.0 \\ 0 & -5.5 & 0 & 5.0 \\ 0 & 2.5 & 6 & 14.0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -0.250 & 0.5 & 3.0 \\ 0 & 1 & 0 & -0.9091 \\ 0 & 2.5 & 6 & 14.0 \end{bmatrix} R_2/(-5.5)$$

$$\begin{array}{l} R_1 - (-0.250)R_2 \\ \\ R_3 - (2.5)R_2 \end{array} \begin{bmatrix} 1 & 0 & 0.5 & 2.7727 \\ 0 & 1 & 0 & -0.9091 \\ 0 & 0 & 6 & 16.2728 \end{bmatrix} \Rightarrow R_3/(6) \begin{bmatrix} 1 & 0 & 0.5 & 2.7727 \\ 0 & 1 & 0 & -0.9091 \\ 0 & 0 & 1 & 2.7121 \end{bmatrix}$$

$$R_1 - R_3(0.5) \begin{bmatrix} 1 & 0 & 0 & 1.4167 \\ 0 & 1 & 0 & -0.9091 \\ 0 & 0 & 1 & 2.7121 \end{bmatrix}$$

Solution. $(x_1, x_2, x_3) = (1.4167, -0.9091, 2.7121)$

5. (5%) Find the sol ition of the augmented matrix below, which is the representation of a set of 3x3 linear equations with the unknown variables (x_1, x_2, x_3) after the elimination phase using Gaussian Elimina ion Method.

$$\begin{bmatrix} -4.000 & 3.000 & -2.000 & -10.000 \\ 0.000 & -4.500 & 0.000 & 6.000 \\ 0.000 & 0.000 & 6.000 & 19.667 \end{bmatrix}$$

$x_3 = \frac{19.667}{6} = 3.2778$

$x_2 = 6/(-4.5) = -1.3333$

$x_1 = [-10 - 3(-1.3333) + 2(3.2778)] / (-4) = -0.1389$

$(x_1, x_2, x_3) = (-0.1389, -1.3333, 3.2778)$

6. (5%) The matrices below are the coefficient matrix of a linear 3x3 equation in the form [A]{x}={y} and ts inverse. Calculate the condition number of this system of equations using the first norm, and comment whether it is well-conditioned or ill-conditioned.

$$[A] = \begin{bmatrix} -7.000 & 2.000 & -2.000 \\ 1.000 & -6.000 & 0.500 \\ 2.000 & 1.000 & 7.000 \end{bmatrix} \quad , \quad [A]^{-1} = \begin{bmatrix} -0.150 & 0.042 & -0.046 \\ -0.021 & -0.159 & 0.005 \\ 0.046 & 0.011 & 0.155 \end{bmatrix}$$

$\Sigma|a| = 10 \quad 9 \quad 9.5$ $\qquad \Sigma|a| = 0.217 \quad 0.212 \quad 0.206$

Condition Number = $10 \times 0.217 = \underline{2.17}$

Well-conditioned

7. (15%) Using Gauss-Seidel Method, carry out two iterations of calculation for the following set of 3x3 linear equations. Start with (x_1, x_2, x_3) = (0,0,0)

$$\begin{aligned} -5x_1 + x_2 - x_3 &= -10 \\ x_1 + 2x_2 + 10x_3 &= 10 \\ 2x_1 - 8x_2 + x_3 &= 11 \end{aligned}$$

Iteration	x_1	x_2	x_3
0	0.000	0.000	0.000
1	2	-0.875	0.975
2	1.630	-0.846	1.006

$x_1 = \frac{-10}{-5} - \frac{x_2}{-5} + \frac{x_3}{-5}$

$= 2 + 0.2x_2 - 0.2x_3$

$x_2 = -\frac{1}{8}(11 - 2x_1 - x_3)$

$= -1.375 + 0.25x_1 + 0.125x_3$

$x_3 = \frac{1}{10}(10 - x_1 - 2x_2)$

$= 1 - 0.1x_1 - 0.2x_2$

$x_1 = 2 + 0.2x_2 - 0.2x_3$

$x_2 = -1.375 + 0.25x_1 + 0.125x_3$

$x_3 = 1 - 0.1x_1 - 0.2x_2$

i = 1. $x_1 = 2$

$x_2 = -1.375 + 0.25(2) = -0.875$

$x_3 = 1 - 0.1(2) - 0.2(-0.875) = 0.975$

i = 2. $x_1 = 2 + 0.2(-0.875) - 0.2(0.975) = 1.630$

$x_2 = -1.375 + 0.25(1.63) + 0.125(0.975) = -0.846$

$x_3 = 1 - 0.1(1.63) - 0.2(-0.846) = 1.006$

8. (10%) Given 4 data pairs, interpolate the value of y at x = 1 using Lagrangian Polynomial.

x_i	0.0	2.0	3.0	4.0
y_i	1.00	7.00	26.50	65.00

$$y(1) = \frac{(1-2)(1-3)(1-4)}{(0-2)(0-3)(0-4)}1.0 + \frac{(1-0)(1-3)(1-4)}{(2-0)(2-3)(2-4)}7.0 + \frac{(1-0)(1-2)(1-4)}{(3-0)(3-2)(3-4)}26.50$$

$$+ \frac{(1-0)(1-2)(1-3)}{(4-0)(4-2)(4-3)}65.0$$

$$= 0.25 + 10.5 + (-26.50) + 16.25 = 0.5$$

9. (10%) Complete the table using Neville's Method for column P_{i3} and interpolate for y at x = 2 using 2nd degree polynomial interpolation. Note that the data set have been arranged in the order closest to x = 2.

i	x_i	$\|x-x_i\|$	$f_i = P_{i0}$	P_{i1}	P_{i2}	P_{i3}
0	2.4	0.4	12.579	3.872	6.395	6.929
1	3.0	1.0	25.641	13.963	5.060	6.770
2	0.8	1.2	-0.051	17.524	10.874	7.241
3	3.4	1.4	38.028	24.175	16.324	
4	-0.4	2.4	0.427	-0.501		
5	-1.0	3.0	0.659			

x = 2. y = 6.395 using 2nd degree polynomial interpolation.

$$P_{03} = \frac{(2-2.4)5.06 + (3.4-2.0)6.395}{3.4-2.4} = 6.929.$$

$$P_{13} = \frac{(2-3.0)(10.874) + (-0.4-2.0)5.06}{(-0.4-3.0)} = 6.770.$$

$$P_{23} = \frac{(2-0.8)16.324 + (-1.0-2.0)10.874}{(-1.0-0.8)} = 7.241$$

The ordina y difference table with evenly spaced data below has been calculated for pairs of data set. If the interpolation is carried out for the 3^{rd} degree polynomial using an appropriate row of the table that centered the data set for x = 1.2, calculate the estimated error using the **Next-Term Rule** for the interpolation at x = 1.2

i	x_i	$f_i = \Delta^0 f_i$	$\Delta^1 f_i$	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$	$\Delta^5 f_i$	$\Delta^6 f_i$
0	1.0	1.00	0.47	2.26	9.38	13.88	9.38	2.25
1	1.5	1.47	2.73	11.63	23.25	23.25	11.63	
2	2.0	4.20	14.36	34.88	46.50	34.88		
3	2.5	18.56	49.24	81.38	81.38			
4	2.0	67.80	130.62	162.76				
5	2.5	198.42	293.38					
6	3.0	491.80						

Use row $i=1$, $h=0.5$, $x_0=0.5$

$s = \frac{1.2-0.5}{0.5} = 1.4$

Estimate error $= \frac{s(s-1)(s-2)(s-3)\Delta^4 f_1}{4!} = \frac{1.4(1.4-1)(1.4-2)(1.4-3)23.25}{4\times 3\times 2\times 1}$

$= 0.5208$

BONUS

B1.(5%) Write the Matlab command lines required to solve for the following set of 3x3 linear equations to obtain the numerical solution.

$$-5x_1 + x_2 - x_3 = -10$$
$$x_1 + 2x_2 + 10x_3 = 10$$
$$2x_1 - 8x_2 + x_3 = 11$$

1)
```
a = [-5,1,-1; 1 2,10; 2,-8,1]
b = [-10; 10; 11]      or    c = [-10, 10, 11].
x = a\b                x=a\c'
x = a^-1*b,  x = a^-1*c'
x = inv(a)*b ,  x = inv(a)*c'
```

2).
```
[x1,x2,x3] = solve('-5*x1+x2-x3=-10','x1+2*x2+10*x3=10',
               '2*x1 -8*x2 + x3 =11')
x = numeric([x1,x2,x3])
```