MATH 1007E – Test 4

Wednesday, November 14th, 2012

NAME: $\boxed{\text{Solutions}}$

STUDENT NUMBER: ____________________

This test has 4 questions (worth a total of 20 marks). Calculators are <u>not</u> allowed. You have 50 minutes. Write your answers in the spaces provided and put a box around the final answers where appropriate.

1. (10 points) Consider the function $f(x) = (\ln x)^2$. Sketch the graph of the function by determining the important information (domain, symmetry, asymptotes, intervals of increase/decrease, local extrema, intervals of concavity, points of inflection).

$f(x) = (\ln x)^2$ defined only for $x > 0 \Rightarrow$ no symmetry

$\lim_{x \to 0^+} (\ln x)^2 = \infty \Rightarrow x = 0$ is a vertical asymptote

$\lim_{x \to \infty} (\ln x)^2 = \infty \Rightarrow$ no horizontal asymptote

$f'(x) = \frac{2 \ln x}{x}$, $f'(x) = 0$ if $x = 1$

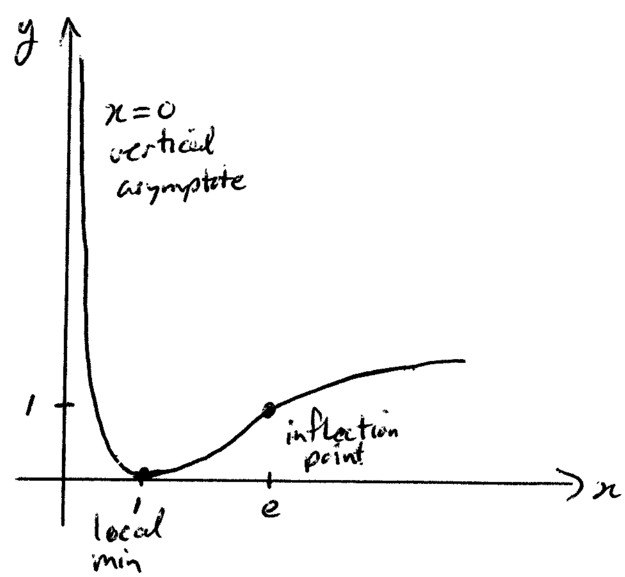
if $0 < x < 1$, $f'(x) < 0$, $f(x)$ decreasing
if $x > 1$, $f'(x) > 0$, $f(x)$ increasing
} local min at $x = 1$ or at $(1, 0)$

$f''(x) = -\frac{2}{x^2} \ln x + \frac{2}{x^2} = \frac{2}{x^2}(1 - \ln x)$, $f''(x) = 0$ if $x = e$

if $0 < x < e$, $f''(x) > 0$, $f(x)$ concave up

if $x > e$, $f''(x) < 0$, $f(x)$ concave down

So inflection pt at $x = e$ or at $(e, 1)$



2. (4 points) Find the (first) derivatives of the follwing functions.

(a) $f(x) = \arcsin(e^{x^2})$

$$f'(x) = \frac{1}{\sqrt{1-(e^{x^2})^2}}(e^{x^2})(2x)$$

$$= \boxed{\frac{2xe^{x^2}}{\sqrt{1-e^{2x^2}}}}$$

(b) $y = e^{\arcsin(x^2)}$

$$\frac{dy}{dx} = e^{\arcsin(x^2)} \frac{1}{\sqrt{1-(x^2)^2}} (2x)$$

$$= \boxed{\frac{2xe^{\arcsin(x^2)}}{\sqrt{1-x^4}}}$$

3. (3 points) Find the linearization of $f(x) = \arctan(4x)$ at $x = 0$ and use it to estimate $\arctan(0.4)$.

$f(0) = \arctan(0) = 0$, $\quad f'(x) = \frac{4}{1+(4x)^2} \Rightarrow f'(0) = 4$

$L(x) = f(0) + f'(0)(x-0) = 0 + 4(x-0) = \boxed{4x}$

then $\arctan(0.4) \approx L(0.1) = 4(0.1) = \boxed{0.4}$

4. (3 points) Find the absolute maximum and minimum of $f(x) = x^3 - 3x$ on the interval $[-2, 4]$.

$f'(x) = 3x^2 - 3 = 3(x^2-1)$ $\quad$ so $f'(x) = 0$ if $x = \pm 1$ (critical pts)

$f(-2) = (-2)^3 - 3(-2) = -8+6 = -2$

$f(-1) = (-1)^3 - 3(-1) = -1+3 = 2$

$f(1) = (1)^3 - 3(1) = 1-3 = -2$

$f(4) = (4)^3 - 3(4) = 64-12 = 52$

$\therefore$ $\boxed{\text{max is } 52}$ (at $x=4$) and $\boxed{\text{min is } -2}$ (at $x=-2$ and $x=1$)