Solutions - Version 1

A

Question 1. (4 points) Find and classify the critical points of the function
$f(x,y) = x^3 + y^2 - 2xy$.

2 for work

$f_x = 3x^2 - 2y$

$f_y = 2y - 2x = 2(y-x)$

so $f_y = 0$ only if $x = y$

so then $f_x = 3x^2 - 2x = x(3x-2) = 0$ if $x = 0, 2/3$

thus there are 2 critical pts $(0,0)$ and $(2/3, 2/3)$

$f_{xx} = 6x, \quad f_{xy} = -2, \quad f_{yy} = 2$

so $D(x,y) = f_{xx} f_{yy} - (f_{xy})^2 = 12x - 4$

$D(0,0) < 0 \Rightarrow$ $\boxed{(0,0) \text{ is a saddle pt}}$ 1

$D(2/3, 2/3) > 0$, $f_{xx}(2/3, 2/3) > 0$

$\Rightarrow$ $\boxed{(2/3, 2/3) \text{ is a local min}}$ 1

(A)

Question 2. (4 points) Use Lagrange Multipliers to find the absolute maximum and minimum values of the function $f(x,y) = (x+y)^2$ subject to the constraint $x^2+y^2 = 2$.

3 for work

solve $\nabla f = \lambda \nabla g$, $g = c$ or $f_x = \lambda g_x$, $f_y = \lambda g_y$, $g(x,y) = x^2+y^2 = 2$

$f_x = \lambda g_x \Rightarrow 2(x+y) = 2\lambda x \Rightarrow x+y = \lambda x$

$f_y = \lambda g_y \Rightarrow 2(x+y) = 2\lambda y \Rightarrow x+y = \lambda y$

so either $x = y$ or $\lambda = 0$ and $x = -y$

$x^2+y^2 = 2 \Rightarrow 2x^2 = 2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

and there are 4 pts $(1,1)$, $(1,-1)$, $(-1,1)$ and $(-1,-1)$

$f(1,1) = f(-1,-1) = 4$ max

$f(-1,1) = f(1,-1) = 0$ min

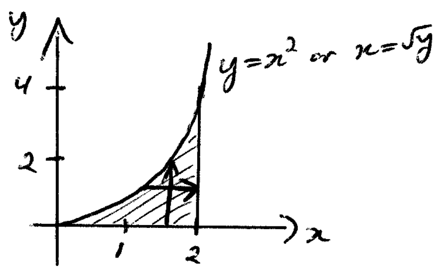
so $\boxed{\text{max is } 4}$ and $\boxed{\text{min is } 0}$ (1)

A

Question 3. (4 points) Evaluate the following integral

$$\int_0^4 \int_{\sqrt{y}}^2 \frac{2}{1+x^3}\, dx\, dy\,.$$

have to switch the order of integration



x goes $\sqrt{y}$ to 2

Then y goes 0 to 4

so then y goes from 0 to x^2

and then x is 0 to 2

$$\int_0^4 \int_{\sqrt{y}}^2 \frac{2}{1+x^3}\, dx\, dy = \int_0^2 \int_0^{x^2} \frac{2}{1+x^3}\, dy\, dx$$ 1

$$= \int_0^2 \frac{2}{1+x^3}\left(y\Big|_0^{x^2}\right) dx$$

$$= \int_0^2 \frac{2x^2}{1+x^3}\, dx$$

2 for work

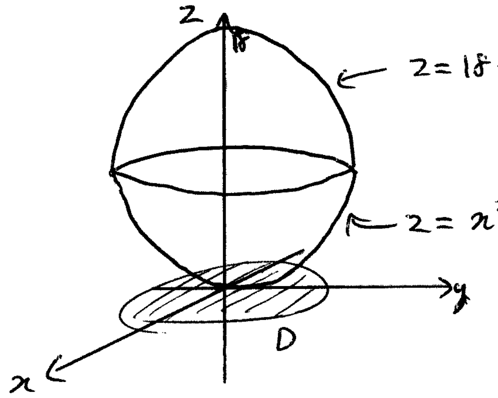
$$= \frac{2}{3}\ln(1+x^3)\Big|_0^2$$

$$= \boxed{\frac{2}{3}\ln 9}$$ 1

$$\simeq \boxed{1.4648}$$

Ⓐ

Question 4. (4 points) Find the volume of the solid enclosed by the paraboloids $z = 18 - x^2 - y^2$ and $z = x^2 + y^2$.



$z = 18 - x^2 - y^2 = 18 - r^2$

$z = x^2 + y^2 = r^2$

intersection

$18 - x^2 - y^2 = x^2 + y^2$

$x^2 + y^2 = 9$

So D is disk of radius 3

use polar coordinates

Volume $V = \int_0^{2\pi} \int_0^3 (18 - r^2 - r^2)\, r\,dr\,d\theta$

$= 2\pi \int_0^3 (18 - 2r^2) r\,dr$

$= 2\pi \int_0^3 (18r - 2r^3)\,dr$

$= 2\pi \left(9r^2 - \frac{1}{2} r^4 \Big|_0^3 \right)$

$= 2\pi \left(81 - \frac{81}{2} \right)$

$= \boxed{81\pi}$ 1

$\simeq \boxed{254.5}$

3 for work

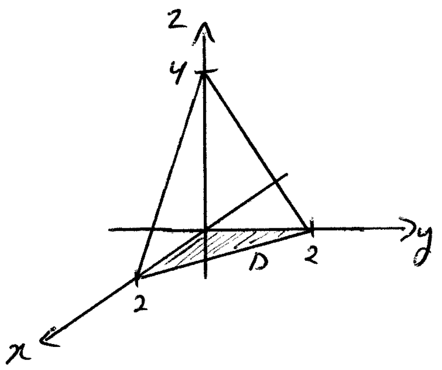
Also, view the volume as a triple integral:

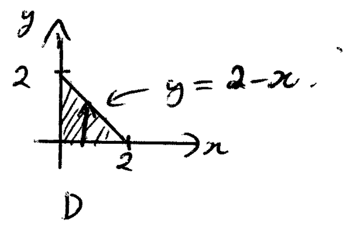
$V = \int_0^{2\pi} \int_0^3 \int_{r^2}^{18-r^2} dz\, r\,dr\,d\theta = \int_0^{2\pi} \int_0^3 (18 - r^2 - r^2) r\,dr\,d\theta$

$= \ldots$

A

Question 5. (4 points) Find the volume of the tetrahedron bounded by the coordinate planes and the plane $2x + 2y + z = 4$.





volume is $V = \int_0^2 \int_0^{2-x} \int_0^{4-2x-2y} dz\, dy\, dx$

3 for work

$$= \int_0^2 \int_0^{2-x} (4-2x-2y)\, dy\, dx$$

$$= \int_0^2 \left((4-2x)y - y^2 \Big|_0^{2-x} \right) dx$$

$$= \int_0^2 \left(2(2-x)^2 - (2-x)^2 \right) dx$$

$$= \int_0^2 (2-x)^2\, dx$$

$$= -\frac{1}{3}(2-x)^3 \Big|_0^2$$

$$= \boxed{8/3}$$

1