Assignment 1

Question 1

Correcting the defect will cost $2 million. Not correcting the defect will only cost $1 million. Economics predicts that the auto maker will not correct the defect.

Question 2

False.

The application of taxes on goods affects all players:
- Firms (higher cost of production, affects demand)
- Consumers (affects price)
- Government (revenue generated)

Question 3

$$q = 14L + 24L^2 - 2L^3$$

a) $MP_L = 14 + 48L - 6L^2$

The marginal product of labour is the additional output produced by an additional unit of labour, holding all other factors constant (course notes 2 - slide 5)

$$AP_L = \frac{14L + 24L^2 - 2L^3}{L}$$

$$\Leftrightarrow AP_L = 14 + 24L - 2L^2$$

The average product of labour is the ratio of output to the amount of labour employed (course notes 2 - slide 5).

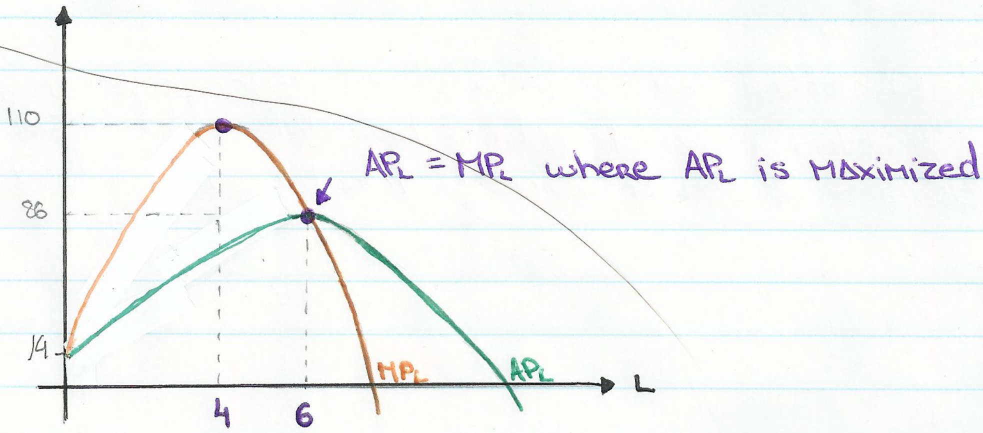
b) Calculate the first order derivative of the MP_L to obtain information on the shape of the curve and important points.

$$\frac{dMP_L}{dL} = 48 - 12L \begin{cases} > 0 \text{ when } L < 4. \ MP_L \text{ is increasing} \\ = 0 \text{ when } L = 4. \ MP_L \text{ is constant} \\ < 0 \text{ when } L > 4. \ MP_L \text{ is decreasing} \end{cases}$$

Replicate the analysis for AP_L.

$$\frac{dAP_L}{dL} = 24 - 4L \begin{cases} > 0 \text{ when } L < 6. \ AP_L \text{ is increasing} \\ = 0 \text{ when } L = 6. \ AP_L \text{ is constant} \\ < 0 \text{ when } L > 6. \ AP_L \text{ is decreasing} \end{cases}$$

c)



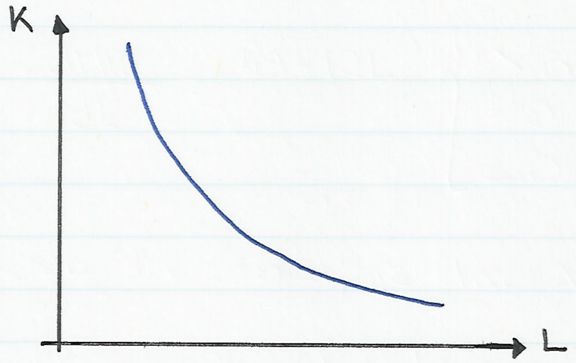
Question 4

$q = K^{0.2}L^{0.8}$

a) This is the long-run production function because none of the factors of production are held constant.

The short-run function is $q = \bar{K}^{0.2}L^{0.8}$

b) An isoquant graphically summarizes the efficient combinations of inputs that will produce a specific level of output (course notes 2 - slide 11).



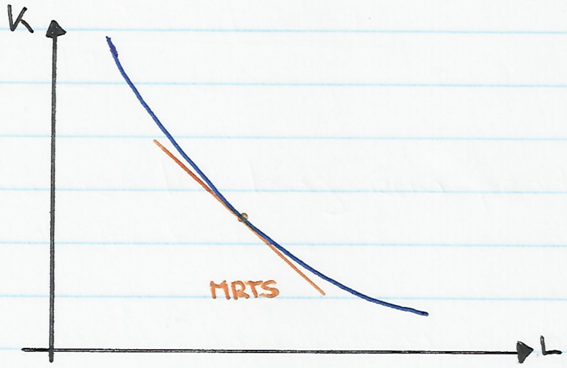
c) The slope of the isoquant is the marginal rate of technical substitution.

$$MRTS = -\frac{MP_L}{MP_K}$$

$$\Rightarrow \quad MRTS = -\frac{0.8L^{-0.2}K^{0.2}}{0.2L^{0.8}K^{-0.8}}$$

$$\Leftrightarrow \quad MRTS = 4\frac{K}{L}$$

d)



THE MARGINAL RATE OF TECHNICAL SUBSTITUTION REPRESENTS THE NUMBER OF UNITS OF K THE FIRM CAN REPLACE with AN extra unit of labour (course notes 2 - slide 16)

e) Let σ be the elasticity of substitution.

$$\sigma = \frac{d\,K/L}{d\,MRTS} \cdot \frac{MRTS}{K/L} \qquad \text{(Eq 1)}$$

As previously calculated $MRTS = -4\frac{K}{L}$.

We can derive $\frac{dMRTS}{d(K/L)} = -4$

Therefore $\frac{d(K/L)}{dMRTS} = -\frac{1}{4}$

Replace in Eq 1: $\sigma = -\frac{1}{4} \cdot \frac{-4\,K/L}{K/L}$

$$\Leftrightarrow \sigma = 1$$

The elasticity of substitution represents a percentage change in the capital-labour ratio, given a one percent change in MRTS. It measures the ease with which a firm can substitute capital for labour (course notes 2 - slide 18).

* Side note *

The elasticity of substitution of the Cobb-Douglas function is always equal to 1.

PROOF:

Let $\lambda L^{\alpha} K^{B}$ be a Cobb-Douglas function.

$\alpha, B > 0$

$\alpha + B = 1$

λ is a constant

$$MRTS = -\frac{\lambda \alpha L^{\alpha-1} K^{B}}{\lambda B L^{\alpha} K^{B-1}} = -\frac{\alpha}{B} \frac{L^{-B} K^{B}}{L^{\alpha} K^{-\alpha}} = -\frac{\alpha}{B} \frac{K^{\alpha} K^{B}}{L^{\alpha} L^{B}} = -\frac{\alpha}{B} \frac{K}{L}$$

Let σ be the elasticity of substitution

$$\sigma = \frac{d\,K/L}{d\,MRTS} \cdot \frac{MRTS}{K/L}$$

Given that $MRTS = -\frac{\alpha}{B} \frac{K}{L}$, $\frac{d\,MRTS}{d\,K/L} = \frac{-\alpha}{B}$

Therefore $\frac{d\,K/L}{d\,MRTS} = \frac{-B}{\alpha}$

Then $$\sigma = \frac{-B}{\alpha} \cdot \frac{-\frac{\alpha}{B} \frac{K}{L}}{\frac{K}{L}} = 1$$