

1. *Confidence interval for  $\mu$  when  $\sigma$  is known*

Suppose  $n = 9$  people are selected at random from a large population. Assume the heights of the people in this population are normal, with mean  $\mu = 68.71$  inches and  $\sigma = 3$  inches. Simulate the results of this selection 20 times and in each case find a 90% confidence interval for  $\mu$ . The following commands may be used:

MTB > random 9 c1-c20;

SUBC> normal 68.71 3.

MTB > zinterval 0.90 3 c1-c20

- a. [3] How many of your intervals contain  $\mu$ ? I had 16, but any number greater than 14 is acceptable.
- b. [3] What is the probability that 100 (not 20) such intervals would contain  $\mu$ ? 0.90.
- c. Do all the intervals have the same width? [1] Yes. Why [1] (what is the theoretical width)?  $2z_{0.05}(3/\sqrt{9}) = 2 * (1.645) = 3.29.$
- d. [2] Suppose you constructed 89% intervals instead of 90%. Would they be narrower or wider? Narrower.
- e. [2] How many of your intervals contained the value 71? I had 7(any number in [5,15] is acceptable).
- f. Suppose you took samples of size  $n = 4$  instead of  $n = 9$ . Would you expect more or fewer intervals to contain 71? [2] More. What about 68.71? [2] Same. What about the width of the intervals for  $n = 4$ : Would they be narrower or wider than for  $n = 9$ ? [2] Wider.

2. *Confidence interval for  $\mu$  when  $\sigma$  is NOT known*

Repeat the simulation of Question 3 but now assume  $\sigma$  is unknown and use the tintervals command to get the 20 90% intervals:

MTB > random 9 c1-c20;

SUBC > normal 68.71 3.

MTB > tinterval 0.90 c1-c20

- a. [3] How many of your intervals contain  $\mu$ ? I had 17 but any number > 14 is acceptable.
- b. Would you expect all 20 of the intervals to contain  $\mu$ ? [2] No. Why? [2] Expected is  $(20)(0.90)=18.$
- c. Do all the intervals have the same width? [2] No. Why (what is the theoretical width)? [2]  $2(t_{0.05})s/\sqrt{n}$ , where  $s$  is changed from sample to sample .
- d. [3] Suppose you took 95% intervals instead of 90%. Would they be narrower or wider? Wider.

e. [2] How many of your intervals contain the value 71? I had 6, but any number between 5 and 16 is acceptable.

f. Suppose you took samples of size  $n = 64$  instead of  $n = 9$ . Would you expect more or fewer intervals to contain 71? [1] Fewer. What about 68.71? [1] Same. What about the width of the intervals for  $n = 64$ : Would they be narrower or wider than for  $n = 9$ ? [1] Narrower.

3. *Hypothesis testing for  $\mu$  when  $\sigma$  is known*

Imaging choosing  $n = 16$  women at random from a large population and measuring their heights. Assume that the heights of the women in this population are normal with  $\mu = 63.8$  inches and  $\sigma = 3$  inches. Suppose you then test the null hypothesis  $H_0 : \mu = 63.8$  versus the alternative that  $H_a : \mu \neq 63.8$ , using  $\alpha = 0.10$ . Assume  $\sigma$  is known. Simulate the results of doing this test 30 times as follows:

MTB > random 16 c1-c30;

SUBC > normal 63.8 3.

MTB > ztest 63.8 3 c1-c30

a. [3] In how many tests did you reject  $H_0$ . That is, how many times did you make an “incorrect decision”? I had 3 p-values less than 0.10, but any number  $\leq 8$  is acceptable.

b. [2] Are the  $p$ -values all the same for the 30 tests? No.

c. Suppose you used  $\alpha = 0.001$  instead of  $\alpha = 0.10$ . Does this change any of your decisions to reject or not? [2] Yes, some may report No. In general, should the number of rejections *increase* or *decrease* if  $\alpha = 0.001$  is used instead of  $\alpha = 0.10$ ? [2] Decrease.

d. Now assume that the population really has a mean of  $\mu = 63$ , instead of 63.8, and carry out the above 30 simulations, (thus, use the above minitab commands with ‘normal 63.8 3’ changed to ‘normal 63 3’). Once again, using  $\alpha = 0.10$  and assuming  $\sigma$  known, in how many tests did you reject  $H_0$ ? [2] I had 12 p-values less than 0.10, but any number in [3,22] is acceptable.

A rejection of  $H_0$  in part (a) is a “correct decision”. True or False? [1] False

A rejection of  $H_0$  in part (d) is a “correct decision”. True or False? True [1]

4. *Hypothesis testing for  $\mu$  when  $\sigma$  is NOT known*

Repeat Question 1, using *ttest* instead of *ztest*, and answer parts (a), (b), and (c) again. (Thus ‘ztest 63.8 3 c1-c30’ changes to ‘ttest 63.8 c1-c30’)

a. [3] In how many tests did you reject  $H_0$ . That is, how many times did you make an “incorrect decision”? I had 4 but any number  $\leq 9$  is Okay .

b. [3] Are the  $p$ -values all the same for the 30 tests? No.

c. Suppose you used  $\alpha = 0.00008$  instead of  $\alpha = 0.10$ . Does this change any of your decisions to reject or not? [2] Yes, some may report No. In general, should the number of rejections *increase* or *decrease* if  $\alpha = 0.00008$  is used instead of  $\alpha = 0.10$ ? [2] Decrease.

**ALSO do the following questions:**

5. In an attempt to compare the starting salaries for university graduates who majored in education and the social sciences, random samples of 100 recent university graduates were selected from each major and the following sample information was obtained:

Major	Mean	ST. Dev.
Education	\$50,554	\$2225
Social Science	\$48,348	\$2375

a. [5] Conduct an appropriate hypothesis test at the 5% level of significance to determine if there is a difference in the average starting salaries for all university graduates who majored in education and the social sciences.

Solution: The hypotheses are

$$H_0 : \mu_1 - \mu_2 = 0, \quad H_a : \mu_1 - \mu_2 \neq 0$$

Since our sample sizes are greater than 30, the test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(50554 - 48348) - 0}{\sqrt{\frac{2225^2}{100} + \frac{2375^2}{100}}} = 6.78$$

At  $\alpha = 0.05$ , the critical values are  $z = \pm 1.96$ .  $H_0$  is rejected since  $6.78 > 1.96$

b. [3] Calculate the p-value.

Solution:

$$\text{p-value} = 2P(Z > 6.78) \approx 0$$

c. [4] Find the 95% confidence interval for the confidence interval for the difference in the average starting salaries for all university graduates who majored in education and the social sciences.

Solution: Compute 95% confidence interval as follows

$$(\bar{x}_1 - \bar{x}_2) \pm z_{0.025} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (50554 - 48348) \pm 1.96 \sqrt{\frac{2225^2}{100} + \frac{2375^2}{100}} \rightarrow (1568, 2844)$$

6. [8] Online ads targeting children were said to have decreased at least 56% between May and August 2000. A sample of 100 ads selected during this period showed a 45% decrease in online ads targeting children. State the null and alternative hypotheses and carry out the test at the 1% level of significance.

Solution:

We wish to test  $H_0 : p = 0.56$  vs.  $H_a : p < 0.56$ .

$$\text{Test statistic is } z = \frac{\hat{p} - p_0}{\sqrt{p_0(1 - p_0)/n}} = \frac{0.45 - 0.56}{\sqrt{0.56(1 - 0.56)/100}} = -2.22$$

We are unable to reject  $H_0$  since  $-z_{0.01} = -2.33$  and test statistic is not less than this value

7. A fast food franchiser is considering building a restaurant at a certain location. According to a financial analysis, a site is acceptable only if the number of pedestrians passing the location averages more than 100 per hour. A random sample of 50 hours produced  $\bar{x} = 110$  and  $s = 12$  pedestrians per hour.

a. [5] Do these data provide sufficient evidence to establish that the site is acceptable? Use  $\alpha = 0.05$ .

$H_0 : \mu = 100$  vs.  $H_a : \mu > 100$ , since  $n$  is large enough, test statistic is

$$z = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{110 - 100}{12/\sqrt{50}} = 5.89,$$

null hypothesis is rejected since  $5.89 > 1.645 = z_{0.05}$

**b.** [3] What are the consequences of Type I and Type II errors? Which error is more expensive to make?

Type I error is rejecting  $H_0$  when it is true and the result of this error is to construct the site when there are not enough pedestrians passing the location. (bankruptcy). Type II error is to accept  $H_0$  when it is not true, so the site does not construct in location more than 100 pedestrians. (missing a good location). Type I error is more expensive. (It is possible type II error be more expensive. Accept if it supports with good explanation)

**c.** [2] Considering your answer in part (b), should you select  $\alpha$  to be large or small? Explain.

According to part (b)  $\alpha$  should be a small number.

**d.** [3] What assumptions about the number of pedestrians passing the location in an hour are necessary for your hypothesis test to be valid? Pedestrians selected randomly.

8. [7] An assembly line supervisor suspects that a difference exists between the proportions  $p_1$  and  $p_2$  of defective items produced by two different machines. Experience has shown that the proportion of defective items for each of the two machine is in the range 0.015 and 0.03. If the supervisor wants to estimate the difference in the proportions to within 0.005 of the true difference, with probability 0.95, what is the smallest number of items that must be randomly sampled from the output produced by each machine. The supervisor wants  $n_1 = n_2 = n$ .

Solution:

Take  $p_1 = p_2 = 0.03$ ,  $Z_{0.025} = 1.96$ ,  $n_1 = n_2 = n$

$$1.96\sqrt{\frac{p_1q_1}{n} + \frac{p_2q_2}{n}} \leq 0.005 \rightarrow 1.96\sqrt{\frac{2(0.03)(0.97)}{n}} \leq 0.005 \rightarrow n \geq 8943.2 \rightarrow n = 8944$$