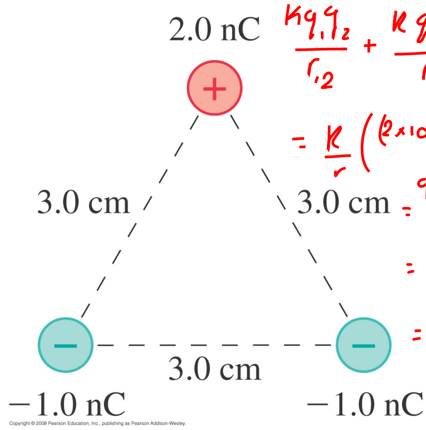
PHYS 1004
INTRODUCTORY ELECTROMAGNETISM AND WAVE MOTION
2013 Winter Term

Assigned Problems for Tutorial #4:
Electric Potential and Potential Energy, Capacitance, Current and Resistance

1. What is the electric potential energy of the group of charges in the figure below?



$\frac{Kq_1q_2}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}}$

$= \frac{K}{r}\left((2\times10^{-9})(-1\times10^{-9}) + (2\times10^{-9})(-1\times10^{-9}) + (-1\times10^{-9})(-1\times10^{-9})\right)$

$= \frac{9\times10^{9}}{3\times10^{-2}}\left(-2\times10^{-18} + -2\times10^{-18} + 1\times10^{-18}\right)$

$= \frac{9\times10^{9}}{3\times10^{-2}}(-3\times10^{-18})$

$= -9\times10^{-7}$ J

2. Two point charges 2.0 cm apart have an electric potential energy of $-180\ \mu$J. The total charge is 30 nC. What are the two charges? $PE = \frac{KQq}{r} \rightarrow -180\times10^{-6} = \frac{9\times10^{9}(Qq)}{2\times10^{-2}}$ Also, $Q_1 + Q_2 = 30\times10^{-9} \rightarrow Q_2 = 30\times10^{-9} - Q_1$

$(q_1-q_2)^2 = (q_1+q_2)^2 - 4(q_1q_2) \rightarrow (30\times10^{-9})^2 - 4(-4\times10^{-16}) \rightarrow 9\times10^{-16} + 1.6\times10^{-15} \rightarrow 2.5\times10^{-15}$ $\therefore q_1 - q_2 = 5\times10^{-8}$ Now $q_1 - q_2 = 5\times10^{-8}$, $q_1 + q_2 = 30\times10^{-9}$ $q_1 = 6\times10^{-8}$ C, $q_2 = -1\times10^{-8}$ C

3. The electric potential along the x-axis is $V = 100x^2$ V, where x is in metres. What is E_x at

$E_x = \frac{dV}{dx} \rightarrow \frac{200x}{1} \rightarrow 200x$

   a. $x = 0$ m? 0

   b. $x = 1$ m? 200

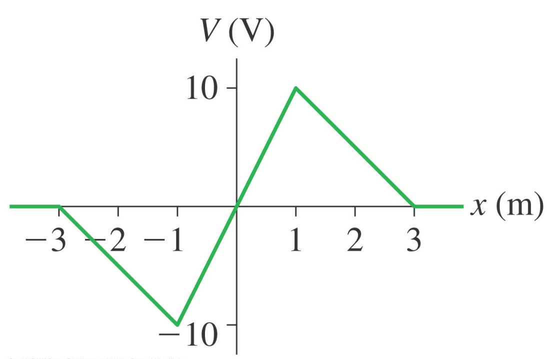
4. The figure below shows a graph of V versus x in a region of space. The potential is independent of y and z. What is E_x at

$E_x = \frac{V}{x}$

   a. $x = -2$ m? $E_x = -5$

   b. $x = 0$ m? $E_x = 0$

   c. $x = 2$ m? $E_x = 5$



5. Two 3.0-cm-diameter aluminum electrodes are spaced 0.50 mm apart. The electrodes are connected to a 100 V battery.

a. What is the capacitance? $C = \frac{\varepsilon_0 A}{d} \rightarrow \frac{(8.85\times10^{-12})(\pi(1.5\times10^{-2})^2)}{0.5\times10^{-3}} = 1.25\times10^{-11}\,F$

b. What is the magnitude of the charge on each electrode?

$C = \frac{Q}{V} \rightarrow Q = CV \rightarrow (1.25\times10^{-11})(110\,V) \rightarrow 1.25\times10^{-9}\,C$

6. Capacitor 2 has half the capacitance and twice the potential difference of capacitor 1. What is the ratio, U_{C1}/U_{C2}, of the energy stored in the capacitors?

$C_1 = C_1$, $V_1 = V_1$, $U_1 = \frac{1}{2}C_1V_1^2$ | $C_2 = \frac{1}{2}C_1$, $V_2 = 2V_1$, $U_2 = \frac{1}{2}C_2V_2^2$ | $\frac{U_1}{U_2} = \frac{\frac{1}{2}C_1V_1^2}{\frac{1}{2}C_2V_2^2}$ then sub to get $\frac{U_1}{U_2} = \frac{1}{2}$

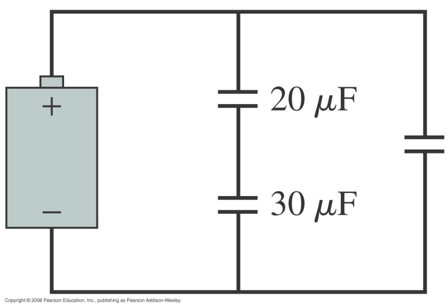
7. Two 4.0 cm × 4.0 cm metal plates are separated by a 0.20-mm-thick piece of Teflon (dielectric constant $\kappa = 2.1$ and dielectric strength $E_{max} = 6.0\times10^7$ V/m).

a. What is the capacitance? $C = \kappa\varepsilon_0\frac{A}{d} \rightarrow 2.1(8.85\times10^{-12})\left(\frac{0.0016}{2\times10^{-4}}\right) = 8.5\times10^{-11}\,F$

b. What is the maximum potential difference between the plates?

$V_{max} = Dd \rightarrow (6\times10^7)(0.2\times10^{-3}) \rightarrow 12000\,V$

8. What is the equivalent capacitance of the three capacitors in the figure below?



First reduce series combination:

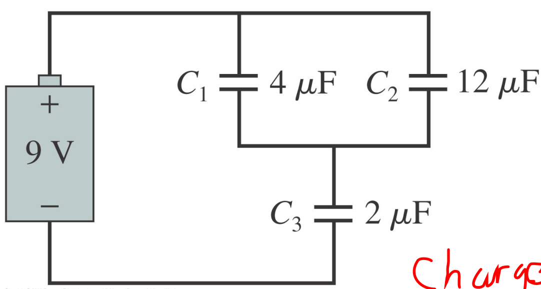
$\frac{1}{C_{eq.}} = \frac{1}{20} + \frac{1}{30} = 0.083$

$C_{eq} = 12\,\mu F$

reduce parallel

$12 + 25 = 37\,\mu F$

9. What are the charge on and the potential difference across each capacitor in the figure below?



First reduce parallel combination:

$C_{11} = C_1 + C_2 = 4 + 12 = 16\,\mu F$

Now reduce series combination:

$C_{series} = \frac{C_3C_{11}}{C_3 + C_{11}} = \frac{2(16)}{2+(6)} = 1.8\times10^{-6}\,F = 1.8\,\mu F$

Total charge:

$Q_{tot} = C_{series}V = (1.8\,\mu F)(9V) = 1.6\times10^{-5}\,C$

$V_1 = \frac{Q_{tot}}{C_3} = \frac{1.6\times10^{-5}}{2\times10^{-6}} = 8\,V$

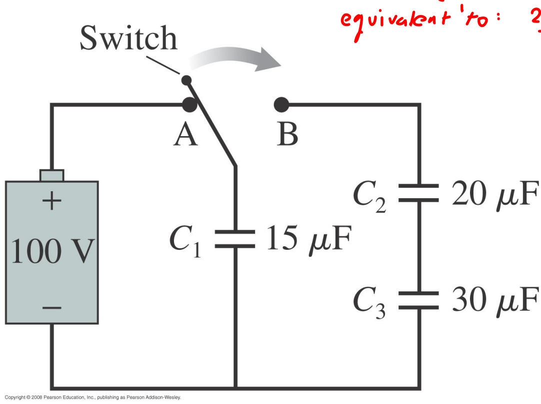
$V_2 = \frac{Q_{tot}}{C_{11}} = \frac{1.6\times10^{-5}}{16\times10^{-6}} = 1\,V$

Charges in parallel

$Q_{4\mu F} = C_1V_{11} = (4\times10^{-6})(1V) = 4\times10^{-6}\,C$

$Q_{25\mu F} = C_2V_{11} = (12\times10^{-6})(1V) = 12\times10^{-6}\,C$

10. Initially, the switch in the figure below is in position A and capacitors C_2 and C_3 are uncharged. Then the switch is flipped to position B. Afterward, what are the charge on and the potential difference across each capacitor?



11. In a classic model of the hydrogen atom, the electron moves around the proton in a circular orbit of radius 0.053 nm.

a. What is the electron's orbital frequency?

b. What is the effective current of the electron?

12. The starter motor of a car engine draws a current of 150 A from the battery. The copper wire ($n_e = 8.5 \times 10^{28}$ m^{-3}) to the motor is 5.0 mm in diameter and 1.2 m long. The starter motor runs for 0.80 s until the car engine starts.

a. How much charge passes through the starter motor?

b. How far does an electron travel along the wire while the starter motor is on?

13. Variations in the resistivity of blood can give valuable clues about changes in various properties of the blood. Suppose a medical device attaches two electrodes into a 1.5-mm-diameter vein at positions 5.0 cm apart. What is the blood resistivity if a 9.0 V potential difference causes a 230 μA current through the blood in the vein?

14. A hollow metal sphere has inner radius a, outer radius b, and conductivity σ. The current I is radially outward from the inner surface to the outer surface.

a. Find an expression for the electric field strength inside the metal as a function of the radius r from the centre.

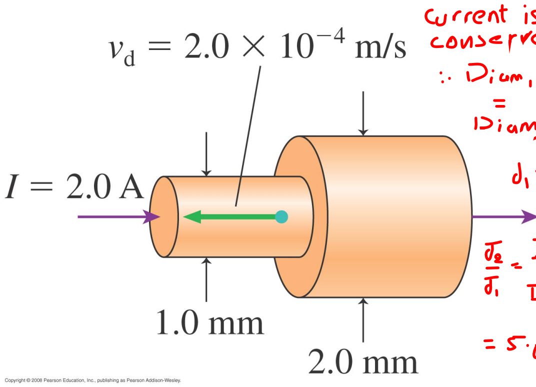
b. Evaluate the electric field strength at the inner and outer surfaces of a copper sphere if $a = 1.0$ cm, $b = 2.5$ cm, and $I = 25$ A.

15. The total amount of charge that has entered a wire at time t is given by the expression $Q = (20\ C)(1 - e^{-t/(2.0\ s)})$, where Q is in Coulombs, t is in seconds and $t \geq 0$.

a. Find an expression for the current in the wire at time t.

b. What is the maximum value of the current?

16. Two wires in the figure below are made of the same material. What are the current and the electron drift speed in the 2.0-mm-diameter segment of the wire?



Current is conserved

∴ Diam₁ Current = Diam₂ Current $I_1 = I_2 = 2.0$ A

$J_1 = nev_{d_1}$ $J_2 = nev_{d_2}$

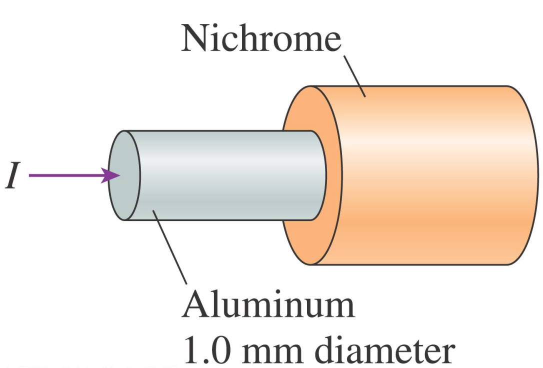
$\frac{J_2}{J_1} = \frac{I_2/A_2}{I_1/A_1} = \frac{A_1}{A_2} \Rightarrow v_{d_2} = \frac{A_1}{A_2} v_{d_1} = \left(\frac{D_1}{D_2}\right)^2 v_{d_1} = \left(\frac{1.0\,mm}{2.0\,mm}\right)^2 (2.0\times10^{-4}\,m/s)$

$= 5.0\times10^{-5}$ m/s

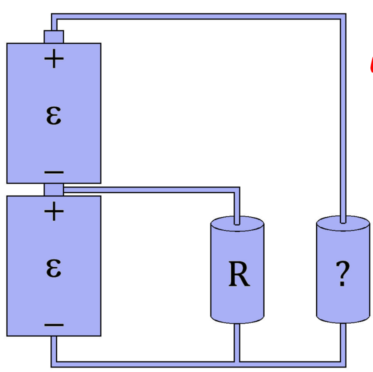
17. What diameter should the nichrome wire in the figure below be in order for the electric field strength to be the same in both wires? The resistivities of aluminum and nichrome are $\rho_{Al} = 2.8 \times 10^{-8}$ Ω m and $\rho_{Ni} = 1.5 \times 10^{-6}$ Ω m, respectively.

$E = \frac{I}{A\sigma} \rightarrow \frac{E_{nic}}{E_{alu}} = \left(\frac{I_{nic}}{I_{alu}}\right)\left(\frac{A_{alu}}{A_{nic}}\right)\left(\frac{\sigma_{alu}}{\sigma_{nic}}\right) = \left(\frac{D^2_{alu}}{D^2_{nic}}\right)\left(\frac{\sigma_{alu}}{\sigma_{nic}}\right)$

$D_{nic} = \sqrt{\frac{\sigma_{alu}}{\sigma_{nic}}}\, D_{alu} \rightarrow \sqrt{\frac{3.5\times10^{7}\,\Omega^{-1}m^{-1}}{6.7\times10^{5}\,\Omega^{-1}m^{-1}}}\,(1.0\,mm) = 7.22$ mm



18. The batteries in the figure below are identical. Both resistors have equal currents. What is the resistance of the resistor on the right?



$i_1 = i_2$

$i_1 = \frac{V_1}{R}$

$\frac{V_1}{R_1} = i_2 = \frac{2V_1}{R_2}$

$\therefore R_2 = 2R$

19. Two 10-cm-diameter metal plates 1.0 cm apart are charged to ±12.5 nC. They are suddenly connected together by a 0.244-mm-diameter copper wire ($\rho = 1.7 \times 10^{-8}$ Ω m) stretched taut from the centre of one plate to the centre of the other.

a. What is the maximum current in the wire?

$C = \frac{\epsilon_0 A}{d} \rightarrow \frac{8.85\times10^{-12} \times \pi \times 0.0025}{0.01} = 6.95\times10^{-12}$ F

$Q = CV$, $V = \frac{Q}{C} = \frac{12.5\times10^{-9}}{6.95\times10^{-12}} = 1798$ V

$R = \frac{\rho \ell}{A} = \frac{1.7\times10^{-8} \times 0.01}{\pi (1.22\times10^{-4})^2} = 0.004314$ Ω

$I = \frac{\mathcal{E}}{R} = \frac{1798}{0.004314} = 4.17\times10^{5}$ A

b. Does the current increase with time, decrease with time, or stay the same? Explain.

decreases w/ time as the C discharges.

c. What is the total amount of energy dissipated in the wire?

$E = \frac{1}{2}CV^2 \rightarrow \frac{1}{2}(6.95\times10^{-12})(1798\,V)^2 \rightarrow 1.12\times10^{-5}$

20. You've decided to protect your house by placing a 5.0-m-tall iron ($\rho = 9.7 \times 10^{-8}\ \Omega\ \text{m}$) lightning rod next to the house. The top is sharpened to a point and the bottom is in good contact with the ground. From your research, you've learned that lightning bolts can carry up to 50 kA of current and last up to 50 μs.

a. How much charge is delivered by a lightning bolt with these parameters?

$Q = I\Delta t \to (50\times10^{3}A)(50\times10^{-6}) \to 2.5C$

b. You don't want the potential difference between the top and bottom of the lightning rod to exceed 100 V. What minimum diameter must the rod have?

$R = \rho\frac{L}{A} \to R = \frac{V}{I} = \rho\frac{L}{A} \to A = L\rho\frac{I}{V} \to d^2 = \frac{4}{\pi}(5m)(9.7\times10^{-8})\left(\frac{50\times10^{3}A}{100V}\right)$

$d^2 = 3.088\times10^{-4}$

$d = 0.0176\,m$

$= 1.76\,cm$