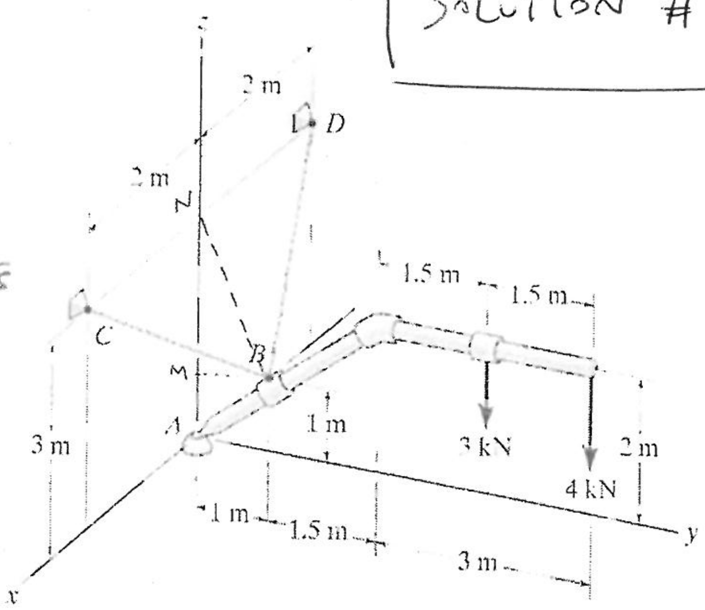
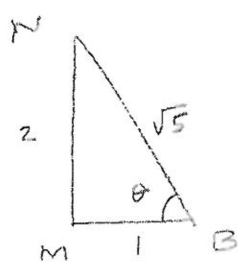
SOLUTION #1

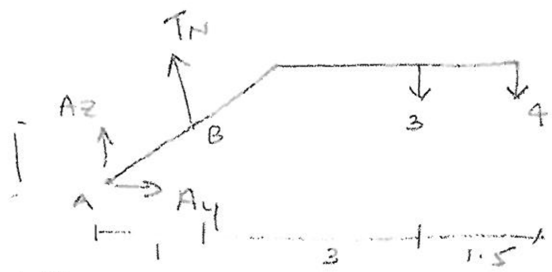
Q1. The pipe assembly supports the vertical loads shown. Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables BC and BD.





$BN = \sqrt{2^2+1^2} = \sqrt{5}$

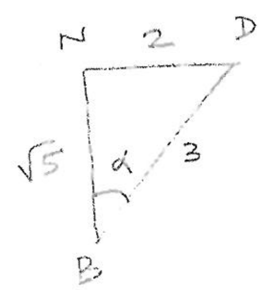
Considering yz plane



$\circlearrowleft \Sigma M_A \quad (T_N \cos\theta)(1) + (T_N \sin\theta)(1) - 3(4) - 4(5.5)$

$T_N \ (\sin\theta + \cos\theta) = 34$

$T_N = \frac{34\sqrt{5}}{3}$



$BD = \sqrt{5+2^2} = 3$

Due to Symmetry

$T_c = T_D = T$

$A_x = 0$

$2T\cos\alpha = T_N$

$T = \frac{34\sqrt{5}}{3} \cdot \frac{1}{2} \cdot \frac{3}{\sqrt{5}} = 17$

$\Rightarrow T_c = T_D = 17 \text{ k}$

$\rightarrow \quad A_y = T_N \cos\theta = \frac{34}{3} = 11.3 \text{ kN}$

$\uparrow \quad A_z = 7 - T_N \sin\theta = 7 - \frac{34(2)}{3} = -15.7 \text{ kN}$

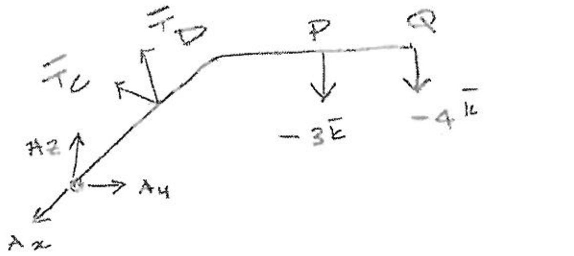
$A_x = 0 \ ; \quad A_y = 11.3 \text{ kN} \ ; \quad A_z = -15.7 \text{ kN}$

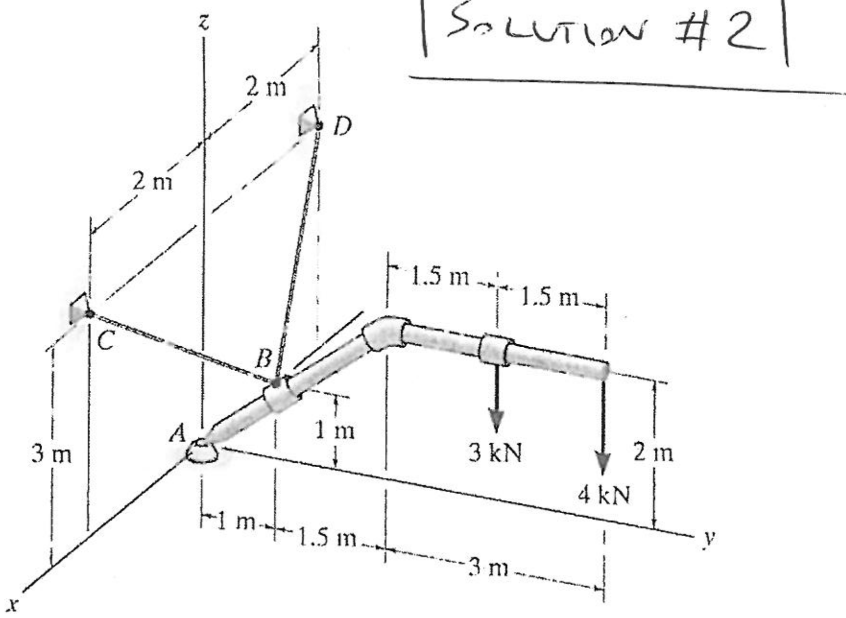
$T_{BC} = T_{BD} = 17 \text{ kN}$

[2]

SOLUTION #2

Q1. The pipe assembly supports the vertical loads shown. Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables BC and BD.





$\bar{r}_{BC} = 2\bar{i} - 1\bar{j} + 2\bar{k}$, $|\bar{r}_{BC}| = \sqrt{9} = 3$

$\bar{u}_{BC} = \frac{2}{3}\bar{i} - \frac{1}{3}\bar{j} + \frac{2}{3}\bar{k}$; $\bar{T}_C = T_C \cdot \bar{u}_{BC}$

$\therefore \bar{u}_{BD} = -\frac{2}{3}\bar{i} - \frac{1}{3}\bar{j} + \frac{2}{3}\bar{k}$ $\quad \bar{T}_D = T_D \cdot \bar{u}_{BD}$

$\Sigma \bar{M}_A = \bar{r}_{AB} \times \bar{T}_{BC} + \bar{r}_{AB} \times \bar{T}_{BD} + \bar{r}_{AP} \times (-3\bar{k}) + \bar{r}_{AQ} \times (-4\bar{k}) =$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 1 & 1 \\ \frac{2T_C}{3} & -\frac{T_C}{3} & \frac{2T_C}{3} \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 1 & 1 \\ -\frac{2T_D}{3} & -\frac{T_D}{3} & \frac{2T_D}{3} \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 4 & 2 \\ 0 & 0 & -3 \end{vmatrix} + \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & 5.5 & 2 \\ 0 & 0 & -4 \end{vmatrix} = \bar{0}$$

Considering '$\bar{k}$' terms $\quad -\frac{2T_C}{3}\bar{k} + \frac{2T_D}{3}\hat{k} = \bar{0}$

$\Rightarrow T_C = T_D$

(Alternatively : $T_C = T_D$ due to Symmetry)

Considering '$\bar{i}$' terms

$\left(\frac{2T_C}{3} + \frac{T_C}{3}\right)\bar{i} + \left(\frac{2T_D}{3} + \frac{T_D}{3}\right)\bar{i} + (-12\bar{i}) + (-22\bar{i}) = 0$

$T_C + T_D = 34$;

$T_C = 17$ kN

$T_D = 17$ kN

$\swarrow \Sigma F_x = 0$

$A_x + T_C\left(\frac{2}{3}\bar{i}\right) - T_D\left(\frac{2}{3}\bar{i}\right) = 0 \quad \Rightarrow \quad A_x = 0$

$\rightarrow \Sigma F_y = 0$

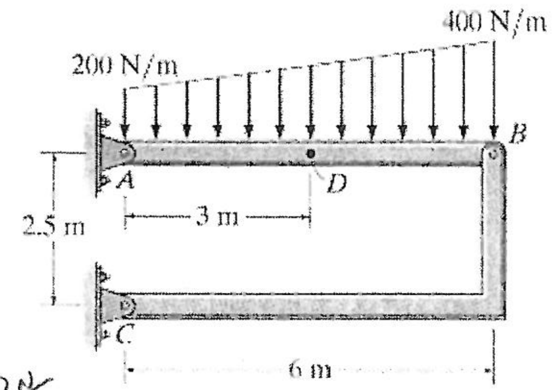
$A_y + T_C\left(-\frac{1}{3}\right) + T_D\left(-\frac{1}{3}\right) = 0 \quad \Rightarrow \quad A_y = \frac{34}{3}$ kN $= 11.33$ kN

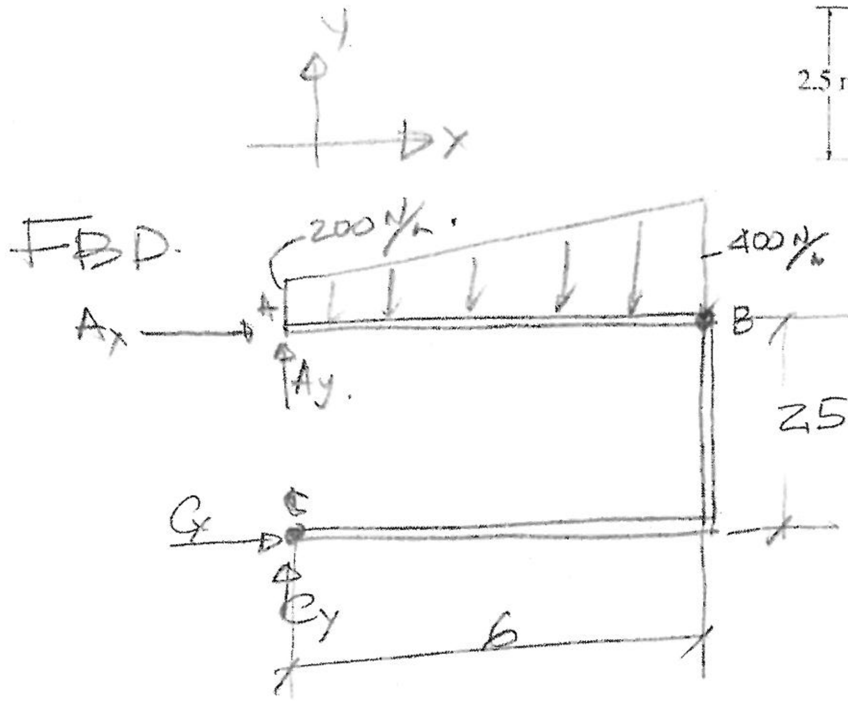
$\uparrow \Sigma F_z = 0$

$A_z + \frac{2T_C}{3} + \frac{2T_D}{3} - 7 = 0 \qquad A_z = -15.66$ kN [2]

$A_z = 7 - \frac{47}{3}$

Q2. Determine the normal force, shear force, and the bending moment at a section passing through point D of the frame.

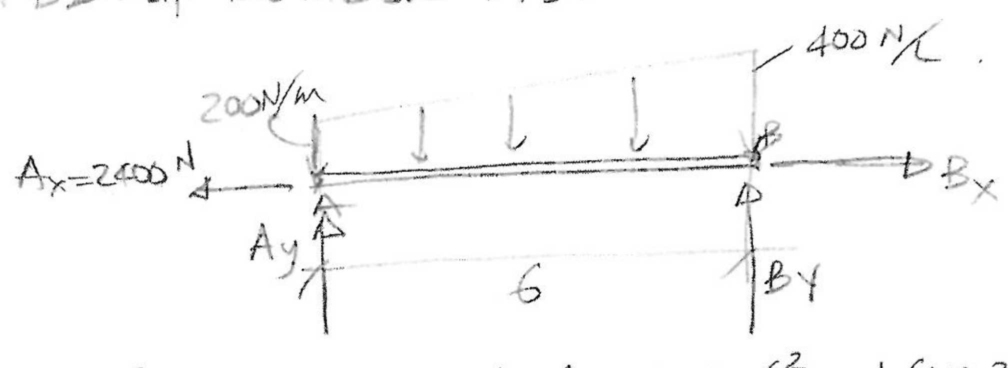




$$\Sigma M_C = 0 \Rightarrow -\frac{200 \cdot 6^2}{2} - \frac{1}{2}(400-200) \cdot \frac{6^2}{3} \cdot 2 - A_x \cdot 2.5 = 0.$$

$$\Rightarrow A_x = -2400\ N = 2.40\ kN.$$

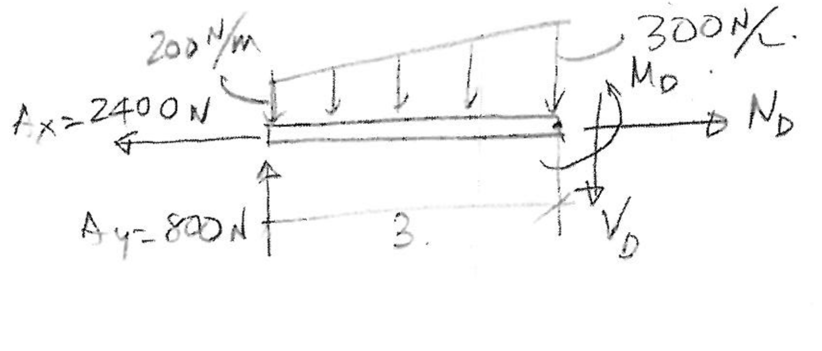
FBD. of MEMBER AB.



$$\Sigma M_B = 0 \Rightarrow -A_y 6 + \frac{200 \cdot 6^2}{2} + \frac{1}{2}(400-200)\frac{6^2}{3} = 0$$

$$A_y = 800\ N$$

MAKE SECTION @ D AND CONSIDER SECTION AD



$$\Sigma F_x = 0 \Rightarrow -2400 + N_D = 0$$

$$\underline{\underline{N_D = 2400\ N}} = 2{,}40\ kN$$

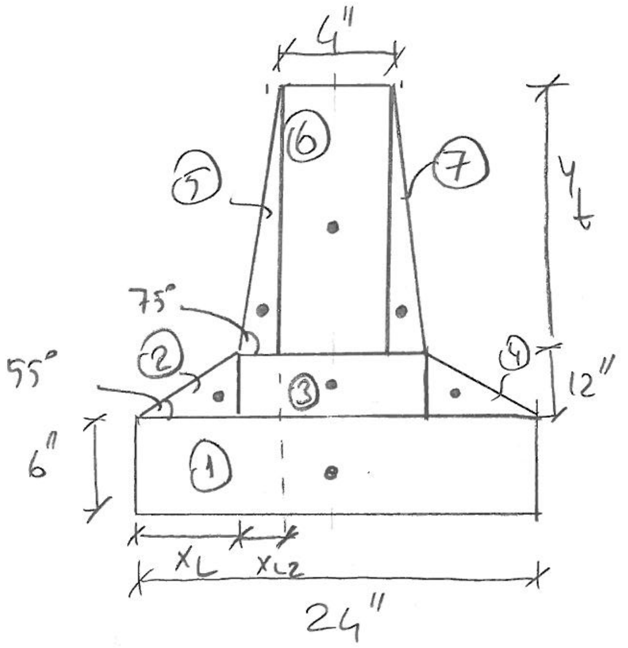
$$\Sigma F_y = 0 \Rightarrow 800 - \frac{1}{2}(200+300) \cdot 3 - V_D = 0$$

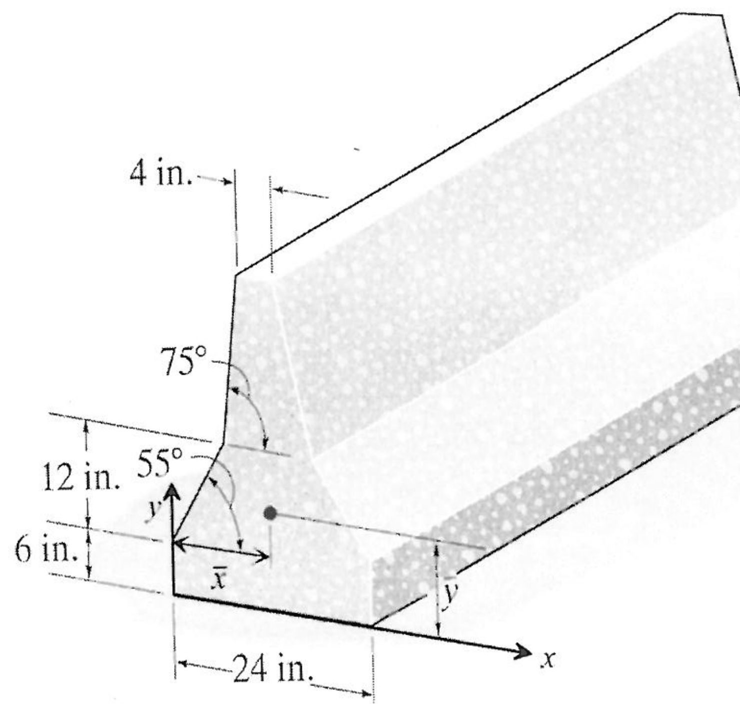
$$\underline{\underline{V_D = 50\ N}}$$

$$\Sigma M_D = 0 \Rightarrow -800 \cdot 3 + 200 \cdot \frac{3^2}{2} + \frac{1}{2}(300-200)\frac{3^2}{3} + M_D = 0$$

$$M_D = 1350\ Nm = 1{,}35\ kNm$$

Q3. A concrete barrier commonly used in highway construction is shown in the figure. Determine the centroid $(\bar{x}, \bar{y})$ of the cross-sectional area, if the origin is at the bottom left corner of the barrier as shown in the Figure.





$$\tan 55 = \frac{12}{x_L} \Rightarrow x_L = \frac{12}{\tan 55} = 8.4025''$$

$$x_L + x_{L2} + 2 = \frac{24}{2} \Rightarrow x_{L2} = 12 - 2 - 8.4025 = 1.5975''$$

$$\tan 75 = \frac{y_t}{x_{L2}} \Rightarrow y_t = x_{L2} \tan 75 = 1.5975 \tan 75 = 5.962''$$

Section	A_i (in^2)	$\bar{x}_i$ (in)	$\bar{y}_i$ (in)	$A_i \bar{x}_i$ (in^3)	$A_i \bar{y}_i$ (in^3)
1	144	12	3	1728	432
2	50.415	5.602	10	282.425	504.15
3	86.34	12	12	1036.08	1036.08
4	50.415	18.398	10	927.535	504.15
5	4.762	9.4625	19.987	45.08	95.180
6	23.848	12	20.981	286.176	500.355
7	4.762	14.5325	19.987	69.204	95.180

$\Sigma A_i = 364.542$

$\Sigma A_i \bar{x}_i = 4374.50$

$\Sigma A_i \bar{y}_i = 3167.10$

$$\bar{x} = \frac{\Sigma A_i \bar{x}_i}{\Sigma A_i} = \frac{4374.50}{364.642} = 12''$$

$$\bar{y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{3167.10}{364.642} = 8.69''$$

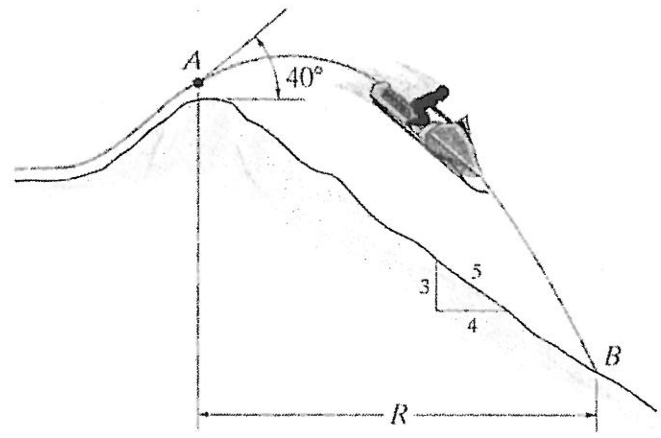
[4]

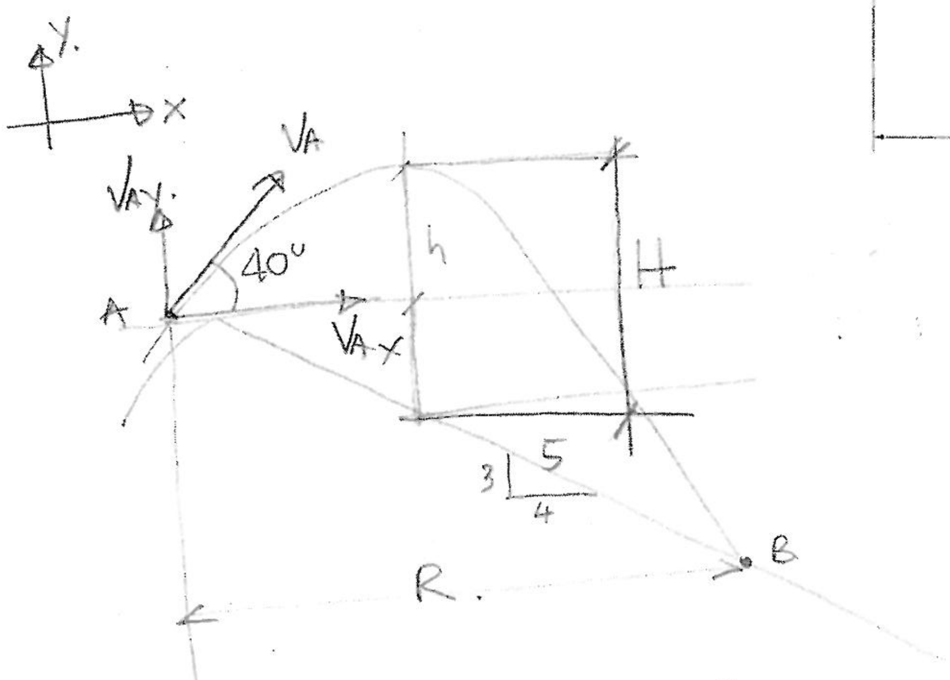
Q4. The snowmobile is travelling at a speed of $V_o = 10\ m/s$ when it leaves the embankment at A at angle $\theta = 40°$. The angle of the inclined slope can be approximated as shown in the figure. Determine

(a) the time of flight from A to B

(b) the range R of the trajectory

(c) the height above the ground level when it is at the highest elevation.





$V_A = 10 m/s \Rightarrow$ $V_{Ax} = V_A \cos 40° = 10 \cdot \cos 40° = 7{,}66\ m/s$

$V_{Ay} = V_A \sin 40° = 10 \cdot \sin 40° = 6{,}427\ m/s$

X-MOTION

$a_x = 0$

$V_x = V_{Ax} + a_x t \Rightarrow V_x = V_{Ax} = 7{,}66\ m/s$

$X = X_A + V_{Ax} t + \frac{1}{2} a_x t^2 \Rightarrow X = X_A + V_{Ax} t$

$X_B = R = 0 + 7{,}66 t$ ——— ①

Y-MOTION

$a_y = -g$

$V_y = V_{Ay} - gt = 6{,}427 - 9{,}81 t$ ——— ②

$Y = Y_A + V_{Ay} t - \frac{1}{2} g t^2 = 0 + 6{,}427 t - \frac{9{,}81}{2} t^2$

$Y_B = -\frac{3}{4} R = 6{,}427 t - \frac{9{,}81}{2} t^2$

FROM ① $R = 7{,}66 t$

[5]

$$t\left(6{,}427 + \frac{3}{4}\cdot 7{,}66 - \frac{9{,}81}{2}t\right) = 0.$$

$\therefore$ $t = 0$ s

$t = 2{,}48$ s.

a) Time of flight $= \underline{\underline{2{,}48\,s}}$.

b) Range $R = 7{,}66t = 7{,}66 \cdot 2{,}48 = \underline{\underline{19\,m}}$

c) At peak elevation $V = 0$.

From (2)

$$6{,}427 - 9{,}81t = 0$$

$$\Rightarrow t = 0{,}655\ s$$

$$h = 6{,}427 \cdot 0{,}655 - \frac{9{,}81 \cdot 0{,}655^2}{2}$$

$$= \underline{\underline{2{,}1\,m}}.$$

$$X(\text{at } 0{,}655) = 7{,}66 \cdot 0{,}655 = 5{,}017\,m$$

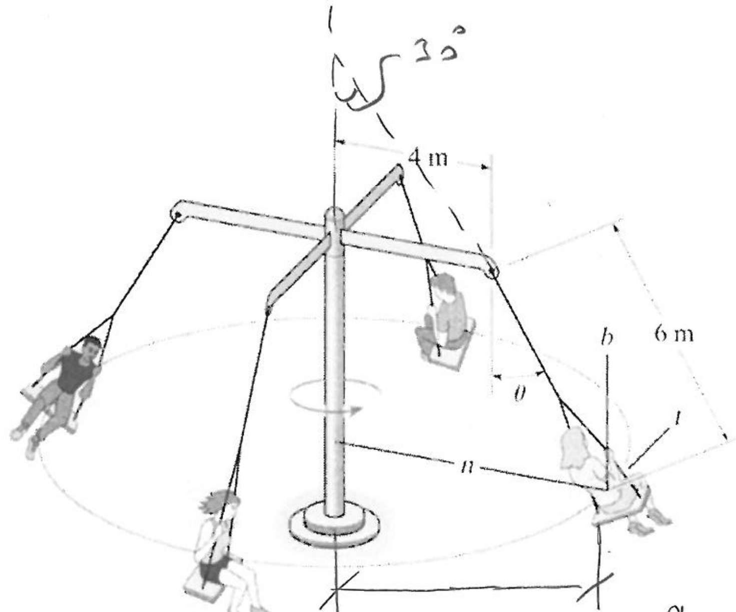
$\therefore$ Height above ground level H is

$$H = \frac{3}{4} \cdot 5{,}017 + 2{,}1 = \underline{\underline{5{,}86\,m}}.$$

Q5. Each chair of an amusement park ride has a mass of 80kg (including the passenger) and rotates at a constant speed. The 6m long cables are inclined at an angle of $\theta = 30°$ to vertical and each arm of the ride is 4m long. Determine

(a) the constant speed of the passengers

(b) the normal and tangential components of the force exerted by the chair on the passenger if the mass of the passenger is 50kg.

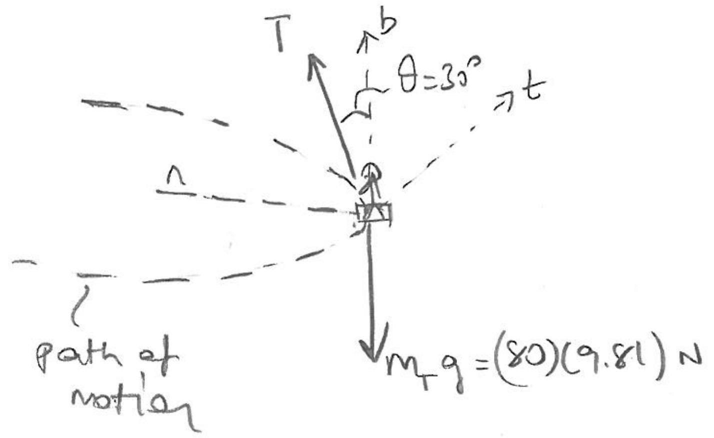


$\rho = 4 + 6\sin 30$

$\rho = 7\,m$

$a_b = 0$

FBD of the passenger and the chair :



$\sum \vec{F}_b = 0 \Rightarrow T\cos 30 - m_T g = 0$

$T = \frac{(80)(9.81)}{\cos 30} \Rightarrow \boxed{T = 906.2N}$

$\sum \vec{F}_n = 0 \Rightarrow T\sin 30 = m_T a_n$

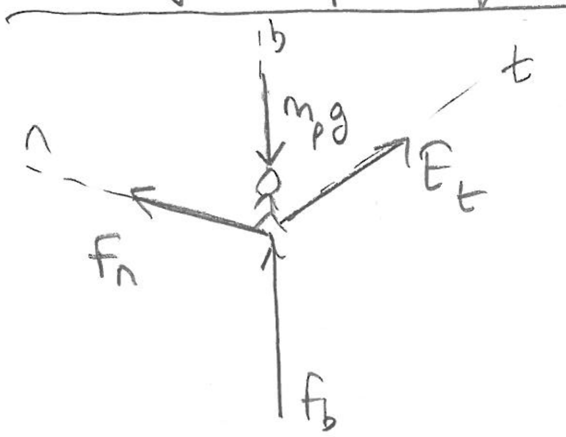
$(906.2)\sin 30 = (80)\, a_n$

$a_n = 5.66\ m/s^2$

$a_n = \frac{v^2}{\rho} = \frac{v^2}{4+6\sin 30} = 5.66$

a) $\boxed{v = 6.3\ m/s}$ ANSWER

b) FBD of the passenger alone :



$\sum \vec{F}_n = m_p \vec{a}_n$

$F_n = (50)(5.66)$

$\boxed{F_n = 283\ N}$ ANSWER

$\vec{F}_t = m_p \vec{a}_t$

$v = \text{constant} \Rightarrow \therefore a_t = 0 \Rightarrow \boxed{F_t = 0}$ ANSWER

[6]

Q6. Two students A and B, each weighing 150 lb jump off the bridge with zero initial velocity holding to an elastic cord. The stiffness of the cord is 80 lb/ft.

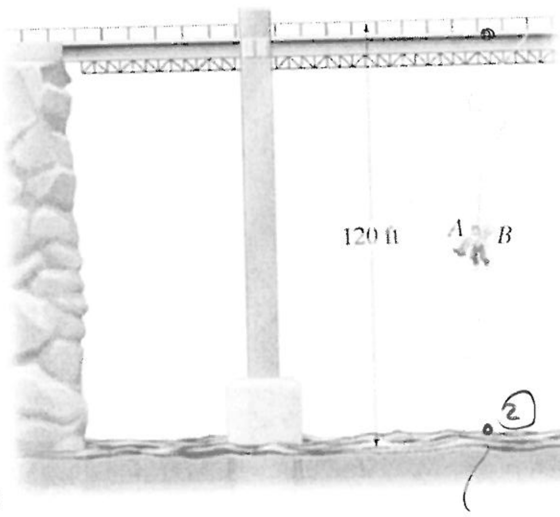
(a) Determine the unstretched length of the cord required if they want to just reach the surface of the river.

(b) At the instant they touch the river, if student B lets go off the cord and student A holds on to the cord and rebounds upwards, determine the maximum height he will reach.

(c) What is the maximum acceleration of A during the entire stunt.

① $(v_1 = 0)$

$h_1 = 120\,ft$



$h_2 = 0$ REF.

$(v_2 = 0$ when they reach the river)

a) Conservation of energy b/w positions ① and ②:

$$T_1 + V_{g_1} + V_{e_1} = T_2 + V_{g_2} + V_{e_2}$$

$$\cancel{\tfrac{1}{2} m v_1^2} + \underbrace{(M_A + M_B) g}_{W_A + W_B = 150 + 150 = 300\,lb}\, h_1 + \cancel{\tfrac{1}{2} k s_1^2} = \cancel{\tfrac{1}{2} m v_2^2} + \cancel{(M_A + M_B) h_2 g} + \tfrac{1}{2} k s_2^2$$

$$(300)(120) = \tfrac{1}{2}(80) s_2^2 \Rightarrow s_2 = 30\,ft \text{ (strech)} \Rightarrow \boxed{l_0 = 120 - 30 = 90\,ft}$$

ANSWER
UNSTRECHED LENGTH

b) If the position where the maximum height reached is ③ :

$$T_2 + V_{g_2} + V_{e_2} = T_3 + V_{g_3} + V_{e_3} \Rightarrow \cancel{0} + \cancel{0} + \tfrac{1}{2} k s_2^2 = \cancel{0} + (150) h_3 + \cancel{0}$$

$v_2 = 0$ $h_2 = 0$ $v_3 = 0$ W_A $s_3 = 0$

assuming the cord does not strech on the way back.

$$\tfrac{1}{2}(80)(30)^2 = 150 h_3$$

$h_3 = 240\,ft \Rightarrow$ This is large such that cord will strech on the way up $\Rightarrow s_3 \neq 0$

$$\therefore \tfrac{1}{2}(80)(30)^2 = 150 h_3 + \tfrac{1}{2}(80) s_3^2$$

$$\tfrac{1}{2}(80)(30)^2 = 150 h_3 + \tfrac{1}{2}(80)(h_3 - 120 - 90)^2$$

120: height of the bridge

90: unstreched length of the cord

$\boxed{h_3 = 219\,ft}$ ANSWER

c)

$$T - mg = ma$$

$$\boxed{a = \frac{T - mg}{m}}$$

$\therefore$ maximum $\bar{a}$ will occur when T is max and m is minimum.

@ position # 2 for m_A

$$\therefore a = \frac{(80)(30) - 150}{\left(\frac{150}{32.2}\right)}$$

150: W_A

$\frac{150}{32.2}$: M_A

$$\boxed{a_{max} = 483\,ft/s^2}$$

[7]

ANSWER