Final Exam ECON 3600, Winter 2012

NAME:

ID Number:

Signature:

Write your NAME and STUDENT ID NUMBER at the top of your exam and on each exam booklet that you use.

Before handing in your exam, sign each exam/booklet that you want graded.

This is a closed book exam. No formula sheets or notes are allowed.

Only a non-graphing calculator (the Sharp EL-243SB) is allowed.

Regular Examination Time: 3 hours (Double Examination Time: 6 hours)

There are 5 questions worth a total of 100 points and 5 bonus points.

Pay attention to the time you have left. If a problem is too difficult, skip it and move on, coming back to it if you have time.

It is in your interest to always show your work. We cannot give you partial credit if there is only a wrong answer with no explanation.

Question 1. Short-Answer Questions (20 points total)

State whether each of the following claims is true or false (or cannot be determined). For each explain your answer in (at most) **one short** paragraph. Each part is worth 5 points of which 4 points are the explanation. Explaining an example or counter example is sufficient. Absent this, a nice concise intuition is sufficient: you do not need to provide a formal proof. Points will be deducted for incorrect explanations.

a. (5 Points) “In Stackelberg competition, first Firm 1 decides how much to produce, second Firm 2 (knowing how much Firm 1 is producing) makes their production decision. Thus, Firm 2 will have higher profits than Firm 1 because Firm 2 can use this extra information to its advantage."

b. (5 Points) “In extensive form games (i.e. with a game tree), all Nash equilibria are also subgame perfect Nash Equilibria because, by definition, no player has an incentive to change strategies in a Nash Equilibria."

c. (5 Points) “If a players’ discount factor, δ, is close to one then we can sustain cooperation as an SPE of an infinitely repeated prisoner’s dilemma using a grim trigger strategy.”

d. (5 Points) “In a War of Attrition, players are unlikely to both fight in the first round, but if they do both fight in the first round, then they are equally likely to both fight in the second round."

Question 2: (15 points) Bargaining

Consider the following discounted, three-period bargaining game. There are two players bargaining over $1. The discount factor of both players is δ, where $0 < \delta < 1$. In this game, Player 1 makers the first offer of how to split the dollar. If Player 2 rejects this offer, then Player 1 makes *another* offer. If Player 2 rejects the second offer, then Player 2 makes the final offer. If Player 1 rejects this final offer, they both get nothing. In other words, Player 1 makes offers in Periods 1 and 2, while Player 2 makes the offer only in Period 3.

a. (5 points) Draw the extensive form (i.e. game tree) of this game.

b. (10 points) Compute the subgame perfect equilibrium (SPE) of this game. Show your work.

Question 3: (15 points) Infinitely Repeated Game

The following game is repeated infinitely and players 1 and 2 have the same discount factors δ and 0 ≤ δ ≤ 1.

		Player 2: C	Player 2: D
Player 1	C	3, 3	-2, 6
	D	6, -2	0, 0

a. (2 points) What is the Nash equilibrium if the game is only played for one period?

b. (6 points) What is the lowest discount factor, δ, such that the Grim Trigger strategy induces the players to cooperate forever in equilibrium?

c. (7 points) What is the lowest discount factor, δ, such that the Tit-for-Tat strategy induces both players to cooperate forever in equilibrium? (The Tit-for-Tat strategy is: Play C to start, then: Play C if either (C,C) or (D,D) were player last turn. Play D if either (C,D) or (D,C) were played last turn.) Does δ have to be larger or smaller in the Tit-for-Tat case compared to the Grim Trigger case to get cooperation?

Question 4. [25 total points] Asymmetree

Consider the following three player game. At the beginning of the game, simultaneously, player A and player B each choose X or Y. If both choose X, then the game ends and the payoff vector is (1,0,0); that is, player A gets 1 and the other two players get 0. If both choose Y then the game ends and the payoff vector is (0, 1, 0); that is, player B gets 1 and the other two players get 0.

If and only if A and B differ in their choices (i.e. one chooses X while the other chooses Y), then player C must guess which of the players chose X. That is, player C chooses between A and B. Player C makes his choice knowing only that the game did not end following the choices of players A and B. If player C guesses correctly which player played X, then player C and the player she (correctly) selected each get a payoff of 2 while the player who chose Y gets 0. If player C guesses incorrectly, then everyone gets 0.

a. (5 points) Draw the game tree for this game, being careful to indicate the players' information sets and payoffs.

b. (5 points) Find a (possibly mixed) NE in which player A chooses Y, or show that no such equilibrium exists.

c. (5 points) Find a (possibly mixed) NE in which player B chooses Y, or show that no such equilibrium exists.

d. (5 points) Find a (possibly mixed) NE in which player C chooses B, or show that no such equilibrium exists.

e. (5 points) Suppose that the game is now changed to one of perfect information with A moving first, then B, and then C (where C only moves if A and B differ in their choices, but where she is able to observe those choices). Explain how you would expect the game to proceed.

Question 5. [25 total points] Entry Game

Firm 1 is thinking about entering a market where Firm 2 is the incumbent. Firm 1 has two strategies: Enter or Don't Enter. If Firm 1 does not enter, Firm 2 gets $7 and Firm 1 gets $0. If Firm 1 enters, Firm 2 can either Fight or Not. First, assume that Firm 2 is a **strong** firm. In this case, if Firm 1 enters and Firm 2 fights, Firm 1 has a payoff of -$2 (that is a loss of $2) and Firm 2 has a payoff of $4. If Firm 1 enters and Firm 2 does not fight, both firms get $3.

a. (5 points) Write out the extensive form of this game and explain the subgame perfect NE.

Second, assume that Firm 2 is a **weak** firm. In this case, if Firm 1 enters and Firm 2 fights, Firm 1 has a payoff of -$1 (that is a loss of $1) and Firm 2 has a payoff of $2. If Firm 1 enters and Firm 2 does not fight, both firms get $3. If Firm 1 does not enter, Firm 2 gets $5 and Firm 1 gets $0.

b. (5 points) Write out the extensive form of this game and explain the subgame perfect NE.

Now assume that Firm 1 does not know whether Firm 2 is a **strong** firm or a **weak** firm, but Firm 1 believes that there is a 50% chance Firm 2 is a **strong** firm and a 50% chance Firm 2 is a **weak** firm.

c. (5 points) Write out the extensive form of this game and explain the subgame perfect NE.

Now assume Firm 2 is able to send a signal to Firm 1. It can choose to burn some of its capital. If it burns $X of capital, it loses $X in payoffs in every outcome (e.g. if a weak Firm 2 burned $5 of capital, it would have a payoff of $0 if Firm 1 does not enter, a payoff of -$3 if Firm1 enters and Firm 2 Fights, and a payoff of -$2 if Firm 1 enters and Firm 2 does not fight). Firm 1's payoffs are unaffected by this burning of capital, but Firm 1 observes how much capital is burned before Firm 1 makes a decision to enter or not.

d. (5 points) Consider the following strategies: {Firm 1 do not enter if $1 is burned, otherwise enter. Strong Firm 2's burn $1 and fight. Weak Firm 2's do not burn anything and do not fight.} (With the belief by Firm 1 that seeing $1 burned means it is for sure a strong Firm 2 and not seeing this means it is for sure a weak Firm 2.) Explain whether this is or is not a separating equilibrium.

e. (5 points) Aside from the strategy profile in part d (which may or may not be an equilibrium), either find a separating equilibrium or explain why such an equilibrium cannot exist.

(Bonus Question on Next Page)

Bonus (5 points) Is there any pooling equilibrium in this game? Either find a pooling equilibrium or explain why such an equilibrium cannot exist.