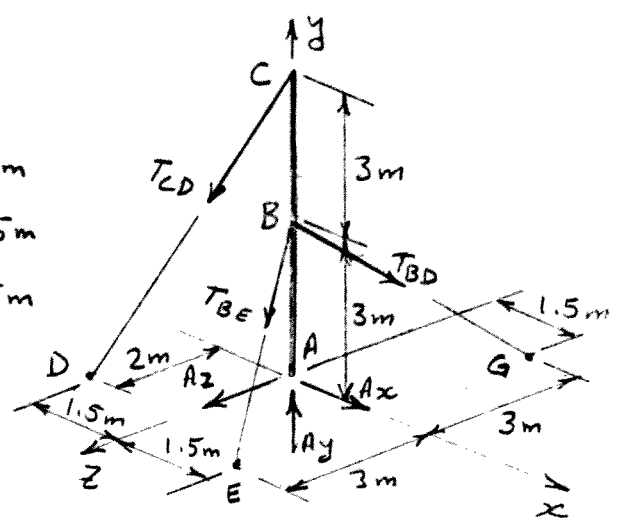


GNG 1105
ENGINEERING MECHANICS
FINAL EXAM
SOLUTIONS

Dec. 13, 2010

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1. a) See FBD to the right.

b) $\vec{BE} = 1.5\vec{i} - 3.0\vec{j} + 3.0\vec{k}$; $BE = 4.5\text{m}$
 $\vec{BG} = 1.5\vec{i} - 3.0\vec{j} - 3.0\vec{k}$; $BG = 4.5\text{m}$
 $\vec{CD} = -1.5\vec{i} - 6.0\vec{j} + 2.0\vec{k}$; $CD = 6.5\text{m}$



$$\vec{T}_{BE} = T_{BE} \vec{\lambda}_{BE} = T_{BE} \frac{\vec{BE}}{BE}$$

$$= \frac{T_{BE}}{4.5} (1.5\vec{i} - 3.0\vec{j} + 3.0\vec{k})$$

$$\vec{T}_{BG} = T_{BG} \vec{\lambda}_{BG} = T_{BG} \frac{\vec{BG}}{BG}$$

$$= \frac{T_{BG}}{4.5} (1.5\vec{i} - 3.0\vec{j} - 3.0\vec{k})$$

$$\vec{T}_{CD} = T_{CD} \vec{\lambda}_{CD} = T_{CD} \frac{\vec{CD}}{CD}$$

$$= \frac{T_{CD}}{6.5} (-1.5\vec{i} - 6.0\vec{j} + 2.0\vec{k})$$

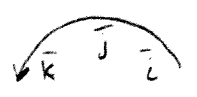
6

c) $\sum \vec{M}_A = 0$

$\sum \vec{M}_A = \vec{r}_{B/A} \vec{T}_{BE} + \vec{r}_{G/A} \vec{T}_{BG} + \vec{r}_{C/A} \vec{T}_{CD}$; where $\vec{r}_{B/A} = 3.0\vec{j}$ and $\vec{r}_{C/A} = 6.0\vec{j}$

$\therefore \sum \vec{M}_A = 3.0\vec{j} \times \frac{T_{BE}}{4.5} (1.5\vec{i} - 3.0\vec{j} + 3.0\vec{k}) + 3.0\vec{j} \times \frac{T_{BG}}{4.5} (1.5\vec{i} - 3.0\vec{j} - 3.0\vec{k})$
 $+ 6.0\vec{j} \times \frac{T_{CD}}{6.5} (-1.5\vec{i} - 6.0\vec{j} + 2.0\vec{k}) = 0$

$\sum \vec{M}_A = -T_{BE}\vec{k} + 2T_{BE}\vec{i} - T_{BG}\vec{k} - 2T_{BG}\vec{i}$
 $+ 1.38T_{CD}\vec{k} + 1.85T_{CD}\vec{i} = 0$



Equate coefficients of \vec{i} & \vec{k} to zero.

(\vec{i}): $2T_{BE} - 2T_{BG} + 1.85T_{CD} = 0$ ———— (1)

(\vec{k}): $-T_{BE} - T_{BG} + 1.38T_{CD} = 0$ ———— (2)

1. Cont'd.

Substitute $T_{CD} = 500\text{ N}$ in eqs. (1) & (2):

$$2T_{BE} - 2T_{BG} + 1.85 \times 500\text{ N} = 0$$

$$2T_{BE} - 2T_{BG} + 925 = 0 \quad \text{--- (1)}$$

and $-T_{BE} - T_{BG} + 1.38 \times 500\text{ N} = 0$

$$-T_{BE} - T_{BG} + 690 = 0 \quad \text{--- (2)}$$

Solving (1) & (2). Multiply (2) by 2 and add to (1)

$$-4T_{BG} + 2305 = 0$$

$$\therefore T_{BG} = \frac{2305}{4} = \underline{\underline{576.25\text{ N}}}$$

ANS.

6 From (2): $-T_{BE} - 576.25 + 690 = 0$

$$\therefore T_{BE} = 690 - 576.25 = \underline{\underline{113.75\text{ N}}}$$

ANS.

Another Method

$$\begin{aligned} \sum \bar{M}_A = & \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 3 & 0 \\ 1.5 & -3 & 3 \end{vmatrix} \times \frac{T_{BE}}{4.5} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 3 & 0 \\ 1.5 & -3 & -3 \end{vmatrix} \times \frac{T_{BG}}{4.5} \\ & + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 6 & 0 \\ -1.5 & -6 & +2 \end{vmatrix} \times \frac{T_{CD}}{6.5} = 0 \end{aligned}$$

$$\sum \bar{M}_A = \frac{9T_{BE}}{4.5} \bar{i} - \frac{4.5T_{BE}}{4.5} \bar{k} - \frac{9T_{BG}}{4.5} \bar{i} - \frac{4.5T_{BG}}{4.5} \bar{k}$$

$$+ \frac{12T_{CD}}{6.5} \bar{i} + \frac{9T_{CD}}{6.5} \bar{k} = 0$$

i.e. $2T_{BE} \bar{i} - T_{BE} \bar{k} - 2T_{BG} \bar{i} - T_{BG} \bar{k} + 1.85T_{CD} \bar{i} + 1.38T_{CD} \bar{k} = 0$

(\bar{i}): $2T_{BE} - 2T_{BG} + 1.85T_{CD} = 0$ --- (1)

(\bar{k}): $-T_{BE} - T_{BG} + 1.38T_{CD} = 0$ --- (2)

Substitute $T_{CD} = 500\text{ N}$ and solve both equations:

$$\therefore T_{BG} = \underline{\underline{576.25\text{ N}}} \text{ and } T_{BE} = \underline{\underline{113.75\text{ N}}} \text{ (check)}$$

ANS.

2. a)

$$\rightarrow \Sigma F_x = 0$$

$$A_x + 10 \text{ kN} = 0$$

$$\therefore A_x = -10 \text{ kN} = 10 \text{ kN} \leftarrow$$

$$\uparrow \Sigma M_E = 0$$

$$10 \text{ kN} \times 3 \text{ m} + 20 \text{ kN} \times 9 \text{ m}$$

$$- 10 \text{ kN} \times 3 \text{ m} - A_y \times 12 \text{ m} = 0$$

$$\therefore 12 A_y = 180$$

$$\text{Hence, } A_y = \frac{180}{12} = 15 \text{ kN} \uparrow$$

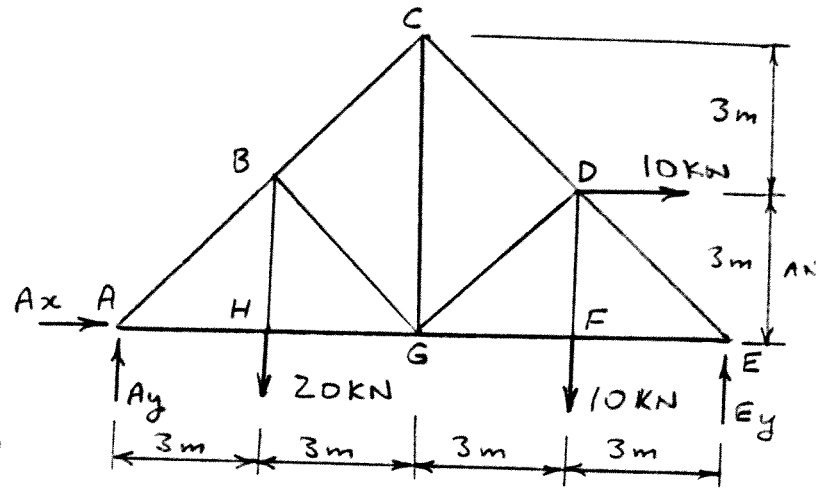
$$\uparrow \Sigma F_y = 0$$

$$E_y - 10 \text{ kN} - 20 \text{ kN} + 15 \text{ kN} = 0$$

$$\therefore E_y = 15 \text{ kN} \uparrow$$

ANS.

ANS.



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b)

$$\uparrow \Sigma M_D = 0$$

$$15 \text{ kN} \times 3 \text{ m} - F_{FG} \times 3 \text{ m} = 0$$

$$\therefore F_{FG} = 15 \text{ kN (T)}, \text{ i.e. } F_{GF} = 15 \text{ kN (T) ANS.}$$

$$\uparrow \Sigma M_G = 0$$

$$15 \text{ kN} \times 6 \text{ m} - 10 \text{ kN} \times 3 \text{ m} - 10 \text{ kN} \times 3 \text{ m} + F_{DC} \times \frac{1}{\sqrt{2}} \times 3 \text{ m} + F_{DC} \times \frac{1}{\sqrt{2}} \times 3 \text{ m} = 0$$

$$90 - 30 - 30 + 4.24 F_{DC} = 0$$

$$\therefore F_{DC} = \frac{-30}{4.24} = -7.07 \text{ kN} = 7.07 \text{ kN (C)}$$

$$\uparrow \Sigma M_E = 0$$

$$\text{A.C.C. } F_{CD} = 7.07 \text{ kN (C)}$$

ANS.

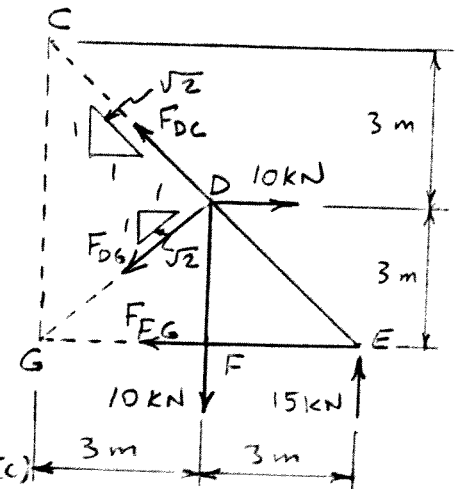
$$10 \text{ kN} \times 3 \text{ m} - 10 \text{ kN} \times 3 \text{ m} + F_{DG} \times \frac{1}{\sqrt{2}} \times 3 \text{ m} + F_{DG} \times \frac{1}{\sqrt{2}} \times 3 \text{ m} = 0$$

$$30 - 30 + 2 F_{DG} \times \frac{1}{\sqrt{2}} \times 3 \text{ m} = 0$$

$$\therefore 2 F_{DG} = 0$$

$$\text{and } F_{DG} = F_{GD} = 0$$

ANS



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Check:

$$\Sigma F_y = 0; 15 \text{ kN} - 10 \text{ kN} - 7.07 \times \frac{1}{\sqrt{2}} = 0$$

$$5 - 5 = 0, \text{ checks } \checkmark$$

3. a)

$$30 \text{ kg} \times 9.81 = 294.3 \text{ N}$$

$$\uparrow \Sigma M_A = 0$$

$$E_x \times 0.9 \text{ m} - 294.3 \text{ N} \times 0.6 \text{ m} - 294.3 \text{ N} \times 1.2 \text{ m} = 0$$

$$\therefore E_x = (176.58 + 353.16) \div 0.9 \text{ m}$$

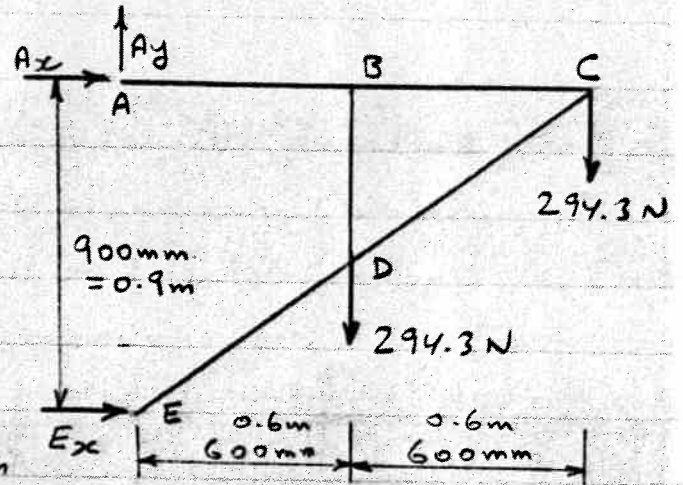
$$E_x = \frac{529.74}{0.9} = 588.6 \text{ N} \rightarrow$$

$$\rightarrow \Sigma F_x = 0$$

$$588.6 \text{ N} + A_x = 0; \therefore A_x = -588.6 \text{ N} = 588.6 \text{ N} \leftarrow \text{ANS.}$$

$$\uparrow \Sigma F_y = 0$$

$$A_y - 294.3 \text{ N} - 294.3 \text{ N} = 0; \therefore A_y = 588.6 \text{ N} \uparrow \text{ANS.}$$

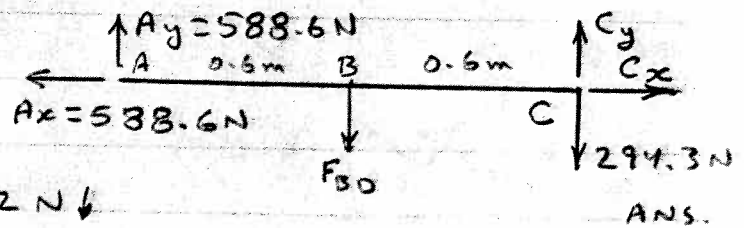
6

b)

$$\uparrow \Sigma M_C = 0$$

$$F_{BD} \times 0.6 \text{ m} - 588.6 \times 1.2 \text{ m} = 0$$

$$\therefore F_{BD} = \frac{588.6 \times 1.2}{0.6} = 1177.2 \text{ N} \downarrow$$



$$\rightarrow \Sigma F_x = 0$$

$$C_x - 588.6 = 0, \therefore C_x = 588.6 \text{ N} \rightarrow \text{ANS.}$$

$$\uparrow \Sigma F_y = 0$$

$$C_y - 294.3 \text{ N} - 1177.2 \text{ N} + 588.6 \text{ N} = 0$$

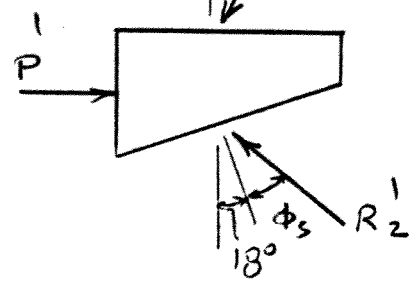
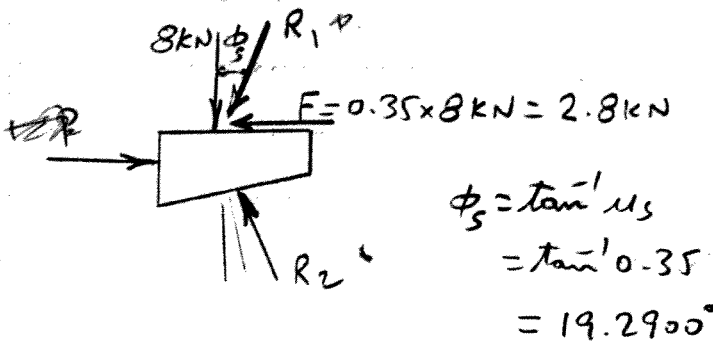
$$\therefore C_y = 882.9 \text{ N} \uparrow \text{ANS.}$$

4.

Since $\mu_s = 0.60$ between the board and the ground is greater than $\mu_s = 0.35$ between all wood surfaces, therefore motion will be impending between all wood surfaces in contact and not between the board and the ground.

Criterion - 2
Motion →

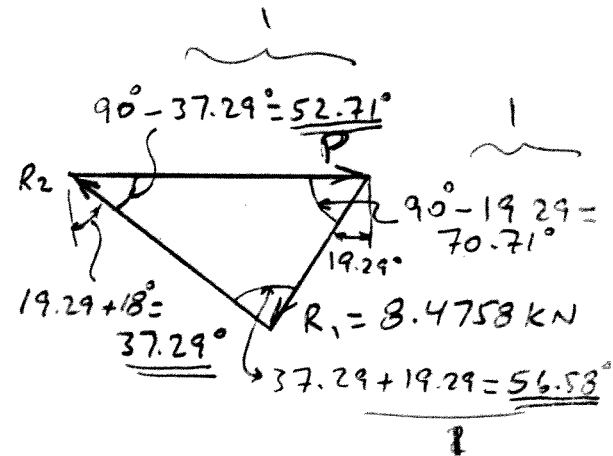
FBD - Top Wedge A



$$\left. \begin{aligned} R_1 \cos 19.2900^\circ &= 8 \text{ kN} \\ \therefore R_1 &= \frac{8 \text{ kN}}{\cos 19.29^\circ} = 8.4758 \text{ kN} \end{aligned} \right\}$$

$$\frac{R_1}{\sin 52.71^\circ} = \frac{P}{\sin 56.58^\circ} \quad \left. \vphantom{\frac{R_1}{\sin 52.71^\circ}} \right\} 2$$

$$\frac{8.4758}{0.7956} = \frac{P}{0.8347}$$



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$$\therefore P = \frac{8.4758 \times 0.8347}{0.7956} = 8.892 \text{ kN} = \underline{\underline{8.89 \text{ kN}}}$$

ANS.

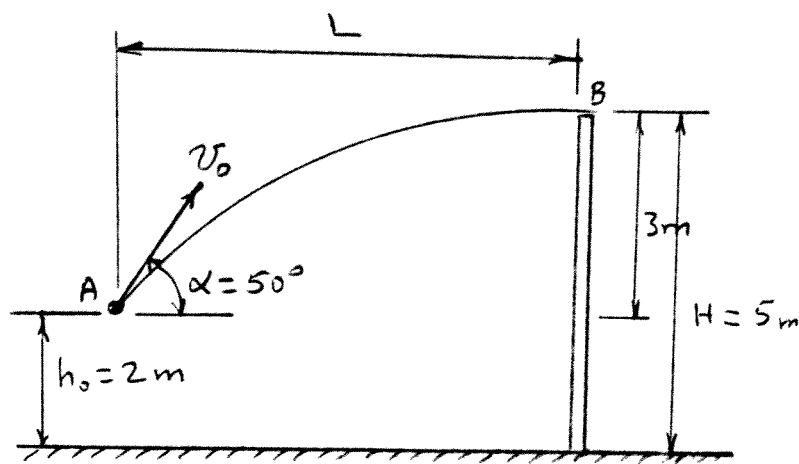
5. a)

$$(v_0)_x = v_0 \cos 50^\circ$$

$$(v_0)_y = v_0 \sin 50^\circ$$

For minimum v_0 , then
B should be the highest
point on the projectile.

$$\text{i.e. } (v_y) \text{ at B} = 0$$



* - Vertical Motion: From A to B.

$$(v_y)_B = (v_0)_y + at, \text{ where } (v_y)_B = 0 \text{ \& } (v_0)_y \text{ at A} = v_0 \sin 50^\circ$$

and $a = -g = -9.81\text{m/s}^2$.

$$\therefore 0 = v_0 \sin 50^\circ - 9.81t$$

$$\therefore t = \frac{v_0 \sin 50^\circ}{9.81} = 0.078 v_0 \text{ sec.}$$

$$\text{Now, } y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$$

$$3\text{m} = 0 + v_0 \sin 50^\circ t - \frac{1}{2} \times 9.81 t^2$$

$$\text{Insert } t = 0.078 v_0$$

$$3 = v_0 \sin 50^\circ \times 0.078 v_0 - \frac{1}{2} \times 9.81 \times (0.078 v_0)^2$$

$$3 = 0.060 v_0^2 - 0.030 v_0^2$$

$$0.03 v_0^2 = 3; \quad \therefore v_0 = \sqrt{\frac{3}{0.03}} = 10\text{m/s}$$

$$\text{and } t = 0.078 \times 10 = 0.78\text{ s.}$$

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b) * - Horizontal Motion

$$x = x_0 + (v_0)_x t \quad * \cos 50 \quad 5.03\text{m}$$

$$L = 0 + 10 \times 0.78 = \underline{\underline{7.8\text{m}}} \text{ Distance from A to the wall.}$$

4

ANS.

END