MAT 1348 – The Most Important Concepts and Questions

- **Logic and Proofs**

  Concepts

  1. propositions, logical connectives, equivalence
  2. tautology, contradiction, contingency
  3. disjunctive normal form
  4. rules of inference, arguments; validity of an argument
  5. counter-example
  6. types of proof

  Methods

  1. truth tables
  2. truth trees
  3. proofs

  Questions

  1. Translate English sentences into compound propositions.
  2. Is a given proposition a tautology/contradiction?
  3. Are given propositions equivalent?
  4. Find a disjunctive normal form of a given proposition.
  5. Is a given argument valid?
  6. Prove a given theorem using a specified method of proof.

- **Sets**

  Concepts

  1. set, cardinality, subset, power set, cartesian product of sets
  2. set operations

  Questions

  1. Find the cardinality of a power set, cartesian product etc.
  2. Prove a set identity (using a rigorous proof, known set identities, or membership tables).

- **Functions**

  Concepts

  1. one-to-one and onto functions; bijections
  2. invertible functions, inverse

3. composition of functions

Questions

1. Determine whether a given function is one-to-one, onto, invertible, or a bijection.
2. Find the inverse of an invertible function.

- **Relations**

  Concepts

  1. reflexive, symmetric, antisymmetric, transitive relations
  2. equivalence relations, equivalence classes, partitions
  3. congruences

  Questions

  1. Determine whether a given relation is reflexive, symmetric, antisymmetric, or transitive.
  2. Prove that a given relation is an equivalence relation.
  3. Find the equivalence classes of a given equivalence relation (or the corresponding partition of the set).

- **Counting**

  Concepts

  1. Product Rule, Sum Rule, Principle of Inclusion – Exclusion
  2. Pigeonhole Principle
  3. combinations, permutations
  4. binomial coefficients, Binomial Theorem

  Questions

  1. How many...?
  2. What is the smallest number of ... to guarantee...? or Show that in any set of... we can find... such that....
  3. Find the coefficient of ... in the expansion of...

- **Mathematical Induction**

  Concepts

  1. Mathematical Induction, Strong Induction

  Questions

  1. Prove... using Mathematical Induction/Strong Induction.

- **Graphs**

  Concepts

  1. simple graph, multigraph, pseudograph
  2. directed graph, directed multigraph
  3. complete graphs, cycles, paths, complete bipartite graphs
  4. bipartite graphs
  5. subgraphs, isomorphism
  6. walks, trails, paths, cycles
  7. connected graphs
  8. Euler tours and open Euler trails

  Theorems

  1. Handshaking Theorem
  2. equivalent conditions for a graph to be bipartite
  3. necessary and sufficient conditions for a graph to have an Euler tour/open Euler trail

  Questions

  1. How many edges in a graph with a given degree sequence?
  2. Is a given graph bipartite? (If so, give a 2-colouring. If not, find an odd cycle.)
  3. Are given graphs isomorphic? (If so, give an isomorphism. If not, find an invariant in which they disagree.)
  4. Does a graph have an Euler tour/open Euler trail? Explain why.

- **Trees**

  Concepts

  1. tree, forest
  2. rooted tree; root, parent, child, internal vertex, leaf
  3. m-ary tree, full m-ary tree
  4. level of a vertex, height of a tree, balanced tree

  Theorems

  1. Number of edges in a tree with n vertices.
  2. Number of vertices in a full m-ary tree with i internal vertices.
  3. Upper bound on the number of leaves in an m-ary tree of height h.
  4. Lower bound on the number of leaves in a balanced full m-ary tree of height h.

Questions

1. Is a given graph a tree?
2. Determine the number of edges/vertices/internal vertices/leaves in an m-ary tree with a given number of edges/vertices/internal vertices/leaves.