NAME:

STUDENT NUMBER:

ECO 3153
Winter 2012

Final Exam
April 13th, 2012

Instructions:

1. All questions should be answered on the questionnaire. Use the back of the pages as scrap paper.

2. Only nonprogrammable calculators are permitted during this exam.

3. The marks for each question are given in bold following the question. Budget your time accordingly.

4. The maximum grade is **100**.

5. This exam consists of **14** pages and **4** questions. It is your responsibility to ensure that your exam questionnaire is complete.

5. The exam ends at 12:30.
6. Good luck!

Question 1	/30
Question 2	/30
Question 3	/20
Question 4	/20
Total	/100

Question 1

a) List all the axioms imposed on the technology set (Q) when looking at the production of a simple economy problem. **6 points**.

b) You are stuck alone on an island. Only two goods are available xylophones (x) and crème-brulées (y) for consumption. Your production possibility frontier is represented by

$$4x^2 + y^2 \leq 100$$

Your utility function is given by

$$U(x, y) = x^{\alpha} y^{1-\alpha}$$

where $0 < \alpha < 1$.

i) Set up the Lagrangian for the utility maximizing problem **4 points**.

ii) Show that the optimal consumption of xylophones is $x^* = 10\sqrt{\alpha}$. **5 points**.

iii) If you were to invent prices for xylophones and crème-brulées that would be consistent with the solution above, what would be the price ratio ρ_x/ρ_y? **5 points**.

iv) You can now trade with the rest of the world. The international prices of xylophones and crème-brulées are 2 and 1 dollars, respectively. Show that the optimal *production* of xylophones and crème-brulées will be $5/\sqrt{2}$ and $10/\sqrt{2}$, respectively. **5 points**.

v) At the international prices, how much xylophones and crème-brulées would you import/export? Hint: The answers are functions of α. **5 points**.

Question 2

Assume you want to maximize the profit of your firm. We have seen in class that the firm profit maximization problem can be broken into two stages: 1) cost minimization and 2) output choice. The cost minimization problem is:

$$\min_{z} \sum_{i=1}^{m} w_i z_i$$

subject to

$$\begin{aligned} q &\leq \phi(\mathbf{z}) \\ q &\geq 0 \\ z_i &\geq 0 \text{ for } i = 1, 2, ..., m \end{aligned}$$

a) Write the Lagrangian equation for this problem. **3 points**.

b) Show that, if we have an interior solution, the marginal rate of technical substitution will equal the input price ratio at the optimum. **5 points**.

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c) Now define the cost function $C(\mathbf{w}, q)$ as

$$\begin{aligned} C(\mathbf{w}, q) &= \min_{z \geq 0, q \leq \phi(\mathbf{z})} \sum_{i=1}^{m} w_i z_i \\ &= \sum_{i=1}^{m} w_i H^i(\mathbf{w}, q) \end{aligned}$$

Show that

$$\frac{\partial C(\mathbf{w}, q)}{\partial w_i} = H^i(\mathbf{w}, q)$$

7 points.

d) Show that the cost function is homogeneous of degree 1 in $\mathbf{w}$. **5 points**.

e) Show that the cost function is concave in prices. **5 points**

f) Assume that the cost function is convex in q. Show that the supply curve (obtained from the second stage of the profit maximization problem), $S(\mathbf{w}, p)$, is upward sloping (in p). **5 points**

Question 3

Let a consumer's indirect utility function be

$$V(p, y) = -\frac{\left(\sum_{i=1}^{n} \sqrt{p_i}\right)^2}{y}$$

where y is the consumer's fixed income.

a) What is the expenditure function $C(p, v)$? **5 points**.

b) What is the Hicksian demand function for good i? **7 points**.

c) What is the Marshallian demand function for good i? **8 points**.

Question 4

You are studying for your ECO3153 final exam. You know that your grade on the final exam, g, depends on your effort level, e, and on the professor genertosity, ε. More specifically, the grade is determined by the following equation

$$g(e) = f(e) + \varepsilon$$

where $f(e)$ is a strictly increasing and stictly concave function of e (i.e. $f'(e) > 0$ and $f''(e) < 0$) and where $f(0) = 0$. The professor's generosity is a random variable such that $E(\varepsilon) = \mu > 0$ and $var(\varepsilon) = \sigma^2$.

Your felicity function has the following shape:

$$u(x) = \beta_0 + \beta_1 g(e) + \beta_2 [g(e)]^2 - C(e)$$

where $\beta_1 > 0$, $\beta_2 < 0$, and $C(e)$ is the cost of exerting effort. $C(e)$ is deterministic (i.e. not random) and increasing and convex in e.

a) Show that your expected utility can be written as

$$E(u(x)) = \beta_0 + \beta_1 \mu + \beta_2 [\sigma^2 + \mu^2] + [\beta_1 + 2\mu\beta_2] f(e) + \beta_2 [f(e)]^2 - C(e)$$

5 points.

b) Assuming an interior solution, show that the condition that will determine the optimal value of e, (e^*) is given by

$$[\beta_1 + 2\mu\beta_2]f'(e^*) + 2\beta_2 f(e^*)f'(e^*) = C'(e^*)$$

5 points.

c) Show the conditions under which you will exert a strictly positive amount of effort (i.e. $e^* > 0$). **5 points**.

d) Write the expression that would determine the effect of increasing the professor's average generosity (say by τ) on the optimal effort level. Assume an interior solution. **5 points**.