It will cost Ralph \$176.20 per hour.

b) Re-arranging the production function: $L = \left(\frac{Q}{K^2}\right)^{4/3}$

Substituting L in Eq 1:

$$C(Q) = 12\left(\frac{Q}{\bar{K}^2}\right)^{4/3} + 10\bar{K}$$

$$C(Q) = 12\left(\frac{Q}{100}\right)^{4/3} + 100$$

c) $$MC = 12 \cdot \frac{4}{3}\left(\frac{Q}{100}\right)^{1/3} = 16\left(\frac{Q}{100}\right)^{1/3}$$

$$AC = \frac{12\left(\frac{Q}{100}\right)^{4/3}}{Q} + \frac{100}{Q} = \frac{12}{100^{4/3}} Q^{1/3} + \frac{100}{Q}$$

$$AVC = \frac{12\left(\frac{Q}{100}\right)^{4/3}}{Q} = \frac{12}{100^{4/3}} Q^{1/3}$$

$$AFC = \frac{100}{Q}$$

Hilroy

d) $\max_{L,K} \ Q = L^{0.75}K^2$ where $12L + 10 = 44{,}000$

Define the Lagrangian as:

$$\mathcal{L} = L^{0.75}K^2 + \lambda(44{,}000 - 12L - 10K)$$

The FOCs are

$$\frac{\partial \mathcal{L}}{\partial L} = 0.75L^{-0.25}K^2 - 12\lambda = 0 \qquad \text{(Eq 1)}$$

$$\frac{\partial \mathcal{L}}{\partial K} = 2L^{0.75}K - 10\lambda = 0 \qquad \text{(Eq 2)}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 44{,}000 - 12L - 10K = 0 \qquad \text{(Eq 3)}$$

Re-arranging Eq 1

$$12\lambda = 0.75L^{-0.25}K^2$$

$$\Leftrightarrow \quad \lambda = \frac{1}{16}L^{-0.25}K^2$$

Re-arranging Eq 2

$$10\lambda = 2L^{0.75}K$$

$$\Leftrightarrow \quad \lambda = \frac{1}{5}L^{0.75}K$$

Therefore, $\frac{1}{16}L^{-0.25}K^2 = \frac{1}{5}L^{0.75}K$

$$\Leftrightarrow \quad \frac{5K}{16} = L$$

Replacing L in Eq 3

$$44{,}000 = 12\left[\frac{5}{16}K\right] + 10K$$

$$\Leftrightarrow \quad 44{,}000 = \frac{15}{4}K + 10K$$

$$\Leftrightarrow \quad 44{,}000 = \frac{55}{4}K$$

$$\Leftrightarrow \quad K = 3{,}200$$

$$\Leftrightarrow \quad L = 1{,}000$$

Ralph will hire 3,200 units of capital and 1,000 units of labour.

Question 10

$$f(L,K) = LK$$

a) $\min_{L,K} \; c(Q) = \omega L + rK$ where $LK = Q$

Define the Lagrangian as:

$$\mathcal{L} = \omega L + rK + \lambda(Q - LK)$$

The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial L} = \omega - \lambda K = 0 \qquad \text{Eq 1}$$

$$\frac{\partial \mathcal{L}}{\partial K} = r - \lambda L = 0 \qquad \text{Eq 2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Q - LK = 0 \qquad \text{Eq 3}$$

Re-arrange Eq 1 : $\lambda = \frac{\omega}{K}$

Re-arrange Eq 2 : $\lambda = \frac{r}{L}$

Therefore $\frac{w}{K} = \frac{r}{L} \iff wL = rK \iff L = \frac{rK}{w}$

Substituting in Eq 3

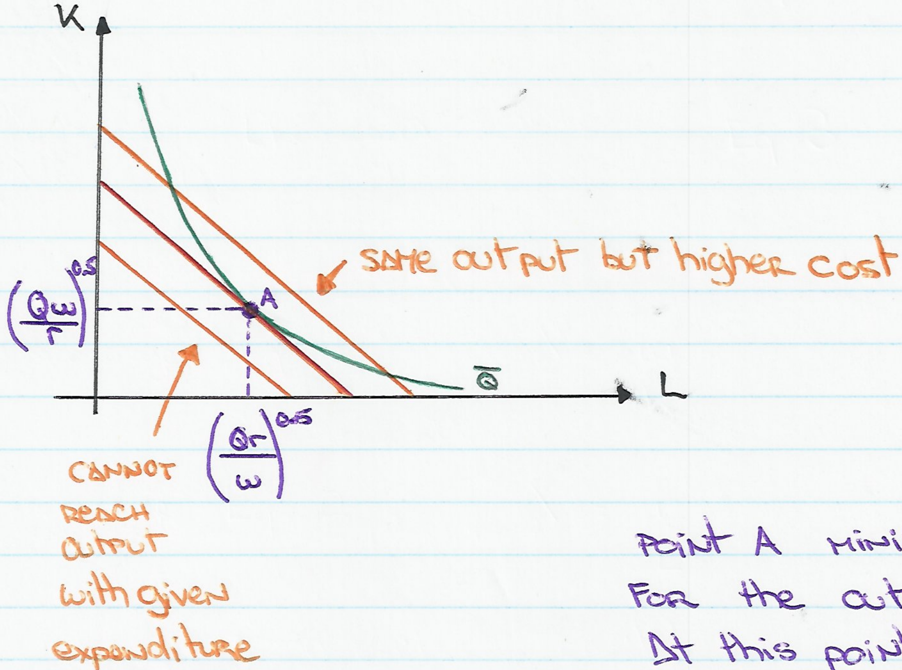
$$Q = \frac{rK}{w} \cdot K$$

$$\iff Q = \frac{r}{w} K^2$$

$$\iff K = \left(\frac{Qw}{r}\right)^{0.5}$$

$$\iff L = \frac{r}{w}\left(\frac{Qw}{r}\right)^{0.5} = \left(\frac{Qr}{w}\right)^{0.5}$$

b)



Point A minimizes cost for the output Q. At this point the isocost is tangent to the isoquant.