Solution to midterm exam Econ 2400D (fall 2012)

Q1)

a) Determinant of the given matrix:

$$7(-4+12) + 1(-4-52) + 0$$

$$= 56 - 56$$

$$= 0$$

The given matrix is $\boxed{\text{singular}}$

b)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Switching row 2 and row 3

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

Therefore the inverse matrix is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} //$$

The given matrix is its own inverse

Q2

a) $$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Matrix form

In order check for existence of a ~~nontrivi~~ nontrivial solution, ~~we~~ we need to find the determinat of the coefficient matrix:

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -1(1-1) - 1(-1-1) + 1(1+1)$$

(expanding across the first row)

$$= 0 + 2 + 2$$

$$= 4 \neq 0$$

Therefore a solution exists.

b) Using Cramer's rule:

$$x^* = \frac{\begin{vmatrix} a & 1 & 1 \\ b & -1 & 1 \\ c & 1 & -1 \end{vmatrix}}{4} = \frac{a(1-1) - 1(-b-c) + 1(b+c)}{4}$$

$$= \frac{2(b+c)}{4}$$

$$= \frac{b+c}{2}$$

Q2 cont'd

$$y^* = \frac{\begin{vmatrix} -1 & a & 1 \\ 1 & b & 1 \\ 1 & c & -1 \end{vmatrix}}{4} = \frac{-1(-b-c)-a(-1-1)+1(c-b)}{4}$$

$$= \frac{\cancel{b}+c+2a+c-\cancel{b}}{4}$$

$$= \frac{2(a+c)}{4}$$

$$= \frac{a+c}{2} \quad //$$

$$z^* = \frac{\begin{vmatrix} -1 & 1 & a \\ 1 & -1 & b \\ 1 & 1 & c \end{vmatrix}}{4} = \frac{-1(-c-b)-1(c-b)+a(1+1)}{4}$$

$$= \frac{b+\cancel{c}-\cancel{c}+b+2a}{4}$$

$$= \frac{a+b}{2} \quad //$$

Q3

a. $\lim_{x\to 4} \dfrac{x^2-16}{4\sqrt{x}-8}$

$= \lim_{x\to 4} \dfrac{(x+4)(x-4)}{4(\sqrt{x}-2)}$

$= \lim_{x\to 4} \dfrac{(x+4)(\sqrt{x}+2)(\sqrt{x}-2)}{4(\sqrt{x}-2)}$

$= \lim_{x\to 4} \dfrac{(x+4)(\sqrt{x}+2)}{4}$ (We can cancel $\sqrt{x}-2$ when $\sqrt{x}-2\neq 0$, i.e. when $x\neq 4$.)

$= \dfrac{(4+4)(2+2)}{4}$

$= \dfrac{32}{4}$

$= 8$ //

b. $\lim_{x\to 4} \dfrac{x^2-9x+20}{x-4}$

$= \lim_{x\to 4} \dfrac{x^2-4x-5x+20}{x-4}$

$= \lim_{x\to 4} \dfrac{x(x-4)-5(x-4)}{x-4}$

$= \lim_{x\to 4} \dfrac{(x-4)(x-5)}{x-4}$

$= \lim_{x\to 4} (x-5)$ (For $x\neq 4$, we can cancel $x-4$ from numerator and denominator)

$= -1$ //

Q4

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{x^3} + 3x^2 \Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - \cancel{x^3}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x \left(3x^2 + 3x\,\Delta x + (\Delta x)^2\right)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \left(3x^2 + 3x\,\Delta x + (\Delta x)^2\right)$$

$$= 3x^2 \quad // \quad \text{(Showed)}$$

Q5

$Q = A(t) K^{\alpha} L^{\beta}$ where $k = K_0 + at$ & $L = L_0 + bt$

$$\frac{dQ}{dt} = A'(t) K^{\alpha} L^{\beta} + A(t) K^{\alpha} \beta L^{\beta-1} \frac{dL}{dt} + A(t) L^{\beta} \alpha K^{\alpha-1} \frac{dK}{dt}$$

$$= A'(t) K^{\alpha} L^{\beta} + b\beta A(t) K^{\alpha} L^{\beta-1} + a\alpha A(t) K^{\alpha-1} L^{\beta} \quad //$$

$$= \left(A'(t) + b\beta A(t)/L + a\alpha A(t)/K\right) K^{\alpha} L^{\beta} \quad //$$

Q6

a.

$$dy = \frac{(x_1+x_2)\,d(2x_1x_2) - 2x_1x_2\,d(x_1+x_2)}{(x_1+x_2)^2}$$

$$= \frac{(x_1+x_2)(2x_1dx_2 + 2x_2dx_1) - 2x_1x_2(dx_1+dx_2)}{(x_1+x_2)^2}$$

$$= \frac{2x_1^2dx_2 + \cancel{2x_1x_2dx_2} + \cancel{2x_1x_2dx_1} + 2x_2^2dx_1 - \cancel{2x_1x_2dx_1} - \cancel{2x_1x_2d}}{(x_1+x_2)^2}$$

$$= \frac{2(x_1^2dx_2 + x_2^2dx_1)}{(x_1+x_2)^2} \quad //$$

b.

$$du = \frac{(x-y)\,d(9y^3) - 9y^3\,d(x-y)}{(x-y)^2}$$

$$= \frac{(x-y)\,27y^2dy - 9y^3(dx-dy)}{(x-y)^2}$$

$$= \frac{27xy^2dy - 27y^3dy - 9y^3dx + 9y^3dy}{(x-y)^2}$$

$$= \frac{27xy^2dy - 18y^3dy - 9y^3dx}{(x-y)^2}$$

$$= \frac{9y^2(3x-2y)dy - 9y^3dx}{(x-y)^2} \quad //$$

Q7

a) Own price elasticity: $\varepsilon_{11} = \frac{dQ_1}{dP_1} \cdot \frac{P_1}{Q_1}$

$= -4 \cdot \frac{P_1}{Q_1}$

b) Crossprice elasticity of Q_1 wrt P_2:

$$\varepsilon_{12} = \frac{dQ_1}{dP_2} \cdot \frac{P_2}{Q_1}$$

$$= -3 \frac{P_2}{Q_1}$$

Since by definition P_2 and Q_1 are both greater than or equal to zero, $\varepsilon_{12} \leq 0$. Therefore good 1 and good 2 are complements.

$$\varepsilon_{13} = \frac{dQ_1}{dP_3} \cdot \frac{P_3}{Q_1}$$

$$= 2 \frac{P_3}{Q_1}$$

Again since $P_3, Q_1 \geq 0$, $\varepsilon_{13} \geq 0$. Therefore an increase in P_3 will ~~do~~ lead to increase in Q_1. Therefore good 1 and good 3 are substitutes.

c) At $P_1 = 5$, $P_2 = 7$, $P_3 = 3$, $Y = 11000 \Rightarrow$

$$Q_1 = 50 - 4 * 5 - 3 * 7 + 2 * 3 + 0.001 * 11000$$
$$= 26$$

Therefore $\varepsilon_{12} = -3 \frac{7}{26}$; $\varepsilon_{13} = 2 \frac{3}{26}$

$= -0.81$; $= 0.23$

$\therefore$ A 10% price increase ~~for~~ good 2 will lead to 8.1% decrease in Q_1.

$\therefore$ A 10% increase in P_3 will lead to 2.3% increase in Q_1.

8.

a) $3x^4 - 7y^5 - 86 = 0$

$3*4x^3 - 7*5\, y^4 \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{12\, x^3}{35\, y^4}$ //

b) $y = (4x^5 - 1)^7$

$\frac{dy}{dx} = 7\,(4x^5-1)^6 \cdot \frac{d}{dx}(4x^5-1)$ [chain rule]

$= 7\,(4x^5-1)^6\; 5*4\,x^4$

$= 140\, x^4\,(4x^5-1)^6$ //

Q9

a)

$$Y = C + I_0 + G_0$$
$$= C_0 + bY_d + I_0 + G_0$$
$$= C_0 + b(Y-T) + I_0 + G_0$$
$$= C_0 + bY - b(T_0 + tY) + I_0 + G_0$$
$$\Rightarrow Y(1-b+bt) = C_0 - bT_0 + I_0 + G_0$$
$$\therefore Y^* = \frac{C_0 + I_0 + G_0 - bT_0}{1-b(1-t)}$$
//

b)

(i) Govt spending G_0:

$$\frac{\partial Y^*}{\partial G_0} = \frac{1}{1-b(1-t)}$$

Since $0 < b, t < 1$, $b(1-t)$ is between 0 and 1. Thus, $\frac{\partial Y^*}{\partial G_0} > 1$

A 1-unit increase in G_0 will lead to more than 1-unit increase in equilibrium national income (by the amount of $\frac{1}{1-b(1-t)}$).

(ii) $$\frac{\partial Y^*}{\partial T_0} = -\frac{b}{1-b(1-t)}$$

Given $0 < b, t < 1$, $\frac{\partial Y^*}{\partial T_0} < 0$.

A 1-unit increase in lumpsum tax will lead to $\frac{b}{1-b(1-t)}$ unit decrease in eqm national income.

Q9 Cont'd

b) (iii)

$$\frac{\partial Y^*}{\partial t} = \frac{(1-b(1-t)) * 0 - (C_0+I_0+G_0-bT_0)\, b}{(1-b(1-t))^2}$$

$$= - \frac{b(C_0+I_0+G_0-bT_0)}{(1-b(1-t))^2}$$

Here $\frac{\partial Y^*}{\partial t} < 0$

A 1-unit increase in the tax rate will lead to $\frac{b(C_0+I_0+G_0-bT_0)}{(1-b(1-t))^2}$ unit decrease in Y^*. //

Q10

* Give up to 75% if they cannot show this mathematically, but can explain using diagrams. see chiang page 159-160 (esp Fig 7.3).

Given:

Total cost $C = C(Q)$

$$\therefore \text{Average cost} = \frac{C}{Q} = \frac{C(Q)}{Q}$$

Slope of the average cost curve:

$$\frac{d(AC)}{dQ} = \frac{d}{dQ}\left(\frac{C(Q)}{Q}\right)$$

$$= \frac{Q\,C'(Q) - C(Q)\cdot 1}{Q^2}$$

$$= \frac{C'(Q)}{Q} - \frac{C(Q)}{Q^2}$$

$$= \frac{1}{Q}\left[C'(Q) - \frac{C(Q)}{Q}\right]$$

$$= \frac{1}{Q}\left[MC - AC\right]$$

When MC curve lies above the AC curve, $MC > AC$.
Therefore $MC - AC > 0 \Rightarrow \frac{d(AC)}{dQ} > 0 \Rightarrow$ AC curve's slope will be positive.

When MC curve intersects AC curve, $MC = AC$
Therefore $\frac{dAC}{dQ} = 0 \Rightarrow$ Slope of the AC curve is zero.

When MC curve lies below the AC curve, $MC < AC$.
Therefore $MC - AC < 0 \Rightarrow \frac{dAC}{dQ} < 0 \Rightarrow$ AC curve's slope will be negative.

//(Showed)