Question 1 (20 marks)

Use Gaussian elimination and back substitution to obtain a solution to the system of equations given below. **Use row pivoting where this is appropriate** and show all steps. The space provided continues on the following page. Look at all of it before beginning your answer. You may or may not need all of the boxes provided.

$$2x - 10y + 4z = 54$$
$$3x + 20y - z = -73$$
$$-6x + 12y + 3z = -60$$

Initial Augmented Matrix:

3	20	-1	-73
2	-10	4	54
-6	12	3	-60

3	20	-1	-73
0	$-\frac{70}{3}$	$\frac{14}{3}$	$\frac{308}{3}$
6	52	1	-206

3	20	-1	-73
0	52	1	-206
0	0	$\frac{133}{26}$	$\frac{133}{13}$

Show your back substitution work here:

$\frac{133}{26}$ $z = \frac{133}{13} \Rightarrow z = 2$

$52y + (2)(1) = -206 \Rightarrow y = -4$

$3x + (20)(-4) + (-1)(2) = -73 \Rightarrow x = 3$

Final Answer: $x =$ 3 $y =$ -4 $z =$ 2

Is your answer correct? How can you tell?

$2(3) - (10)(-4) + (4 \times 2) = 54$ ✓

$-6(3) + 12(-4) + 3 \times 2 = -60$ ✓

$3(2) + 20(-4) - 2 = -73$ ✓

How would you find the solution if you had access to Matlab and were required to use LU decomposition (with the permutation matrix)? Give all of the necessary commands.

```
A=[2 -10 4;3 20 -1;-6 12 3]; b=[54;-73;-60];
[L,U,P]=lu(A);
d=L\(P*b)
sol=U\d

fprintf('x=%f y=%f z=%f',sol(1),sol(2),sol(3));
```

Question 2 (20 Marks)

An experiment has produced a series of 12 data points. Throughout this question you may assume that these data points are stored in two vectors called x and y. $x(1)$ and $y(1)$ are the first data point and so on.

Professor Perkins is convinced that the data is best represented by a curve of the form $y = k_1 * (x / (k_2 + x))$.

Professor Jones, on the other hand, is equally certain that a curve of the form $y = a_1 * e^{-2x} + a_2 * e^{-5x} + a_3$ would be better.

You have been called in to resolve the dispute.

(a) Give all of the Matlab code necessary to compute the best values for the unknown constants (k_1 and k_2) in Professor Perkins' formula and complete the output statement provided.

```
Xt = 1/x;
yt = 1/y;
p = polyfit(xt, yt, 1)
K1 = 1/p(2);
k2 = p(1)/p(2)
fprintf ('The best k1 and k2 for Prof Perkins formula are %f and %f\n', K1, K2);
```

(b) Give all of the Matlab code necessary to compute the best values for the unknown constants (a_1, a_2 and a_3) in Professor Jones' formula and complete the output statement provided.

```
x = [. . . .]; y = [. . . .]'

z(:,1) = exp(-2*x);
z(:,2) = exp(-5*x);
z(:,3) = 1
Sol = z\y

fprintf ('The best a1, a2 and a3 for Jones formula are %f, %f and %f\n', z(1), z(2), z(3));
```

(c) Briefly explain how you would go about deciding which of the two formulas best fits the data. No Matlab code is required but you should provide all necessary equations.

I woul go and calculate the correlation coffecient, for both curves and check which one is closest to 1

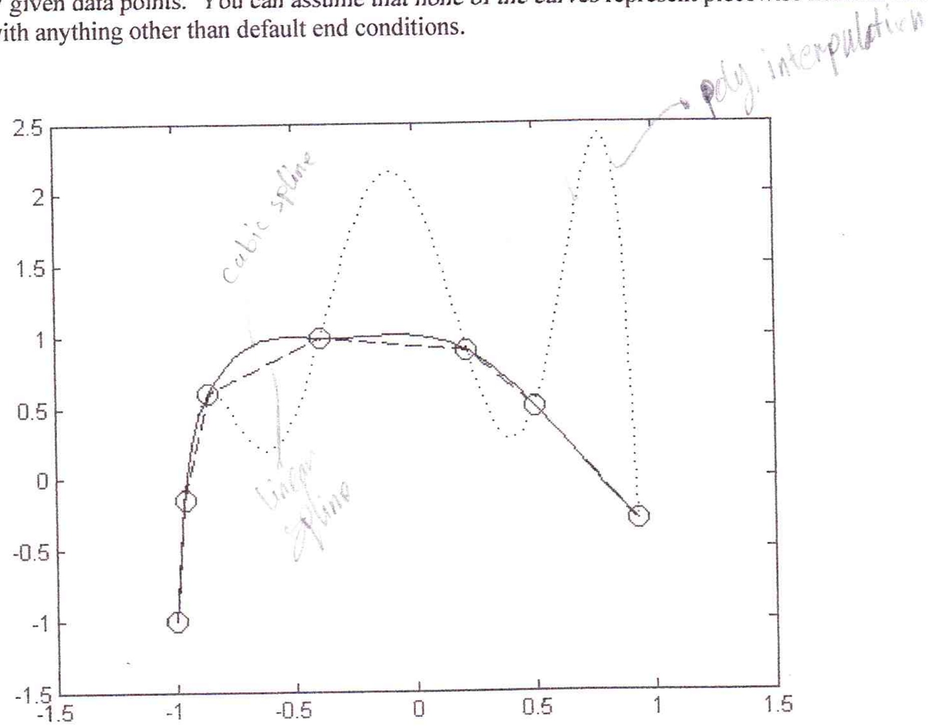
(d) Suppose you think that both Professor Perkins and Professor Jones are crazy and that the data is best represented by a simple second order polynomial. Give all of the Matlab code necessary to find this polynomial and complete the output statement provided.

```
x=[...];    y=[...];
P= Polyfit(x,y,2);
```

```
fprintf ('The best quadratic is %f x^2 + %fx + %f\n', p(1), p(2),p(3));
```

Question 3 (16 Marks)

The three curves (solid, dotted, and dashed) on the graph below represent three basic techniques for interpolating between 7 given data points. You can assume that none of the curves represent piecewise hermite cubic interpolation or a spline with anything other than default end conditions.



(a) Suppose that the data is stored in a 7 by 2 array called *data*. The first column contains the *x* values for the 7 data points and the second column contains the *y* values. Give all of the Matlab code required to generate two ROW vectors (to be called *x* and *y*) containing the *x* and *y* values. These vectors can be assumed throughout the remaining sections of this question.

Load data.txt

x = dat~~ta~~a(:,1)

y = data(:,2)

(b) Name the technique represented by the solid curve and give all of the Matlab code required produce a plot of the curve with the data points superimposed (i.e. a plot like one above but without the dashed and dotted lines).

it is a Cubic spline

Xin = -1 : 0.01 : 1 ;

yout = spline(x, y, xin) ;

Plot = (x, y, 'o', xin, yout -);

(c) Name the technique represented by the dashed curve and give all of the Matlab code required to produce a vector (to be called *myAnswer*) containing interpolated *y* values for *x* from -0.6 to 0.9 in steps of 0.1.

it islinear spline

```
xin = -0.6: 0.1: 0.9
myanswer = interp1(x,y,xin, 'linear');
```

(d) Name the technique represented by the dotted curve, give all of the Matlab code required to locate and determine its maximum value, and complete the output statement provided.

Polynomial interpulation

```
P= polyfit(x,y,7);
fitt= @(x) polyval(P,x);
g= @(x) -fitt(x);
xmax = fminbnd(g, 0.5, 1);
max y = f(xmax)
```

fprintf ('The maximum value occurs at x = %f, the maximum value is %f\n, Xmax, max y);

(e) Give the Matlab code required to estimate the value of *y* at *x* = -0.6 using all three techniques and complete the output statement provided.

```
y1 = spline(x,y, -0.6);
y2 = interp1(x,y, -0.6);
y3 = fitt(0.6);
```

fprintf ('Solid line gives %f, dashed curve gives %f, dotted curve gives %f\n', y1, y2, y3);

Question 4 (12 Marks)

Use polynomial interpolation and all of the data points below to manually estimate the value of $f(0.5)$.

x	-1	2	3	5
$f(x)$	-12	0	24	174

$\to 9$ $\quad n=3$

$\to 186$ $\quad n=4$

You may use either of the two possible techniques. You do not have to fully simply the polynomial. Just go as far as you have to get an answer.

-1	-12	4	5	~~3.25~~ 2
2	0	24	~~17~~	
3	24	75		
5	174			

$$\frac{(x-2)(x-3)(x-5)}{(-1-2)(-1-3)(-1-5)}\cdot(-12)+0$$

$$+\frac{(x+1)(x-2)(x-5)}{(3+1)(3-2)(3-5)}\cdot(24)$$

$$+\frac{(x+1)(x-2)(x-3)}{(5+1)(5-2)(5-3)}\cdot 174$$

$$@0.5 \Rightarrow \frac{-45}{16}-\frac{243}{8}+\frac{433}{16}=-6$$

$$x^2-1x-2$$

$$P(x)=-12+4(x+1)+5(x+1)(x-2)+\frac{13}{2}(x+1)(x-2)(x-3)$$

$$\cancel{-12+4x+5x^2-1x-8+2x^3-8x^2+2x+12}$$

$x^2 - x - 2$

$x^3 - 3x^2 - x + 3x - 2x + 6$

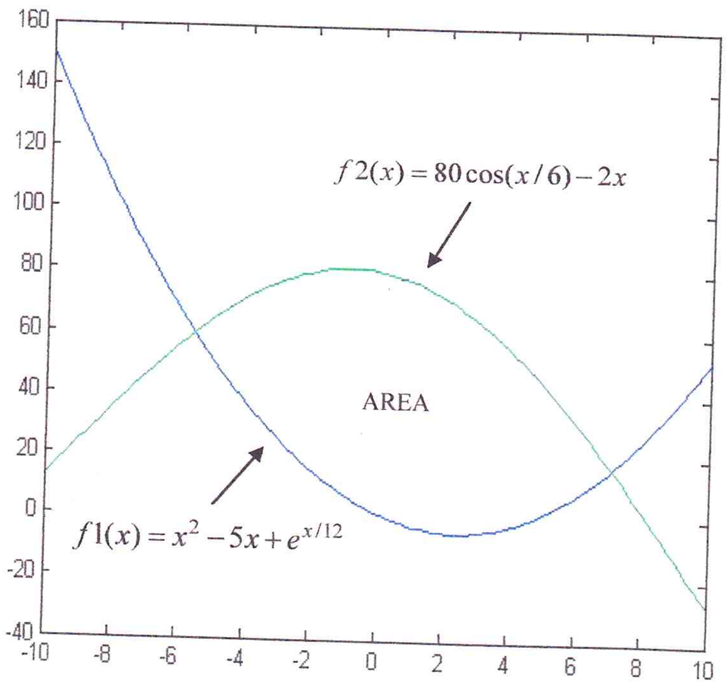
$x^3 - 3x^2$

$$\boxed{2x^3-3x^2+x-6}$$

$$@0.5=-6$$

Question 5 (12 Marks)

Give all of the Matlab code necessary to compute and output the area enclosed by the two curves in the plot below (i.e. the fully enclosed area with "AREA" written inside it). You MAY NOT just estimate the points at which the curves cross but must instead use appropriate techniques to precisely determine the x values at which the lines intersect.




```
F = @(x) (80*cos(x/6) - 2*x) -(x^2 - 5*x + exp(x/12))

left= fzero(F,-8,-4);
right=fzero(F,6,8);

Area = quad(F,left,right);

fprintf('the area is %f',Area);
```

Question 6 (20 Marks)

The table below gives values of some unknown function.

x	1.0	3.0	5.0	7.0	9.0
$f(x)$	-0.69315	3.6492	22.907	61.385	121.83
For parts	a,b,c	c	a,b	c	a,b,c

(a) Use trapezoidal integration and only the first and last data points (the points with an "a" below them), estimate

$$I = \int_1^9 f(x)dx$$

$\frac{8}{2}(-0.69315 + 121.83)$

$I =$ 484.5474

(b) Repeat the process using the first, third, and fifth data points (the points with a "b" below them).

$n=4$

$\frac{4}{2}(-0.69315 + 2 \times 22.907 + 121.83$

$I =$ ~~[illegible]~~

333.9017

(c) Repeat the process using all of the data points

$I =$ 148.509 (297.01

(d) Use your answers to parts (a), (b), and (c) to produce the best possible estimate of the value of the integral. Show your work and give the approximate relative error.

8	484.5474	283.6864	73.5797
4	333.9017	86.71433	
2	148.509		

$I =$ 73.5797 error = 0.1784

(e) Use as many of the data points as you find useful to estimate the derivative of the function at $x = 5$. The best possible estimate is required.

$[-121.83 + 8 \times 61.385 - 8 \times 3.6492 + (-0.69315)]$

$f'(5) =$ 339.36

(f) Suppose that you were told that the mystery function is actually $f(x) = \ln(x/2)x^2$. Use 2 point Gaussian quadrature to estimate

$$I = \int_1^9 f(x)dx$$

$$\approx \left(\frac{8}{2}\right)\left(1 f\left(\frac{10}{2} + \frac{8}{2}\cdot\frac{-1}{\sqrt{3}}\right) + 1 f\left(\frac{10}{2} + \frac{8}{2}\frac{1}{\sqrt{3}}\right)\right)$$

$$\approx \left(\frac{8}{2}\right)\left(f\left(\frac{15-4\sqrt{3}}{3}\right) + f\left(\frac{15+4\sqrt{3}}{3}\right)\right)$$

$$\approx 285.5595915$$