# Laboratory 7 – Matrix Operations with MATLAB
# Assigned week of November 12-16, 2012
# Due Week of November 19-23, 2012

## I — Introduction:

Laboratory exercises 2, 5, and 6 dealt with measurements and how to manage "unstructured measured data". We now turn our attention to so-called "structured" data. In particular, this laboratory introduces a very important engineering software tool for managing engineering data – MATLAB (short for MATrix LABoratory).

The focus of this laboratory is on linear systems (systems of linear equations). You have all solved two equations with two unknowns and likely three equations with three unknowns, but it is unlikely that you have solved 20 equations with 20 unknowns. The number of equations in engineering analysis can reach this number and in some cases the number can go much higher, e.g. the number of linear (and sometimes nonlinear) equations in finite element analysis (FEA), an important engineering analytical system, can reach millions. This year you take a course in matrix methods for solving linear equations—MATH 1104 (Linear Algebra).

Linear equations occupy a special place in engineering: Ohms law (V = IR) is a linear equation; Newton's second law of motion (F = ma) is a linear equation, and many other fundamental relationships are approximately linear. These are relatively simple relationships that can be manipulated manually, however, there are many linear systems of equations where manual manipulation becomes prohibitive and software help is needed.  For the most part, linear systems are set up in matrix form and MATLAB has become the software of choice for their solution, at least in recent years.

This laboratory is primarily an exercise in manipulating matrices using MATLAB.  It is recommended that you read through the MATLAB support material found on WebCT and in your textbook. You can then perform the operations in the MATLAB command window, or with an m-file.  Be sure to keep (save) your m-file work for your report.

We will begin by asking you to manipulate some vectors and matrices with MATLAB and end by asking you to set up and solve some related engineering problems.

## II — Problem Statement:

**Part 1:** Vectors are a special kind of matrix that can be represented by a single column, or row of numbers (components).

1. Consider the following vectors

$$\vec{H} = \begin{bmatrix} 8 \\ 3 \\ -6 \end{bmatrix}, \quad \vec{I} = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}, \quad \vec{J} = \begin{bmatrix} 9 & 4 & -7 \end{bmatrix} \quad \vec{K} = \begin{bmatrix} 6 \\ 4 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{L} = \begin{bmatrix} 10 \\ 9 \end{bmatrix},$$

and the following scalars

$p$ = 0.32, $q$ = 2.25 and $r$ = −0.4.

Perform the following operations in the MATLAB command window and suggest a geometric interpretation for each allowable operation:

(a) $\vec{H}.\vec{I}$

(b) $p\vec{K}$

(c) $\vec{L}.\vec{H}$

(d) $\vec{J}^T - \vec{K}$

You should be able to do these computations from your lecture background and from the support provided in the MATLAB support material. If MATLAB returns an error, explain why.

2. The CCGS Panda, a search boat for the Canadian Coast Guard, has been undergoing tests to verify the usefulness of a new navigational system. The system is supposed to guide the boat along a pre-designed course and return it to the starting location. In one particular test, the navigational system was set for the following course (each change in position is represented by a vector):

- starting at the origin (0, 0), the boat traveled to a position 10 km north and 1 km east. This change in position can be represented by the vector $\begin{bmatrix} 1 \\ 10 \end{bmatrix}$;
- the boat then traveled to a new position 10 km east and 2 km north;
- after traveling to a location 7 km east and 20 km south, the boat then went to a site 10 km west and 7 km north;
- in the final portion of this test, the boat was located 4 km east and 10 km north of the previous point.

Use MATLAB plot command to show the path of the CCGS Panda for visualization purposes. L**abel the plot axes and provide a title: your name should appear in the title;** the plot ranges are $0 \leq x \leq 20$ and $-10 \leq y \leq 15$. Then, using MATLAB, add up all of the vectors, and draw conclusions about the results of the test: did the navigational system perform as planned? Determine the total distance traveled by the boat by finding the norm of each vector and calculating the sum in MATLAB.

3. Plot the equation $y = \sin x^2 + \frac{10+2x+x^2}{5+2x^2}$ from $x$ = - 2 to $x$ = 2 using MATLAB. Vary the number of plotted points so that the curve looks smooth. The plot should have the axes labelled and a title that includes your name, *i.e.,* your name should appear in the title of the plot.

**Part 2:** Use MATLAB to answer the following questions.

1. The following matrices are defined as

$$F=\begin{bmatrix} 3 & 9 \\ 7 & -2 \\ 5 & 0 \\ 6 & 8 \\ 9 & 4 \\ 5 & 2 \end{bmatrix}, \quad G=\begin{bmatrix} -9 & 2 & 8 & 5 & -1 & 2 \\ 7 & -1 & -2 & 9 & 3 & 8 \end{bmatrix}, \quad Q=\begin{bmatrix} 4 & -7 & 2 \\ 1 & 0 & 8 \\ -5 & 3 & 5 \end{bmatrix}, \quad U=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

(a) Multiply $F$ and $G$, and then multiply the result by the scalar $r$ defined previously;

(b) Transpose $G$, then perform the matrix operation $W = F - G^T$;

(c) Sort and then plot $W$, using the vector $[1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6]^T$ for the $x$ indices.

(d) Multiply $U$ and $G$;

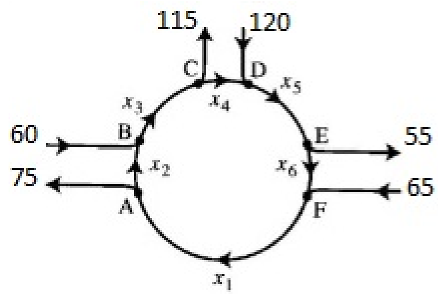
(e) Multiply $F$ and $U$;

(f) Multiply $Q$ and $H$, which was defined previously;

(g) Perform the following operation: $(q \cdot G) + (p \cdot F^T)$.

2. A linear transformation $T(\vec{x})$ maps the "original" vector $\vec{x}$ to its "image" $\vec{b}$ via the following matrix equation: $A\vec{x} = \vec{b}$ where $A$ is called the "standard matrix" (you will study linear transformations in MATH 1104). Consider a particular linear transformation $A = \begin{bmatrix} 9 & 2 & 5 \\ 1 & 4 & 6 \\ 2 & 3 & 1 \end{bmatrix}$. If the image produced with this transformation is $\vec{b} = \begin{bmatrix} 5 \\ 1 \\ 3 \end{bmatrix}$, then find the original vector $\vec{x}$ ?

3. Busy intersections in Europe are often constructed as one-way "roundabouts", such as the traffic network in England displayed below (you will study networks in MATH 1104):




Assuming that the traffic must travel in the directions shown, to find the unknown flow rates in cars/minute, solve the following system of equations:

$$\begin{cases} x_1 - x_2 = 75 \\ x_2 - x_3 = -60 \\ x_3 - x_4 = 115 \\ x_4 - x_5 = -120 \\ x_5 - x_5 = 55 \\ -x_1 + x_6 = -65 \end{cases}$$

Write these equations in matrix form, $\vec{x} = \vec{c}$ , and solve using MATLAB. If MATLAB does not return an answer, explain. Is it possible to solve this system at all? Use the "pseudoinverse" function pinv ( ) to obtain a solution. Check the solution by calculating $B\vec{x} - \vec{c}$ .

4. A steam plant burns two types of coal: anthracite (A) and bituminous (B).
   - For each tonne of A burned, the plant produces 27.6 million Btu of heat, 3 100 g of sulphur dioxide, and 240 g of particulate matter (solid-particle pollutants).
   - For each tonne of B burned, the plant produces 30.2 million Btu, 6 400 g of sulphur dioxide, and 350 g of particulate matter.

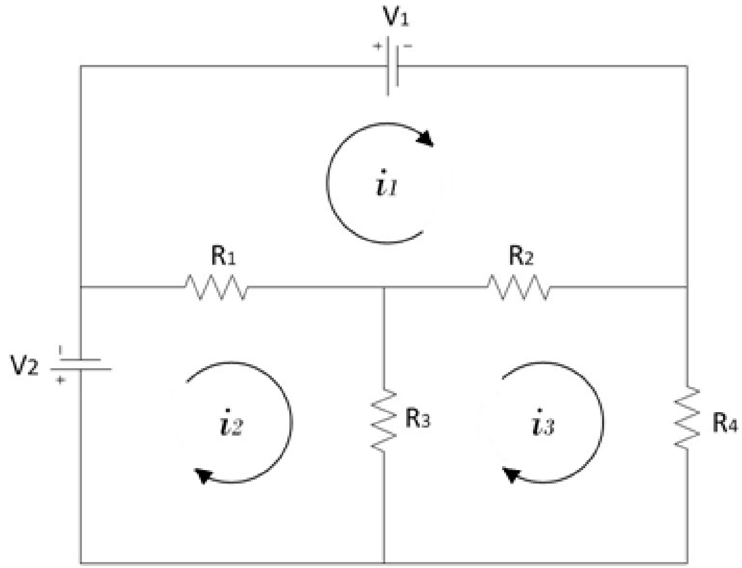
   Over a certain time period, the steam plant produced 162 million Btu of heat, 23 61# g of sulphur dioxide, (where "#" is the last digit of your student number: so if your student number ends with a 9, then you would use 23 619 g of sulphur dioxide), and 1 566 g of particulate matter.

   <u>Determine how many tonnes of each type of coal the steam plant must have burned.</u>

   - Hint: write equations for the heat, sulphur dioxide and particulate matter produced as functions of the variables A and B, corresponding to the tonnes of each type of coal burned. Is this problem under-determined, determined or over-determined? Recall that only determined linear problems can be described with square matrices, and only square matrices can be inverted. From MATLAB HELP: If A is a square matrix, A\B is roughly the same as inv(A)*B, except it is computed in a different way. If A is an m-by-n matrix with m ~= n and B is a column vector with m components, then X = A\B is the solution in the least squares sense to the under- or over-determined system of equations AX = B.

5. You are building a circuit that includes two voltage sources and four resistors connected as shown below:

**YOUR NAME AS TITLE**




Using Kirchhoff's Voltage Law and Ohm's Law, the following equations defining the currents in this circuit can be derived:

$$V_1 + R_2(i_1 - i_3) + R_1(i_1 - i_2) = 0$$
$$V_2 + R_3(i_2 - i_3) + R_1(i_2 - i_1) = 0$$
$$R_2(i_3 - i_1) + R_3(i_3 - i_2) + R_4 i_3 = 0$$

Compute the currents for $R_1 = 7\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 1\ \Omega$, $R_4 = 7\ \Omega$, and $V_1 = 5.\#\#$ V, and $V_2 = 3.\#\#$ V, where "##" are the last two digits of your student number. Comment on the sign of the calculated currents.

Sketch this circuit diagram using IntelliCAD. Use the circuit sketching blocks provided under the "Support Material" on the WebCT to assist you. Draw the arrows in your circuit to agree with the convention for the direction for current flow. <u>All circuit components should be labelled with their respective numerical values (in ohms, amps or volts) on the sketch</u>. **Add your name on the top of the drawing in IntelliCAD.**

6. Make a 3 by 3 matrix with the three elements in the first row equal to the first three numbers of your student number, and the second row equal to the next three numbers, and the third row equal to the last three numbers of your student number; call this matrix A. Perform the following calculation:

$$c = b * A * b^T$$

where $b = [5 \quad 4 \quad 3]$ and report the value "E = c/π" in **"bank"** format.

## III — Steps and Calculations:

Using MATLAB and its functions, perform all the necessary steps to obtain the answers to each of the questions and problems presented above. Use IntelliCAD to sketch the circuit in Part 2 (step 5) to provide visual support to the solutions.

## IV — Report Requirements and Deliverables:

- Using the guidelines presented in Laboratory 1, produce a formal laboratory report that summarizes your findings.
- It is apparent that this is a somewhat different laboratory exercise in the sense that it is more-or-less a series of problems. Nonetheless you should be able to identify a central focus (central objective) to use as guidance for writing your report.
- State briefly the results to all of the problems posed. Discuss the significance of the results in each case.
- Show your MATLAB code for each problem in an appendix(ces).
- Include a short discussion of the advantages and disadvantages of software such as MATLAB for dealing with vectors and matrices.

***Deliverables: The lab assignment includes the following:***

1. **Title page**
2. **One-page report**
3. **The MATLAB scripts properly labeled for each part and section (e.g. solution of "Part 2.6" or "Part 2 question 6" followed by the script)**
4. **Circuit drawing with the proper labels showing the results found in Part 2.5 (Question 5 Part 2) and a title that includes your name.**
5. **IMPORTANT: submit an electronic version of your assignment to the P: drive folder***

***File name: "Lab Session_Student number.doc OR .docx" (e.g. "C3_100812345.doc": it is for C3 Lab session)**

**NOTE:** All of the charts/plots should have properly labeled axes and have a title that includes your name.

## V — Submission and Timing:

Your report is to be submitted to the Teaching Assistant within the first 30 minutes of your next laboratory period. **LATE SUBMISSIONS WILL NOT BE ACCEPTED.**

## VI — Marking:

Laboratory submissions will be marked on a 10-point scale: 9-10 (excellent); 7-8 (good); 5-6 (marginal); less than 5 (poor). **Be sure that you are familiar with the University's policy on plagiarism and academic integrity. Your instructors are obligated to report all suspected violations to the Associate Dean's office for investigation (see also chapter 14 at www4.carleton.ca/calendars//ugrad/current_ /regulations).**