MATH1007G – Test 2 – 7:35 pm - 8:25 pm, Feb 9

Name: Student Number:

Total: 15 marks
Closed book, may use only non-programmable, non-graphing calculators!

I. Multiple Choices (1 point each), circle the correct answer.

1. Let $f(x) = x^3 + x^2 - 5x + 9$. Then $f'(1) =$
   (a) 0 (b) 1 (c) 3 (d) -3 (e) 6
   Solution: (a)

2. Let $f(x) = \sin x$. Then $f'(0) =$
   (a) 0 (b) -1 (c) 1 (d) π (e) $\pi/2$
   Solution: (c)

3. Let $h(x) = f(g(x))$, where $f'(1) = 2$,$g'(3) = -5$, $g(3) = 1$, $f'(3) = 7$. Then $h'(3) =$

   (a) -5 (b) 7 (c) 14 (d) -35 (e) -10
   Solution: (e), $h'(3) = f'(g(3))g'(3) = f'(1)(-5) = 2(-5) = -10$.

4. If $f'(x) = \frac{(x-2)(x^2+1)}{x+3}$, then the curve $y = f(x)$ has horizontal tangent line at $x =$
   (a) -3 (b) -2 (c) -1 (d) 1 (e) 2
   Solution: (e)

II. Long Answer Questions, you have to show your work.

5. [4 points = 2+2] Let $f(x) = xe^x$.
   a) Calculate $f'(x)$ and $f''(x)$.
   b) Find the equation of the tangent line to the curve $f(x) = xe^x$ at the point $(1, e)$.
   Solution: a) $f'(x) = e^x + xe^x$, $f''(x) = e^x + e^x + xe^x = 2e^x + xe^x$.
   b) $m = f'(1) = 2e$. The tangent line is

$$y - e = 2e(x - 1), \Rightarrow y = 2ex - e.$$

6. [3 points = 2 + 1] Let $f(x) = \frac{x}{x+1}$.

a) Calculate $f'(x)$ by the Definition of the derivative of $f(x)$.
b) Use quotient rule to check your result.

Solution: a)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+1) - x(x+h+1)}{h(x+h+1)(x+1)} = \lim_{h \to 0} \frac{h}{h(x+h+1)(x+1)}$$

$$= \lim_{h \to 0} \frac{1}{(x+h+1)(x+1)} = \frac{1}{(x+1)^2}.$$

b) By Quotient rule,

$$f'(x) = \frac{1(x+1) - 1x}{(x+1)^2} = \frac{1}{(x+1)^2}.$$

7. [4 points = 2+2] Find y' for each of the following functions:
a) $y = x^x$. (Hint: Use logarithmic differentiation).
Solution:

$$\ln y = \ln(x)^x = x \ln x, \Rightarrow \frac{d}{dx} \ln y = \frac{d}{dx}(x \ln x), \Rightarrow$$

$$\frac{y'}{y} = \ln x + x \left(\frac{1}{x} \right) = \ln x + 1, \Rightarrow$$

$$y' = y\left(\ln x + 1\right) \quad \text{or} \quad x^x \left(\ln x + 1\right).$$

b) $x^3 + y^4 - y = 1$.
Solution: We use implicit differentiation.

$$\frac{d}{dx}(x^3 + y^4 - y) = \frac{d(1)}{dx}, \Rightarrow$$

$$3x^2 + 4y^3 y' - y' = 0. \Rightarrow$$

$$(4y^3 - 1)y' = -3x^2, \Rightarrow$$

$$y' = -\frac{3x^2}{4y^3 - 1}.$$