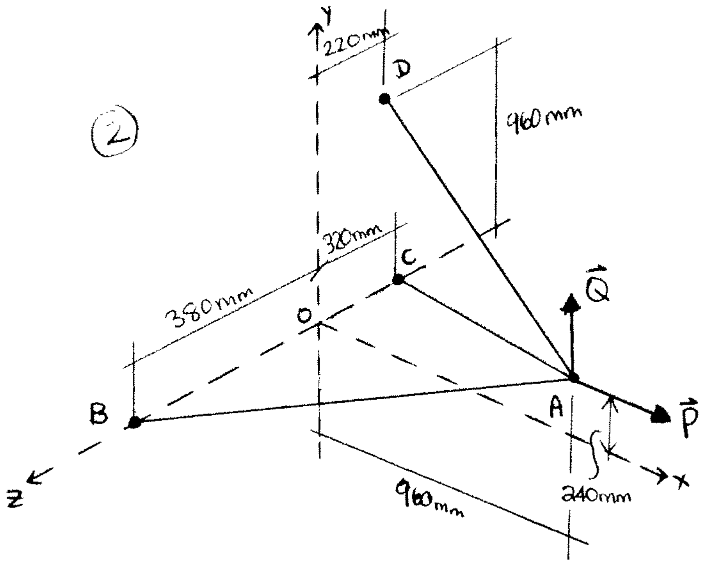
GNG1105C ASSIGNMENT #1 SOLUTIONS

sample 6.3, 6.61
quiz 6.2

1) Problem 2.107



2

Three cables are connected at A, where the forces $\vec{P}$ and $\vec{Q}$ are applied. Knowing that $Q = 90\,N$, Find P for $T_{AD} = 400\,N$

$A = (960, 240, 0)$

$B = (0, 0, 380)$

$C = (0, 0, -320)$

$D = (0, 960, -220)$

$\vec{AB} = -960\hat{i} - 240\hat{j} + 380\hat{k} \qquad |\vec{AB}| = 1060\,mm$

$\vec{AC} = -960\hat{i} - 240\hat{j} - 320\hat{k} \qquad |\vec{AC}| = 1040\,mm$

$\vec{AD} = -960\hat{i} + 720\hat{j} - 220\hat{k} \qquad |\vec{AD}| = 1220\,mm$

$$\vec{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\vec{AB}}{|\vec{AB}|} = \frac{T_{AB}}{1060}\left[-960\hat{i} - 240\hat{j} + 380\hat{k}\right] = \left(-0.905\hat{i} - 0.226\hat{j} + 0.358\hat{k}\right)T_A$$

3

$$\vec{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\vec{AC}}{|\vec{AC}|} = \frac{T_{AC}}{1040}\left[-960\hat{i} - 240\hat{j} - 320\hat{k}\right] = \left(-0.923\hat{i} - 0.231\hat{j} - 0.308\hat{k}\right)T_A$$

$$\vec{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\vec{AD}}{|\vec{AD}|} = \frac{T_{AD}}{1220}\left[-960\hat{i} + 720\hat{j} - 220\hat{k}\right] = \frac{400\,N}{1220\,mm}\left[-960\hat{i} + 720\hat{j} - 220\hat{k}\right]$$

$$= -314.75\hat{i} + 236\hat{j} - 72.13\hat{k}$$

$\vec{P} = P\hat{i}$

$\vec{Q} = Q\hat{j} = 90\hat{j}$

$$\underline{\Sigma F_x = 0}:\quad 0 = \frac{-960}{1060}T_{AB} - \frac{960}{1040}T_{AC} - \frac{960}{1220}(400) + P \qquad ①$$

$$\underline{\Sigma F_y = 0}:\quad 0 = \frac{-240}{1060}T_{AB} - \frac{240}{1040}T_{AC} + \frac{720}{1220}(400) + 90 \qquad ②$$

3

$$\underline{\Sigma F_z = 0}:\quad 0 = \frac{380}{1060}T_{AB} - \frac{320}{1040}T_{AC} - \frac{220}{1220}(400) \qquad ③$$

From ③: $\quad 0 = \frac{380}{1060}T_{AB} - \frac{320}{1040}T_{AC} - \frac{88000}{1220}$

$$0 = T_{AB} - \frac{1060}{380}\,\frac{320}{1040}T_{AC} - \frac{1060}{380}\,\frac{88000}{1220}$$

2

$$0 = T_{AB} - \frac{212}{247}T_{AC} - 201.21 \quad \Rightarrow \boxed{T_{AB} = \frac{212}{247}T_{AC} + 201.21} \quad (3a)$$

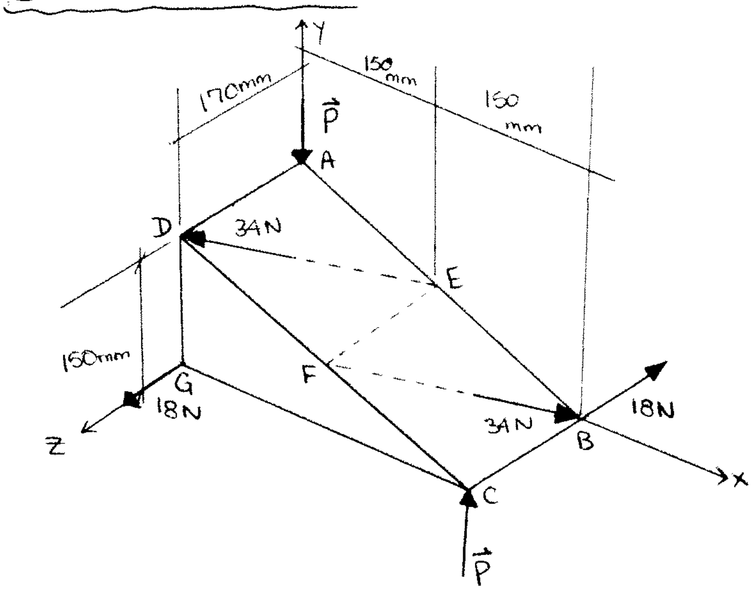
(3a) into ②: $\quad 0 = \frac{-240}{1060}\left(\frac{212}{247}T_{AC} + 201.21\right) - \frac{240}{1040}T_{AC} + 326.07$

$$0 = -0.1943T_{AC} - 45.56 - \frac{240}{1040}T_{AC} + 326.07$$

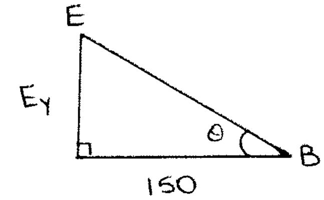
$0.425T_{AC} = 280.51 \quad \Rightarrow \boxed{T_{AC} = 660\,N} \qquad \therefore \boxed{T_{AB} = 768\,N}$

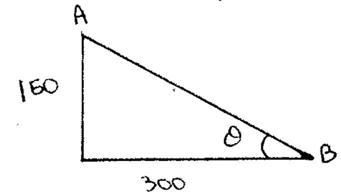
Into ①: $\boxed{P = 1619.5\,N}$

#2 Problem 3.79



$A = (0,150,0)$
$B = (300,0,0)$
$C = (300,0,170)$
$D = (0,150,170)$
$E = (150,75,0)$
$F = (150,75,170)$
$G = (0,0,170)$





$\theta = \tan^{-1}\left(\frac{150}{300}\right) = 26.565°$

$E_y = 75mm$

If P = 90 N, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

$$\Sigma M_O = (\vec{r}_{A/O} \times \overset{-90\hat{j}}{\cancel{P}}) + (\vec{r}_{B/O} \times -18\hat{k}) + (\vec{r}_{C/O} \times 90\hat{j}) + (\vec{r}_{D/O} \times \vec{F}_{ED}) + (\vec{r}_{B/O} \times \vec{F}_{FB}) + (\vec{r}_{G/O} \times 18\hat{k})$$

$\circ\ \vec{F}_{ED} = 34\,N\,\lambda_{ED} = \frac{34}{|\vec{ED}|}\vec{ED}$ $\qquad \vec{ED} = -150\hat{i} + 75\hat{j} + 170\hat{k} \qquad |\vec{ED}| = 238.8\ mm$

$\circ\ \vec{F}_{ED} = \frac{34}{238.8}\left[-150\hat{i} + 75\hat{j} + 170\hat{k}\right]$

$\cdot\ \vec{F}_{FB} = 34N\,\lambda_{FB} = 34\frac{\vec{FB}}{|\vec{FB}|}$ $\qquad \vec{FB} = 150\hat{i} - 75\hat{j} - 170K \qquad |\vec{FB}| = 238.8\,mm$

$\circ\ \vec{F}_{FB} = \frac{34}{238.8}\left[150\hat{i} - 75\hat{j} - 170\hat{k}\right]$

$\circ\ \vec{r}_{A/O} = 150\hat{j}$ $\quad \circ\ \vec{r}_{B/O} = 300\hat{i}$ $\quad \circ\ \vec{r}_{C/O} = 300\hat{i} + 170K$ $\quad \vec{r}_{D/O} = 150\hat{j} + 170K$ $\quad \cdot\ \vec{r}_{G/O} = 170$

① $\vec{r}_{A/O} \times \vec{P} = \begin{vmatrix} i & j & K & i & j \\ 0 & 150 & 0 & 0 & 150 \\ 0 & -90 & 0 & 0 & -90 \end{vmatrix} = (0-0)\hat{i} + (0-0)\hat{j} + (0-0)\hat{k} = 0$

② $\vec{r}_{B/O} \times -18\hat{k} = \begin{vmatrix} i & j & K & i & j \\ 300 & 0 & 0 & 300 & 0 \\ 0 & 0 & -18 & 0 & 0 \end{vmatrix} = (0-0)\hat{i} + (0+5400)\hat{j} + (0-0)\hat{k} = 5400\hat{j}$

③ $\vec{r}_{C/O} \times 90\hat{j} = \begin{vmatrix} i & j & K & i & j \\ 300 & 0 & 170 & 300 & 0 \\ 0 & 90 & 0 & 0 & 90 \end{vmatrix} = (0-15300)\hat{i} + (0-0)\hat{j} + (27000-0)\hat{k}$
$= -15300\hat{i} + 27000\hat{k}$

④ $\vec{r}_{D/O} \times \vec{F}_{ED} = \frac{34}{238.8}\begin{vmatrix} i & j & k & i & j \\ 0 & 150 & 170 & 0 & 150 \\ -150 & 75 & 170 & -150 & 75 \end{vmatrix} = \frac{34}{238.8}\left[(25500 - 12750)\hat{i} + (-25500 - 0)\hat{j} + (0 + 22500)\hat{k}\right]$

$= 1815\hat{i} - 3631\hat{j} + 3204\hat{k}$

⑤ $\vec{r}_{B/O} \times \vec{F}_{FB} = \frac{34}{238.8}\begin{vmatrix} i & j & k & i & j \\ 300 & 0 & 0 & 300 & 0 \\ 150 & -75 & -170 & 150 & -75 \end{vmatrix} = \frac{34}{238.8}\left[(0-0)\hat{i} + (0 + 51000)\hat{j} + (-22500 - 0)\hat{k}\right]$

$= 7261.3\hat{j} - 3204\hat{k}$

⑥ $\vec{F}_{G/O} \times 18\hat{k} = \begin{vmatrix} i & j & k & i & j \\ 0 & 0 & 170 & 0 & 0 \\ 0 & 0 & 18 & 0 & 0 \end{vmatrix} = 0$

$\therefore \Sigma M_o = 0 + 5400\hat{j} + (-15300\hat{i} + 27000\hat{k}) + (1815\hat{i} - 3631\hat{j} + 3204\hat{k}) + (7261\hat{j} - 3204\hat{k})$

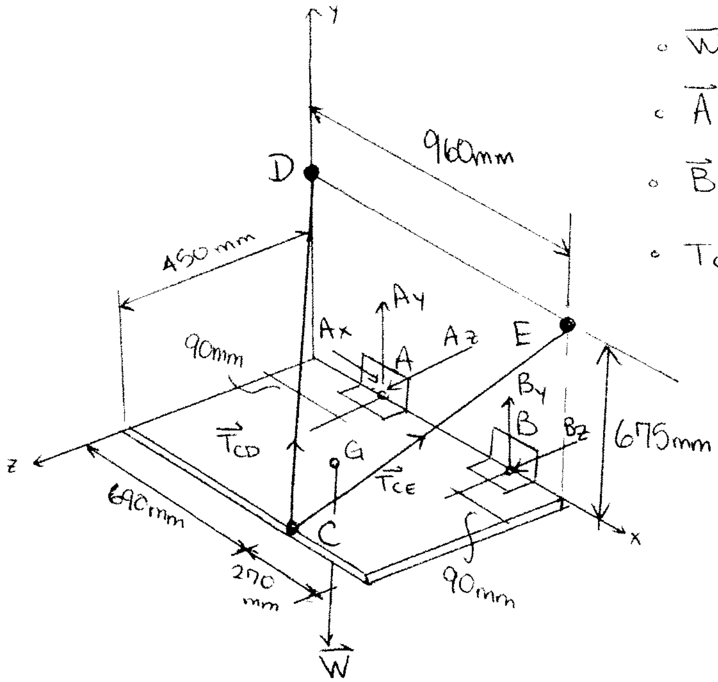
$= -13485\hat{i} + 9030\hat{j} + 27000\hat{k} \rightarrow |\vec{M}_o| = 31502$ N·mm

IF you want directions: $\theta_x = \cos^{-1}\left(\frac{M_{ox}}{|\vec{M}_o|}\right) = 115°$

$\theta_y = \cos^{-1}\left(\frac{M_{oy}}{|\vec{M}_o|}\right) = 73°$

$\theta_z = \cos^{-1}\left(\frac{M_{oz}}{|\vec{M}_o|}\right) = 31°$

#3) Problem 4.115:



$m = 100$ kg

- $\vec{W} = (9.81 \frac{N}{kg} \cdot 100\,kg)\hat{j} = -981\,N\,\hat{j}$
- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
- $\vec{B} = B_y\hat{j} + B_z\hat{k}$
- $T_{CE} = T_{CD} = T$

- $A = (90, 0, 0)$
- $B = (870, 0, 0)$
- $C = (690, 0, 450)$
- $D = (0, 675, 0)$
- $E = (960, 675, 0)$

$\vec{CE} = 270\hat{i} + 675\hat{j} - 450\hat{k}$

$|\vec{CE}| = 855$

$\vec{CD} = -690\hat{i} + 675\hat{j} - 450\hat{k}$

$|\vec{CD}| = 1065$

- $\vec{T}_{CE} = T\lambda_{CE} = \frac{T}{|\vec{CE}|}\vec{CE} = \frac{T}{855}\left[270\hat{i} + 675\hat{j} - 450\hat{k}\right]$
- $\vec{T}_{CD} = T\lambda_{CD} = \frac{T}{|\vec{CD}|}\vec{CD} = \frac{T}{1065}\left[-690\hat{i} + 675\hat{j} - 450\hat{k}\right]$

$\Sigma F_x = 0: \quad 0 = A_x + \frac{270}{855}T - \frac{690}{1065}T$

$0 = A_x - \frac{448}{1349}T$ ①

$\Sigma F_y = 0: \quad 0 = A_y + B_y - 981 + \frac{675}{855}T + \frac{675}{1065}T$ ②

$\Sigma F_z = 0: \quad 0 = A_z + B_z - \frac{450}{855}T - \frac{450}{1065}T$ ③

$\Sigma M_A = 0: \quad 0 = (\vec{r}_{B/A} \times \vec{B}) + (\vec{r}_{C/A} \times \vec{T}_{CE}) + (\vec{r}_{C/A} \times \vec{T}_{CD}) + (\vec{r}_{G/A} \times \vec{W})$

① $\vec{r}_{B/A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 780 & 0 & 0 & 780 & 0 \\ 0 & B_y & B_z & 0 & B_y \end{vmatrix} = 780B_y\hat{k} - 780B_z\hat{j}$

② $\vec{r}_{C/A} \times \vec{T}_{CE} = \frac{T}{855}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 600 & 0 & 450 & 600 & 0 \\ 270 & 675 & -450 & 270 & 675 \end{vmatrix}$

$= (121500\hat{j} + 405000\hat{k} - 303750\hat{i} + 270000\hat{j})\frac{T}{855}$

$= (-303750\hat{i} + 391500\hat{j} + 405000\hat{k})\frac{T}{855}$

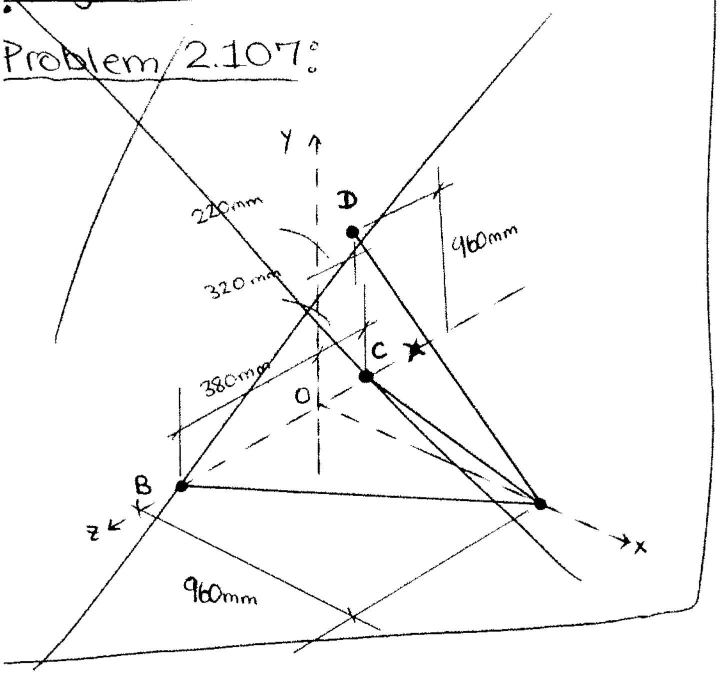
③ $\vec{r}_{C/A} \times \vec{T}_{CD} = \frac{T}{1065}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 600 & 0 & 450 & 600 & 0 \\ -690 & 675 & -450 & -690 & 675 \end{vmatrix}$

$= (-310500\hat{j} + 405000\hat{k} - 303750\hat{i} + 270000\hat{j})\frac{T}{1065}$

$= (-303750\hat{i} - 40500\hat{j} + 405000\hat{k})\frac{T}{1065}$

Assignment #1 - GNG1105C - Solutions

Problem 2.107:



④ $\vec{r}_{G/A} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 390 & 0 & 225 & 390 & 0 \\ 0 & -981 & 0 & 0 & -981 \end{vmatrix}$

$= -382590\,\hat{k} + 220725\,\hat{i}$

$\hat{i}: \quad \frac{-303750}{855}T - \frac{303750}{1065}T + 220725 = 0 \quad \Rightarrow \boxed{T = 345\text{ N}}$

$\hat{j}: \quad -780\,B_z + \frac{391500}{855}T - \frac{40500}{1065}T = 0 \quad \Rightarrow \boxed{B_z = 186\text{ N}}$

$\hat{k}: \quad 780\,B_y + \frac{405000}{855}T + \frac{405000}{1065}T - 382590 = 0 \quad \Rightarrow \boxed{B_y = 113\text{N}}$

T in ①: $\boxed{A_x = 115\text{ N}}$

T and B_y in ②: $\boxed{A_y = 377\text{ N}}$

T and B_z in ③: $\boxed{A_z = 141\text{ N}}$