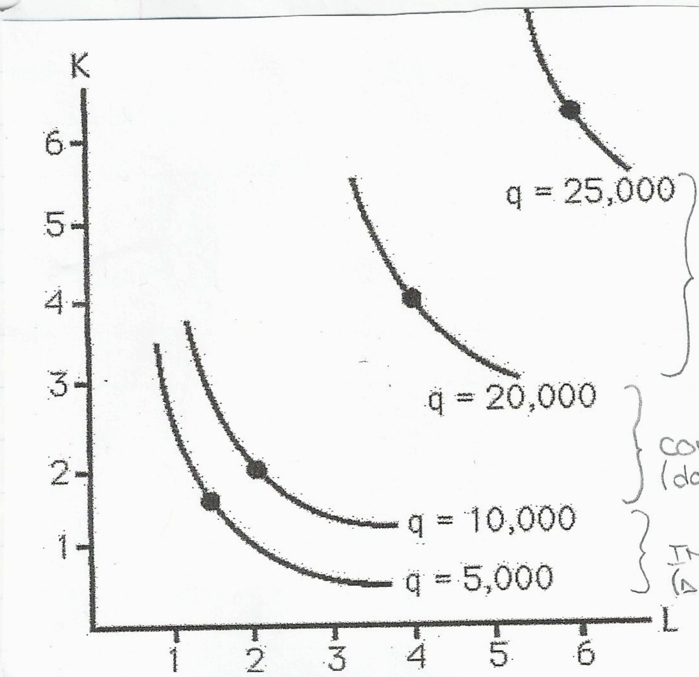
Question 5



decreasing returns to scale (increasing inputs 50% increases output by only 25%)

constant returns to scale (doubling inputs doubles output)

increasing returns to scale (Δ 33% increase in inputs doubles output)

Question 6

$$q = K^{0.5} L^{0.5} \iff q = [KL]^{0.5}$$

i) $MP_L = 0.5K^{0.5}L^{-0.5}$

$$\frac{dMP_L}{dL} = -0.25K^{0.5}L^{-1.5} < 0$$

The first derivative of MP_L is always negative. Therefore, MP_L is always decreasing

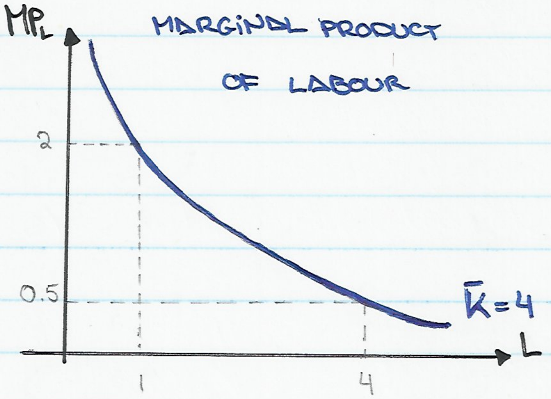
The production function displays diminishing marginal product of labour.

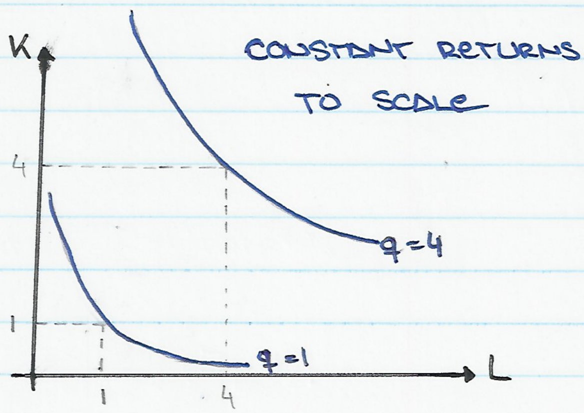
ii) Let α be a positive scalar multiplier. Increasing both K and L by α gives:

$$([\alpha K][\alpha L])^{0.5} = (\alpha^2[KL])^{0.5} = \alpha [KL]^{0.5}$$

When we multiply both inputs by α, the output is also multiplied by α.

Therefore, the production function displays constant returns to scale.





Question 7

$$q = 4LK^{0.5}$$

a) $MP_L = 4K^{0.5}$

$$\frac{dMP_L}{dL} = 0$$

Alison's production function exhibits constant marginal returns, not diminishing marginal returns.

b) Let c represent short-run costs

$$c(L) = w(L)L + 50 \cdot 4$$

$$\Leftrightarrow \quad c(L) = [10 + 2L]L + 200$$

However, re-arranging the production function yields:

$$L = \frac{q}{4[4]^{0.5}} = \frac{q}{8}$$

Substitution L in the cost function, we obtain the short run cost function

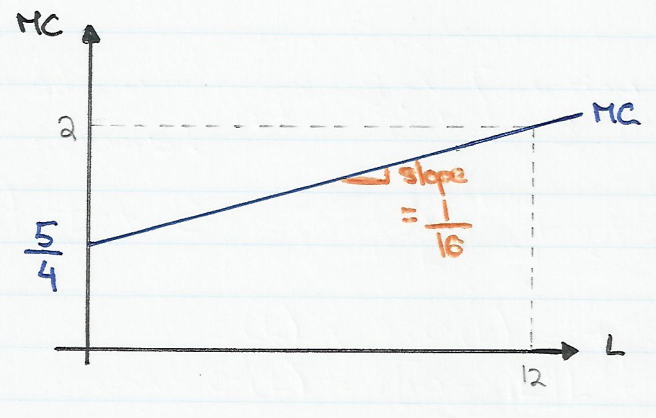
$$c(q) = 10\,\frac{q}{8} + \frac{2q^2}{64} + 200$$

$$\Leftrightarrow c(q) = \frac{5}{4}q + \frac{q^2}{32} + 200$$

Reminder: The cost function is expressed in terms on q.

c) $MC = \frac{5}{4} + \frac{q}{16}$

$\frac{dMC}{dq} = \frac{1}{16} > 0$ marginal cost is always increasing.



d) When w is constant, $MC = \frac{w}{MP_L}$ (course notes 3 - slide 8)

However : $MC = \frac{5}{4} + \frac{q}{16}$

And $\frac{w(L)}{MP_L} = \frac{10 + 2[q/8]}{8}$ $\left[\text{where } L = \frac{q}{8}\right]$

$\Leftrightarrow \frac{w(L)}{MP_L} = \frac{5}{4} + \frac{q}{32}$

Therefore $MC \neq \frac{w(L)}{MP_L}$ because w is variable and dependent on L.

Question 8

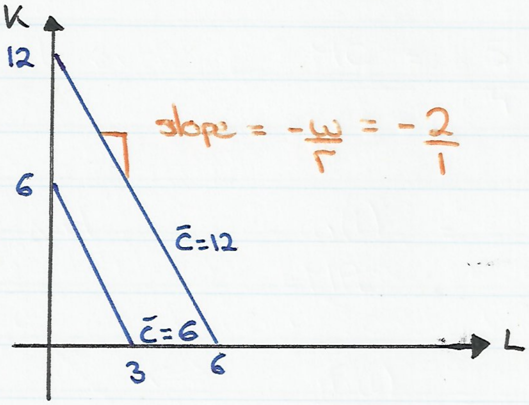
Isocost lines summarize all combinations of inputs that require the same total expenditure (course notes 3 - slide 17)

The isocost function is expressed in terms of production inputs and a set expenditure level

Ex: $\bar{C} = wL + rK$ where w = wage
r = rent

Hilroy

Let $\omega = 2$ and $r = 1$



Question 9

$$f(L, K) = L^{0.75} K^2$$

$\omega = 12$
$r = 10$
$\bar{K} = 10$

a) $400 = L^{0.75} 10^2$

$\Leftrightarrow \quad 4 = L^{0.75}$
$\Leftrightarrow \quad L = (4)^{4/3}$
$\Leftrightarrow \quad L \approx 6.35$

Ralph must employ 6.35 units of labour to serve 400 customers per hour

The cost is $C(L) = 12L + 10\bar{K}$ Eq 1
$\Leftrightarrow \quad C(L) = 12 \cdot 4^{4/3} + 100$
$\Leftrightarrow \quad C(L) \approx 176.20$