Economics 3M03
Sample Test 2 October 17, 2012

1. Two players are about to play in a one shot mixed strategy game. They each intend to choose a mixed strategy that will yield a maximum expected payoff given the decision of the other player. The action set for player 1 is $\{Top, Middle, Bottom\}$ and player 2's action set is $\{Left, Right\}$.

		q	$1-q$
		Left	Right
p	Top	(2,7)	(4,4)
r	Middle	(6, 2)	(2,7)
$1-r-p$	Bottom	(4,7)	(2.5,2)

1a: Write out player 1's expected payoff for any arbitrary mixed strategy profile $\big((p, r, 1-p-r), (q, 1-q)\big)$ where p is the probability that Player 1 chooses top, r is the probability that player 1 chooses Middle and q is the probability that player 2 chooses left.

1b Write out player 1's expected payoff to each of its three actions (pure strategies).

1c: Show the relationship between player 1's expected payoff (your answer to 3a) and its expected payoff to each of its actions (your answer to part (3b).

1d: Draw a graph of player 1's best reply to its beliefs about the likelihood that player 2 will choose left.

1e Based on your answer to Part 3d write out player 1's best reply function to its beliefs about Player 2's choice.

1.f Use your responses to question 3 to study this question: Suppose there is an action which is never a best reply for Player 1 to any beliefs about player 2's decision. Does it follow that this action is weakly dominated by another action for the player?

2. Two players are about to play in a one shot mixed strategy game. They each intend to choose a mixed strategy that will yield a maximum expected payoff given the decision of the other player. The action set for player 1 is $\{Top, Middle, Bottom\}$ and player 2's action set is $\{Left, Right\}$.

		q	$1-q$
		Left	Right
p	Top	(2,7)	(4,4)
r	Middle	(6, 2)	(2,7)
$1-r-p$	Bottom	(4,7)	(2.5,2)

2a: Write out player 1's expected payoff for any arbitrary mixed strategy profile $\big((p,r,1-p-r),(q,1-q)\big)$ where p is the probability that Player 1 chooses top, r is the probability that player 1 chooses Middle and q is the probability that player 2 chooses left.

Answer to 2a Player 1's expected payoff is

$$\begin{aligned} &U\big((p,r,1-p-r),(q,1-q)\big) \\ &= 2pq + 4p(1-q) + 6rq + 2r(1-q) + 4(1-r-p)(q) + 2.5(1-r-p)(1-q) \\ &= p\big[2q + 4(1-q)\big] + r\big[6q + 2(1-q)\big] + (1-r-p)\big[4(q) + 2.5(1-q)\big] \end{aligned} \tag{1.1}$$

2b Write out player 1's expected payoff to each of its three actions (pure strategies).

Answer to 2b

$$\begin{aligned} E_1(T,q) &= 2q + 4(1-q) \\ E_1(M,q) &= 6q + 2(1-q) \\ E_1(B,q) &= 4q + 2.5(1-q) \end{aligned} \tag{1.2}$$

2c: Show the relationship between player 1's expected payoff (your answer to 3a) and its expected payoff to each of its actions (your answer to part (3b).

Answer to 2.c

$$\begin{aligned} & U\left(\left(p,r,1-p-r\right),\left(q,1-q\right)\right) \\ & = p\left[2q+4(1-q)\right]+r\left[6q+2\left(1-q\right)\right]+\left(1-r-p\right)\left[4\left(q\right)+2.5\left(1-q\right)\right] \\ & = pE_1\left(T,q\right)+rE_1\left(M,q\right)+\left(1-r-p\right)E_1\left(B,q\right) \end{aligned} \tag{1.3}$$

2d Draw a graph of player 1's expected payoffs and best reply to its beliefs about the likelihood that player 2 will choose left.

2e Based on your answer to Part 3d write out player 1's best reply function to its beliefs about Player 2's choice.

$$B_1\left(q\right)=\begin{cases} T, 0\le q\le \dfrac{1}{3} \\ \left\{T,M\right\}, q=\dfrac{1}{3} \\ M, \dfrac{1}{3}\le q\le 1 \end{cases} \tag{1.4}$$

2.f Use your responses to question 3 to study this question: Suppose there is an action which is never a best reply for Player 1 to any beliefs about player 2's decision. Does it follow that this action is weakly dominated by another action for the player?

The pure strategy Bottom is never a best reply but neither the pure strategy Top nor the pure strategy Middle weakly dominate Bottom.