Final Exam ECON 3600, Winter 2012

NAME:

ID Number:

Signature:

Write your NAME and STUDENT ID NUMBER at the top of your exam and on each exam booklet that you use.

Before handing in your exam, sign each exam/booklet that you want graded.

This is a closed book exam.

Total time: 3 hours

There are 5 questions worth a total of 100 points.

Pay attention to the time you have left. If a problem is too difficult, skip it and move on, coming back to it if you have time.

It is in your interest to always show your work. We cannot give you partial credit if there is only a wrong answer with no explanation.

Question 1. Short-Answer Questions (20 points total)

State whether each of the following claims is true or false (or cannot be determined). For each explain your answer in (at most) **one short** paragraph. Each part is worth 5 points of which 4 points are the explanation. Explaining an example or counter example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

a. (5 Points) “In Stackelberg competition, first Firm 1 decides how much to produce, second Firm 2 (knowing how much Firm 1 is producing) makes their production decision. Thus, Firm 2 is better off because it can use this extra information to its advantage."

False. Firm 1 has the first mover advantage here. Firm 1 can force Firm 2 to produce less, by producing more, leading to higher profits.

b. (5 Points) “In extensive form games (with a game tree), all Nash equilibria are also subgame perfect Nash Equilibria because, by definition, no player has an incentive to change strategies in a Nash Equilibria."

False. There are non-credible Nash Equilibria, which are not SPE.

c. (5 Points) “If a players’ discount factor δ is close to one then we can sustain cooperation as an SPE of an infinitely repeated prisoner’s dilemma even using a grim trigger strategy.”

True. The higher the discount factor, the more likely it is for players to be able to sustain cooperation.

d. (5 Points) “In a War of Attrition, players are unlikely to both fight in the first round, but if they do both fight in the first round, then they are equally likely to both fight in the second round."

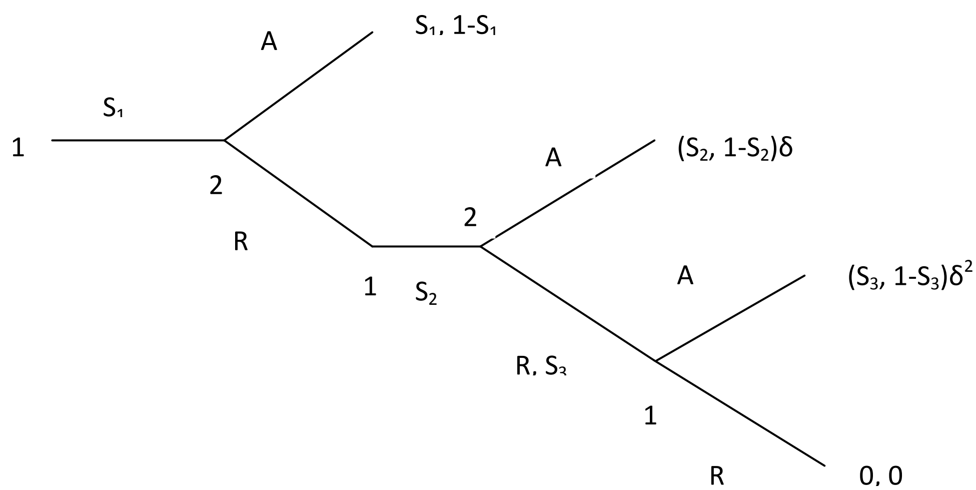
True. If both players are playing a mixed strategy equilibrium, then their probability of fighting is the same each round. (Also full credit if they say False and they explain that under

the condition C>V, they could both fight in the first round under a mixed strategy, but one quits in the second round.)

Question 2: (15 points) Bargaining

Consider the following discounted, three-period bargaining game. There are two players bargaining over $1. The discount factor of both players is δ, where 0 < δ < 1. In this game, Player 1 makers the first offer of how to split the dollar. If Player 2 rejects this offer, then Player 1 makes *another* offer. If Player 2 rejects the second offer, then Player 2 makes the final offer. If Player 1 rejects this final offer, they both get nothing. In other words, Player 1 makes offers in Periods 1 and 2, while Player 2 makes the offer only in Period 3.

a. Draw the extensive form of this game (5 points).



b. Compute the subgame perfect equilibrium of this game. Show your work (10 points).

In the last stage, Player 2 will take everything: 0 for Player 1 and δ^2 for himself. Player 1 must offer at least δ^2 to Player 2 in Period 2: Player 1 gets $\delta - \delta^2$ and Player 2 gets δ^2. Thus, Player 1 still only needs to offer Player 2 δ^2 in the first period. Player 1 will offer Player 2 δ^2 and $1-\delta^2$ for himself in Period 1 and both players will accept.

Question 3: (15 points) Infinitely Repeated Game

The following game is repeated infinitely and players 1 and 2 have the same discount factors δ and 0 ≤ δ ≤ 1.

Player 2

Player 1		C	D
	C	3, 3	-2, 6
	D	6, -2	0, 0

a. (2 points) What is the Nash equilibrium if the game is only played for one period?

D,D

b. (6 points) What is the lowest discount factor, δ, such that the Grim Trigger strategy induces the players to cooperate forever in equilbrium?

(6-3) ≤ (3/(1-δ) - 0)δ → 3 ≤ 3δ/(1-δ) → (1-δ) ≤ δ → 1/2 ≤ δ

c. (7 points) What is the lowest discount factor, δ, such that the Tit-for-Tat strategy induces both players to cooperate forever in equilbrium? (The Tit-for-Tat strategy is: Play C to start, then: Play C if either (C,C) or (D,D) were player last turn. Play D if either (C,D) or (D,C) were played last turn.) Does δ have to be larger or smaller in the Tit-for-Tat case compared to the Grim Trigger case to get cooperation?

(6-3) ≤ (3/(1-δ) - 0 - 3δ/(1-δ))δ → 3 ≤ 3δ → 1 ≤ δ

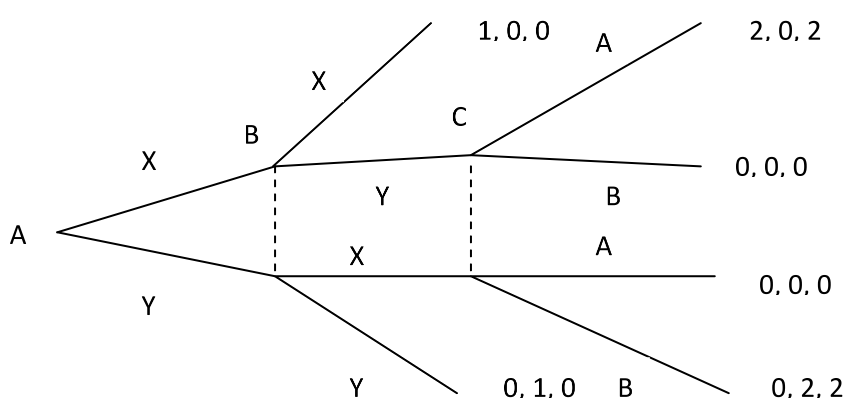
You can only get cooperation if 1 = δ. It is larger than in the Grim Trigger case.

Question 4. [25 total points] Asymmetree

Consider the following three player game. At the beginning of the game, simultaneously, player A and player B each choose X or Y. If both choose X, then the game ends and the payoff vector is (1,0,0); that is, player A gets 1 and the other two players get 0. If both choose Y then the game ends and the payoff vector is (0, 1, 0); that is, player B gets 1 and the other two players get 0.

If and only if A and B differ in their choices (i.e. one chooses X while the other chooses Y), then player C must guess which of the players chose X. That is, player C chooses between A and B. Player C makes his choice knowing only that the game did not end following the choices of players A and B. If player C guesses correctly which player played X, then player C and the player she (correctly) selected each get a payoff of 2 while the player who chose Y gets 0. If player C guesses incorrectly, then everyone gets 0.

a. (5 points) Draw the game tree for this game, being careful to indicate the players' information sets and payoffs.



b. (5 points) Find a (possibly mixed) NE in which player A chooses Y, or show that no such equilibrium exists.

For Player A, X weakly dominates Y. The only possible situation where Player A would play Y over X is if Player B plays Y and Player C plays B. In this situation, Player B would have an incentive to deviate to X, so this cannot be an equilibrium. Thus, there is no NE where Player A plays Y.

c. (5 points) Find a (possibly mixed) NE in which player B chooses Y, or show that no such equilibrium exists.

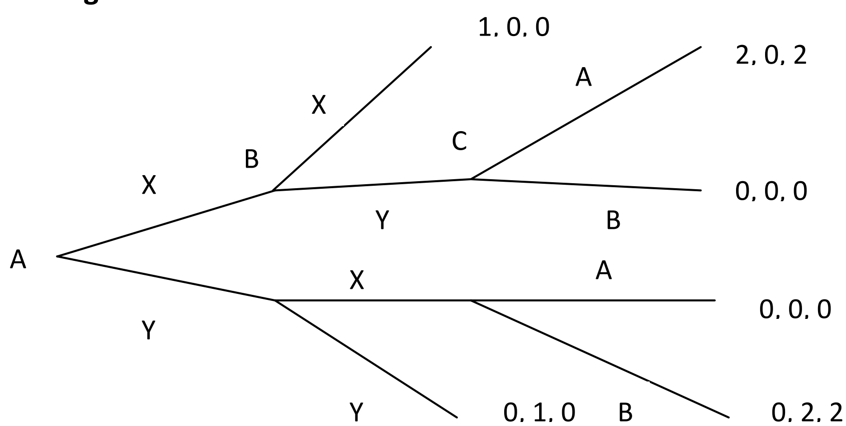
Knowing that A will always play X, Player B is indifferent about playing X or Y and Player C's weakly dominant strategy is to play A. Thus, there is a NE where Player A plays X, Player B plays Y and Player C plays A.

d. (5 points) Find a (possibly mixed) NE in which player C chooses B, or show that no such equilibrium exists.

The NE is Player A plays X, Player B plays X, and Player C plays B. (Notice nobody has an incentive to deviate.)

e. (5 points) Suppose that the game is now changed to one of perfect information with A moving first, then B, and then C (where C only moves if A and B differ in their choices, but where she is able to observe those choices). Explain how you would expect the game to proceed.

The game now looks like:

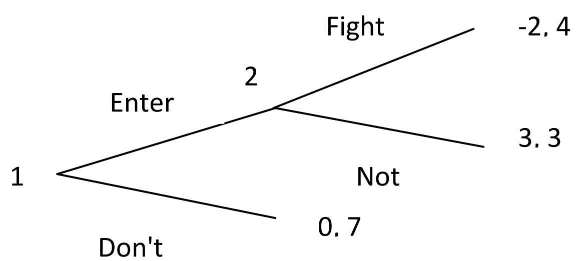


For Player A, X still weakly dominates Y. Thus, A will play X, B will either play X or Y, and C will play A.

Question 5. [25 total points] Entry Game

Firm 1 is thinking about entering a market where Firm 2 is the incumbent. Firm 1 has two strategies: Enter or Don't Enter. If Firm 1 does not enter, Firm 2 gets $7 and Firm 1 gets $0. If Firm 1 enters, Firm 2 can either Fight or Not. First, assume that Firm 2 is a **strong** firm. In this case, if Firm 1 enters and Firm 2 fights, Firm 1 has a payoff of -$2 (that is a loss of $2) and Firm 2 has a payoff of $4. If Firm 1 enters and Firm 2 does not fight, both firms get $3.

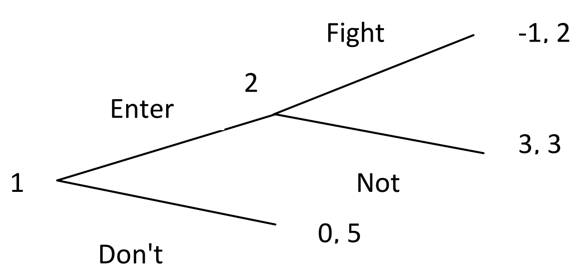
a. (5 points) Write out the extensive form of this game and explain the subgame perfect NE.



The SPE is Don't, Fight.

Second, assume that Firm 2 is a **weak** firm. In this case, if Firm 1 enters and Firm 2 fights, Firm 1 has a payoff of -$1 (that is a loss of $1) and Firm 2 has a payoff of $2. If Firm 1 enters and Firm 2 does not fight, both firms get $3. If Firm 1 does not enter, Firm 2 gets $5 and Firm 1 gets $0.

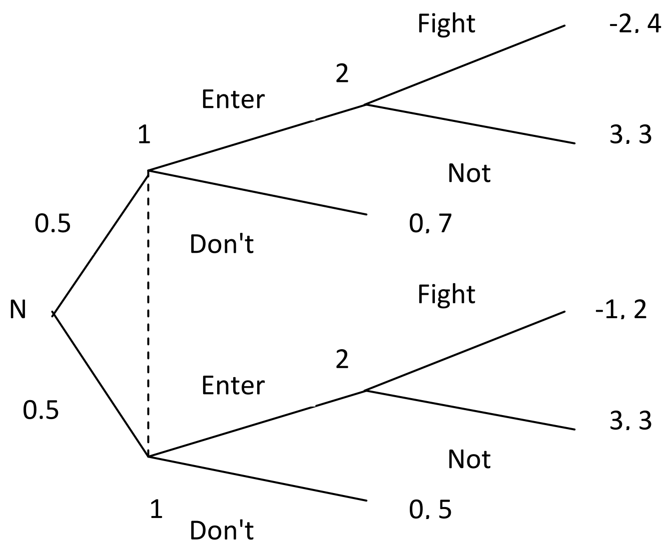
b. (5 points) Write out the extensive form of this game and explain the subgame perfect NE.



The SPE is Enter, Not.

Now assume that Firm 1 does not know whether Firm 2 is a **strong** firm or a **weak** firm, but Firm 1 believes that there is a 50% chance Firm 2 is a **strong** firm and a 50% chance Firm 2 is a **weak** firm.

c. (5 points) Write out the extensive form of this game and explain the subgame perfect NE.



In this case, if Firm 1 enters they have a 50% chance of being Fought (with payoff -2) and 50% chance of Not (with payoff 3). The expected value of entering is 0.5 which is greater than 0, so the SPE is Enter, Fight (if strong firm), Not (if weak firm).

Now assume Firm 2 is able to send a signal to Firm 1. It can choose to burn some of its capital. If it burns $X of capital, it loses $X in payoffs in every outcome (e.g. if a weak Firm 2 burned $5 of capital, it would have a payoff of $0 if Firm 1 does not enter, a payoff of -$3 if Firm1 enters and Firm 2 Fights, and a payoff of -$2 if Firm 1 enters and Firm 2 does not fight). Firm 1's payoffs are unaffected by this burning of capital, but Firm 1 observes how much capital is burned before Firm 1 makes a decision to enter or not.

d. (5 points) Consider the following strategies: {Firm 1 do not enter if $1 is burned, otherwise enter. Strong Firm 2's burn $1 and fight. Weak Firm 2's do not burn anything and do not fight.} (With the belief by Firm 1 that seeing $1 burned means it is for sure a strong Firm 2 and not seeing this means it is for sure a weak Firm 2.) Explain whether this is or is not a separating equilibrium.

This is not an equilibrium because weak Firm 2's would be better off burning $1 and being mistaken for a strong firm.

e. (5 points) Aside from the strategy profile in part d (which may or may not be an equilibrium), either find a separating equilibrium or explain why such an equilibrium cannot exist.

Strong Firm 2's are willing to pay up to $3 to not be mistaken for a weak firm. Weak Firm 2's are willing to pay up to $2 to be mistaken for a strong firm. Thus, the following strategy profile {Firm 1 do not enter if more than $2 is burned, otherwise enter. Strong Firm 2's will burn $2.01 and fight. Weak Firm 2's do not burn anything and do not fight.} represent a separating equilibrium. Give a bonus point if students do correctly point out that the equilibrium also requires Firm 1 to have correct beliefs about who is burning money.

Bonus (5 points) Is there any pooling equilibrium? Either find a pooling equilibrium or explain why such an equilibrium cannot exist.

There cannot be a pooling equilibrium where both burn money because Firm 1 will believe that there is an equal chance of Firm 2 being strong or weak and will, thus, always enter. Either Firm would rather not burn money in this case.

Technically, neither burning money is a pooling equilibrium if Firm 1 believes that any burning of money means that they are facing a strong firm with low probability (and, thus, will enter regardless of the money burning). The belief condition for this to be an equilibrium is the belief probability of a money burning Firm 2 being strong is less than or equal to 3/5.