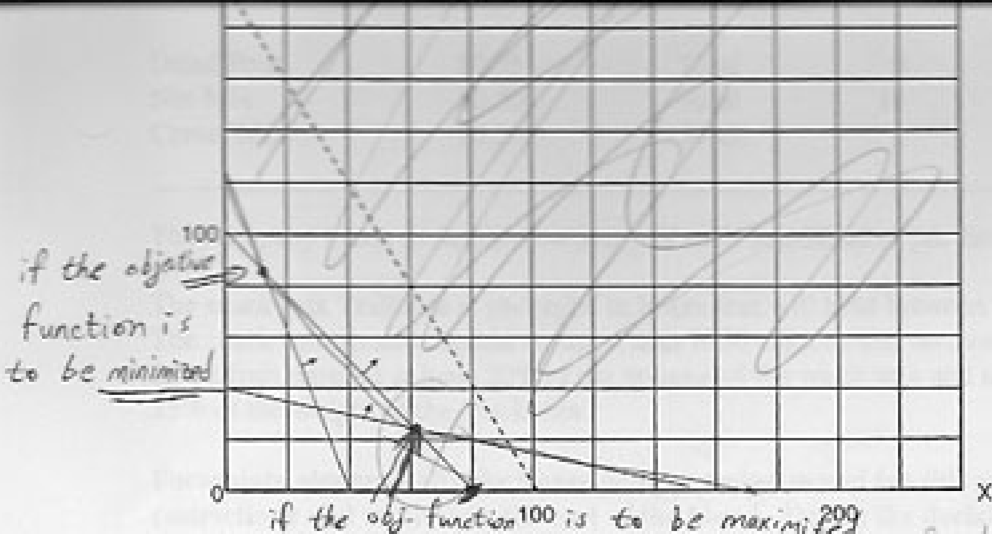

(c) The objective function is to be maximized. (2 points)

unbounded solution

(d) The objective function is to be minimized. (2 points)

2

(e) Constraint T: $3X_1 + X_2 \geq 90$ Constraint S: $3X_1 + 2X_2 \leq 120$ Constraint W: $X_1 - X_2 \geq 0$

All variables are required to be non-negative. Let the objective function be Max: $X_1 + X_2$. The optimal solution for this problem is (24, 24). Is there non-binding constraint(s)? if yes, which one(s). (2 points)

$T: 3(24) + (24) \geq 90 \Rightarrow 96 \geq 90 \Rightarrow$ non-binding

$S: 3(24) + 2(24) \leq 120 \Rightarrow 120 \leq 120 \Rightarrow$ binding

$W: (24) - (24) \geq 0 \Rightarrow 0 \geq 0 \Rightarrow$ ~~non~~-binding

LHS RHS

Constraint T and ~~W~~ are non-binding

1

$\Rightarrow$ RHS = LHS

(f) Divisibility assumption in linear programming implies (1 point)

A) resources can be divided among products
B) products can be divided among customers
C) decision variables may take on integer values
D) decision variables may take on fractional values

1

(g) Consider the following constraints and choose the correct answer: (1 point)

Constraint T: $2X_1^2 + 3X_2 \leq 100$ Constraint S: $2X_1X_2 + 3X_2 \leq 1000$

A) constraint T can be used in a linear program
B) constraint S can be used in a linear program
C) neither can be used in a linear program
D) both may be used in a linear program
E) both may be used in an integer linear program, but not in linear program

1

(h) If a linear programming problem has redundant constraints, then the optimal solution without the redundant constraints will be the same as the optimal solution with the redundant constraints. (1 point)

True 1 False

(i) The algorithm (algebraic method) used to solve any size of linear programming problems is called the ~~Formulation~~ (1 point)

simplex method

Problem # 2 (12 points)

FarmFresh Foods manufactures a snack mix called TrailTime by blending three ingredients: a dried fruit mixture, a nut mixture, and a cereal mixture. Information about the three ingredients **(per 100 grams)** is shown below:

Ingredient	Per 100 grams Cost	Volume	Fat Grams	Calories
Dried Fruit	\$0.35	75ml	0	150
Nut Mix	\$0.50	90ml	10	400
Cereal Mix	\$0.20	250ml	1	50

The company needs to decide how many of each ingredient to put into the TrailTime blend.

The snack mix Trailtime is packaged in boxes that will hold between 750 ml and 1000 ml. The snack mix should contain no more than 1000 calories and no more than 25 grams of fat. Dried fruit must be at least 20% of the volume of the snack mix and nuts must be no more than 15% of the weight of the snack mix.

Formulate algebraically the linear programming model for this problem that meets these restrictions and minimizes the cost of the blend. Define the decision variables, objective function, and constraints. DO NOT SOLVE

X_i: ~~the~~ amount of ingredient (i) added to Trail time per 100 grams

i= Dried Fruit (1), Nut Mix (2), and Cereal Mix (3)

Min Z = $0.35X_1 + 0.5X_2 + 0.2X_3$

Subject to

$750ml \le 75X_1 + 90X_2 + 250X_3 \le 1000ml$ −2

$150X_1 + 400X_2 + 50X_3 \le 1000$ calories

$0X_1 + 10X_2 + X_3 \le 25$ fat grams −2

$X_1 \ge 0.2(X_1 + X_2 + X_3)$ −2

$X_2 \le 0.15(X_1 + X_2 + X_3)$

$X_i \ge 0$

Problem # 3 (14 points)
A manufacturer produces two products, A and B, and is trying to work out his production schedule for the next two months. He currently has on hand 1,000 units of A and 800 units of B and estimates sales over the period as shown in the following table:

Estimated Sales, Units/Months

	A	B
1	2,000	3,000
2	3,000	4,000
Regular Shift Production Cost, $/Unit	$10	$6

if the production facilities were devoted entirely to producing product A, they could turn out a maximum of 4,000 units/month on regular shift operation and up to 1,000 units/month additional on overtime operation. Similarly, if only product B was produced, regular shift capacity would be 6,000 units/month with 2,000 units/months additional available from overtime operation. The production facilities can be devoted to any proportions of products A and B - e.g., 1,000 units of A (= 1/4 capacity) plus 4,500 units of B (= 3/4 capacity) would just use the total regular shift capacity for a month.

Overtime production of product A costs $5/unit more than regular shift production, while for product B the added cost is $3/unit. Management would like to end the two-month period with at least an inventory of 500 units of product A and 1,500 units of product B. It costs $3 to store one unit of Product A for a month and $2 to store one unit of product B for a month.

Formulate algebraically the linear programming model for this production scheduling problem. Define the decision variables, objective function, and constraints. DO NOT SOLVE

~~X_{ij} = the amount of product (i) produced during month (j)~~ 1000 units A

~~i = product A (1), product B (2) ; j = month 1 & 2~~ 800 units B

~~overtime operation in month (i), j = month 1 & 2~~

~~Z = 10X₁₁ + 6X₂₁ + 6X₂₂ + 15 ... 15~~

X_{ijk} = the amount of product (i) produced during operation hours (j)

i = product A (1), product B (2); j = regular (1) and overtime (2)

during month k (K=1, 2)

15

for each month

Min $Z = 10X_{11} + 6X_{21} + 15X_{12} + 13X_{22} + 3X_{11} + 3X_{12} + 2X_{21} + 2X_{22}$

what ending Inventory

Subject to

$X_{11} \leq 8000$ units

$X_{21} \leq 12,000$ units

$X_{12} \leq 2000$ units

$X_{22} \leq 4,000$ units

$X_{ij} \geq 0$

1

Problem # 4 (17 points):

Canadian Market Research (CMR) is a marketing research firm that handles consumer surveys. One of its clients is a national press service that periodically conducts political polls on issues of widespread interest. In a survey for the press service, CMR determines that it must fulfill several requirements in order to draw statistically valid conclusions on the sensitive issue of Canadian immigration laws:

1. Survey at least 2,300 Canadian households in total.
2. Survey at least 1000 households with heads who are 30 years of age or younger.
3. Survey at least 600 households with heads who are between 31 and 50 years of age.
4. Ensure that at least 15% of those surveyed live in the city that is a major destination for immigrants (such as Toronto)
5. Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a destination city.

CMR decides that all surveys should be conducted in person. It Estimates that the costs of reaching people in each age and region category are as follows:

Cost per person Surveyed ($)

Region	Age <= 30	Age 31-50	Age >= 51
Destination City	$7.5	$6.80	$5.50
Not in destination City	$6.90	$7.25	$6.10

CMR's objective is to meet the five sampling requirements at the least possible costs.

The following correct output for this problem is provided below as Excel Solver output. You will need this output to answer the following questions (a) **through (f) below each part of which is to be considered** **independently of all others.**

	D_1	D_2	D_3	N_1	N_2	N_3			
	<= 30 and destination	31-50 and destination	>= 51 and destination	<= 30 and not destination	31-50 and not destination	>= 51 and not destination			
Number of Households	0.00	600.00	140.00	1000.00	0.00	560.00			
Interview Cost	$7.50	$6.80	$5.50	$6.90	$7.25	$6.10		<- Objective	
Constraints									
Total Households	1	1	1	1	1	1	2300.00	>=	2300
<= 30 Households	1			1			1000.00	>=	1000
31-50 Households		1			1		600.00	>=	600
Destination Cities	0.85	0.85	0.85	-0.15	-0.15	-0.15	395.00	>=	0
Limit on >= 51 in Destination			0.8			-0.2	0.00	<=	0
							LHS	Sign	RHS

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B5	Number of Households <= 30 and destination	0.00	0.60	7.5	1E+30	0.6
C5	Number of Households 31-50 and destination	600.00	0.00	6.8	0.45	0.82
D5	Number of Households >= 51 and destination	140.00	0.00	5.5	0.6	29.9
E5	Number of Households <= 30 and not destination	1000.00	0.00	6.9	0.6	0.92
F5	Number of Households 31-50 and not destination	0.00	0.45	7.25	W	X
G5	Number of Households >= 51 and not destination	560.00	0.00	6.1	1.025	0.6

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
[illegible]	Total Households	2300.00	[illegible]	2300	1E+30	[illegible]

(such as Toronto)

5. Ensure that no more than 20% of those surveyed who are 51 years of age or over live in a destination city.

CMR decides that all surveys should be conducted in person. It Estimates that the costs of reaching people in each age and region category are as follows:

<u>**Cost per person Surveyed ($)**</u>

Region	**Age <= 30**	**Age 31-50**	**Age >= 51**
Destination City	$7.5	$6.80	$5.50
Not in destination City	$6.90	$7.25	$6.10

CMR's objective is to meet the five sampling requirements at the least possible costs.

The following correct output for this problem is provided below as Excel Solver output. You will need this output to answer the following questions (a) **through (f) below each part of which is to be considered <u>independently</u> of all others.**

	D_1	D_2	D_3	N_1	N_2	N_3			
	<= 30 and destination	31-50 and destination	>= 51 and destination	<= 30 and not destination	31-50 and not destination	>= 51 and not destination			
Number of Households	0.00	600.00	140.00	1000.00	0.00	560.00			
Interview Cost	$7.50	$6.80	$5.50	$6.90	$7.25	$6.10		<- Objective	
Constraints:									
Total Households	1	1	1	1	1	1	2300.00	>=	2300
<= 30 Households	1			1			1000.00	>=	1000
31-50 Households		1			1		600.00	>=	600
Destination Cities	0.85	0.85	0.85	-0.15	-0.15	-0.15	395.00	>=	0
Limit on >= 51 in Destination			0.8			-0.2	0.00	<=	0
							LHS	**Sign**	**RHS**

Cell	**Name**	**Final Value**	**Reduced Cost**	**Objective Coefficient**	**Allowable Increase**	**Allowable Decrease**
B5	Number of Households <= 30 and destination	0.00	0.60	7.5	1E+30	0.6
C5	Number of Households 31-50 and destination	600.00	0.00	6.8	0.45	0.82
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E5	Number of Households <= 30 and not destination	1000.00	0.00	6.9	0.6	0.92
F5	Number of Households 31-50 and not destination	0.00	0.45	7.25	**W**	**X**
G5	Number of Households >= 51 and not destination	560.00	0.00	6.1	1.025	0.6

Cell	**Name**	**Final Value**	**Shadow Price**	**Constraint R.H. Side**	**Allowable Increase**	**Allowable Decrease**
H8	Total Households	2300.00	5.98	2300	1E+30	700
H9	<= 30 Households	1000.00	0.92	1000	700	1000
H10	31-50 Households	600.00	0.82	600	700	493.75
H11	Destination Cities	395.00	0.00	0	395	1E+30
H12	Limit on >= 51 in Destination	0.00	-0.60	0	560	140

(a) What is the optimal solution to CMR's marketing research problem? How much would such households survey cost? (3 points).

The cost of surveying that number of households will be \$15,166.

$Z = 7.5(0) + 6.8(600) + 5.5(140) + 6.9(1000) + 7.25(0) + 6.1(560)$
$= 0 + 4,080 + 770 + 6,900 + 0 + 3,416$
$= \$15,166$

2

(b) If the cost of surveying one person 51 years of age or older and do not live in a destination city (e.g. N_3) is decreased to \$5.9, will the optimal solution change? Will the cost change? If yes, by how much. Justify. (3 points)

A decrease from 6.1 to 5.9 in the cost of N_3 is within the allowable decrease of 0.6. The optimal solution ~~will change~~ will ~~decrease~~ change the total cost decrease by the amount of $(0.2)(560) = \$112$ so the total cost will be \$15,054 instead of \$15,166. 2.5/3

(c) What is the impact on the optimal solution and the value of the objective function, if CMR wants to increase the sample size to 2500 (i.e. survey at least 2500 Canadian households in total)? Justify. (3 points)

An increase in the sample size to 2500 is within the allowable increase of infinite in this constraint. The shadow price will remain the same. The optimal solution will increase by $5.98(200) = \$1,196$.

2.5/3

(d) What is the impact on the value of the objective function, if we can reduce the minimum required number of respondents age of 30 or younger to 800, provided we raise the number of respondents 31-50 years of age to 900? Justify. (3 points)

Reducing the amount of respondant age of 30 or younger to 800 is within the allowable decrease of 1000 and raising the number of respondants of 31-50 years of age to 900 is within the allowable increase of 700. Both changes won't have any impact on the objective function since both changes are withing the allowable increase or decrease.

0

100% rule → shadow prices

(e) Two numbers have been removed from the objective function sensitivity table by your humble instructor (the letters W and X appear instead of the numbers). What are the correct values of W and X? Justify. (3 points)

~~X = 0.45 because in order for the final value to change~~

W = infinite the more you increase the cost there will be no difference in the final value in the number of households between 31 and 50 years of age who are not in destination.

3

(a) What is the optimal solution to CMR's marketing research problem? How much would such households survey cost? (3 points).

The cost of surveying that number of households will be \$15,166.

$Z = 7.5(0) + 6.8(600) + 5.5(140) + 6.9(1000) + 7.25(0) + 6.1(560)$
$= 0 + 4{,}080 + 770 + 6{,}900 + 0 + 3{,}416$
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2

(b) If the cost of surveying one person 51 years of age or older and do not live in a destination city (e.g. N_3) is decreased to \$5.9, will the optimal solution change? Will the cost change? If yes, by how much. Justify. (3 points)

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0

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A decrease from 6.1 to 5.9 in the cost of N_3 is within the allowable decrease of 0.6. The optimal solution ~~will not change~~ will change ~~decrease~~ the total cost decrease by the amount of (0.2)(560) = \$112 so the total cost will be \$15,054 instead of \$15,166. 2.5/3

(c) What is the impact on the optimal solution and the value of the objective function, if CMR wants to increase the sample size to 2500 (i.e. survey at least 2500 Canadian households in total)? Justify. (3 points)

An increase in the sample size to 2500 is within the allowable increase of infinite in this constraint. The shadow price will remain the same. The optimal solution total cost will increase by 5.98 (200) = \$1,196. ~~which will lead to the~~

optimal will solution change

2.5/3

(d) What is the impact on the value of the objective function, if we can reduce the minimum required number of respondents age of 30 or younger to 800, provided we raise the number of respondents 31-50 years of age to 900? Justify. (3 points)

Reducing the amount of respondant age of 30 or younger to 800 is within the allowable decrease of 1000 and raising the number of respondants of 31-50 years of age to 900 is within the allowable increase of 700. Both changes won't have any impact on the objective function since both changes are withing the allowable increase or decrease

0

100% rule → shadow prices

(e) Two numbers have been removed from the objective function sensitivity table by your humble instructor (the letters W and X appear instead of the numbers). What are the correct values of W and X? Justify. (3 points)

~~X = 0.45 because ... in order for the final value to change~~

W = infinite the more you increase the cost there will be no difference in the final value in the number of households between 31 and 50 years of age who are not in destination.

X = 0.45 because the cost of surveying 31-50/not destination should be less than 31-50 in destinations in order for the final value to change

3

(f) Is the solution to this problem a unique optimal solution or there is a multiple optimal solution? Justify. (2 points)

~~No there is a multiple optimal solut~~

~~It is a unique opti~~

No there is a multiple optimal solutions