Sample Exam for MATH 1005: Differentials Eqns & ∞ Series for Engineers

CARLETON UNIVERSITY

Authorized release by Dr. Sam Dubé, April 4,

FINAL/DEFERRED
EXAMINATION
April 2004

DURATION: 3 HOURS

Department Name & Course Number: Mathematics and Statistics MATH 1005ABCD
Course Instructors : Drs. A. Alaca, S. Dubé (c)

AUTHORIZED MEMORANDA

NON PROGRAMMABLE CALCULATORS ARE ALLOWED

Students **MUST** count the number of pages in this examination question paper **before** beginning to write, and report any discrepancy immediately to a proctor.
This question paper has **14** pages, including this top page.

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In addition to this question paper, students require a **Scantron sheet.**

Family Name: ____________________
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Circle your course section here:

A: Alaca, A. **B: Dubé, S.** **C: Dubé, S.** **D: Alaca, A.**

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ANSWER ALL QUESTIONS (pp.2-14)

PLEASE READ THE FOLLOWING INSTRUCTIONS BEFORE PROCEEDING

- This exam consist of 30 multiple choice questions worth (2) marks each for a total of 60 marks.
- You must use a soft lead pencil (HB#2) to mark the Scantron sheet. Ink cannot be used.
- Fill in your section letter in the "S" column on the Scantron sheet, to the right of the "FIRST NAME" field.
- Answer each question by filling in the appropriate box on the Scantron sheet (in pencil) AND by circling the answer on the question sheet.
- Please make sure your name and student number are filled in accurately on BOTH question paper AND Scantron sheet.

1. Which of the following differential equations is satisfied by the orthogonal trajectories to the curves $y = \dfrac{k}{x^2}$?

$y' = \dfrac{x}{2y}$

$y' = -\dfrac{x}{2y}$

(c) $y' = \dfrac{2y}{x}$

$y' = -\dfrac{2y}{x}$

$y' = -\dfrac{x^2}{k}$

2. Solve the initial value problem $y' = \dfrac{3x + y}{x}$, $y(1) = 0$ in order to obtain the value of $y(e)$.

(a) $y(e) = 3$

(b) $y(e) = e$

(c) $y(e) = e^3$

(d) $y(e) = 3e$

(e) $y(e) = -3e$

3. Solve the initial value problem $x^3 y' + 3x^2 y = \cos x$, $y(\pi) = 0$ in order to obtain the value of $y(2)$.

(a) $y(2) = \sin 2$

(b) $y(2) = \dfrac{\sin 2}{8}$

(c) $y(2) = \dfrac{\sin 2}{4}$

7. Let $w = x^3 + y^3 + z^3$, $x = st$, $y = e^{st}$, $z = s\cos t$.
What is $\dfrac{\partial w}{\partial s}$ at $(s,t) = (1,0)$?

(a) 0

(b) 3

(c) 6

(d) 7

(e) 9

8. Solve the initial value problem $(3x^2y)\,dx + (x^3+1)\,dy = 0$, $y(1) = 1$, in order to obtain $y(2)$.

$y(2) = \frac{1}{7}$

$y(2) = \frac{2}{7}$

(c) $y(2) = \frac{1}{8}$

(d) $y(2) = \frac{2}{9}$

(e) $y(2) = \frac{3}{8}$

9. Consider the differential equation $(3xy + 2y^2)\,dx + (x^2 + 2xy)dy = 0$.
Which of the following is an integrating factor?

x

$\dfrac{1}{x}$

(c) $\dfrac{1}{x^2}$

x^2

(e) e^x

10. The general solution of the differential equation $y'' + 9y = 0$ is

(a) $y = c_1e^{9x} + c_2e^{-9x}$

(b) $y = c_1e^{9x} + c_2xe^{9x}$

(c) $y = c_1e^{3x} + c_2e^{-3x}$

(d) $y = c_1\cos(9x) + c_2\sin(9x)$

(e) $y = c_1\cos(3x) + c_2\sin(3x)$

11. Solve the initial value problem $y'' - 3y' + 2y = 0$, $y(0) = 0$, $y'(0) = 2$ in order to obtain $y(1)$.

(a) $y(1) = -2e^{-2} + 2e^{-1}$

$y(1) = \frac{1}{2}e^{-2} + \frac{1}{2}e^{-1}$

$y(1) = 2e^2 - 2e$

$y(1) = -2e^2 + 2e$

(e) $y(1) = 0$

12. The general solution of the differential equation $y'' - 5y' + 6y = x$ is

(a) $y = c_1e^{3x} + c_2e^{2x} - \frac{x}{6} - \frac{5}{36}$

(b) $y = c_1e^{3x} + c_2e^{2x} - \frac{x}{6} + \frac{5}{36}$

(c) $y = c_1e^{3x} + c_2e^{2x} + \frac{x}{6} - \frac{5}{36}$

(d) $y = c_1e^{3x} + c_2e^{2x} + \frac{x}{6} + \frac{5}{36}$

(e) $y = c_1e^{3x} + c_2e^{2x} - \frac{5x}{36} - \frac{1}{36}$

13. Let $y'' - 6y' + 8y = e^{4x}$. An appropriate trial solution by the method of undetermined coefficients is

(a) $y_p = Ae^{4x}$

(b) $y_p = Axe^{4x}$

$y_p = Ae^{2x}$

$y_p = Axe^{2x}$

(e) $y_p = Ae^{2x} + Be^{4x}$

14. Let $y'' - 3y' + 2y = \cos(e^{-x})$. Given that the form of the particular solution is $y_p = u_1(x)e^x + u_2(x)e^{2x}$, find $u_1(x)$ by using the method of variation of parameters.

(a) $u_1 = e^{-x}\sin(e^{-x})$

(b) $u_1 = \cos(e^{-x})$

(c) $u_1 = -\cos(e^{-x})$

(d) $u_1 = -\sin(e^{-x})$

(e) $u_1 = \sin(e^{-x})$

15. The general solution of the differential equation $x^2y'' + 7xy' + 9y = 0$ is

(a) $y = c_1x^3 + c_2x^{-3}$

(b) $y = x^{-7/2}\left[c_1\cos(\sqrt{13}x) + c_2\sin(\sqrt{13}x)\right]$

(c) $y = c_1x^{-3} + c_2x^{-3}\ln x$

(d) $y = c_1x^3 + c_2x^3\ln x$

(e) $y = x^{-3}\left[c_1\cos(\ln x) + c_2\sin(\ln x)\right]$

16. Consider the system $\begin{cases} x' = 3x - 2y \\ y' = 2x - y \end{cases}$, where $x = x(t)$, and $y = y(t)$.

Then the solution for $x(t)$ is given by

(a) $x(t) = c_1e^{-t} + c_2te^{-t}$

(b) $x(t) = c_1e^{t} + c_2te^{t}$

(c) $x(t) = c_1e^{t} + c_2e^{-t}$

(d) $x(t) = e^{t}\left[c_1\cos t + c_2\sin t\right]$

(e) $x(t) = ce^{-t}$

17. The sum of the series $\sum_{n=1}^{\infty} \frac{3^n}{5^{n+1}}$ is

(a) $\frac{10}{3}$

18. Which of the three series below converge(s)?

1) $\sum_{n=1}^{\infty} \frac{1}{3^n + n}$ 2) $\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$ 3) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

1) and 2)

2) and 3)

(c) 1)

(d) 2)

(e) 3)

19. Which of the three series below converge(s)?

1) $\sum_{n=1}^{\infty} \frac{1}{n^3 + n}$ 2) $\sum_{n=1}^{\infty} \frac{n}{n^2 + n}$ 3) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

1) and 2)

2) and 3)

(c) 1) and 3)

1), 2) and 3)

(e) 1) only

20. Which of the three series below are absolutely convergent?

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ 2) $\sum_{n=0}^{\infty} \frac{\cos n}{4^n}$ 3) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$

(a) 1) and 2)

(b) 2) and 3)

1) and 3)

1), 2) and 3)

None of the above

21. Which of the three series below are conditionally convergent?

1) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+3}}$ 2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\sqrt[3]{n}}{n}$ 3) $\sum_{n=0}^{\infty} \frac{(-1)^n}{5^n}$

1) and 2)

2) and 3)

(c) 1) and 3)

1), 2) and 3)

(e) 3) only

22. Which of the three series below diverge?

1) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ 2) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ 3) $\sum_{n=0}^{\infty} \left(\frac{4n^2+1}{3n^2+7}\right)^n$

(a) 1) and 2)

2) and 3)

1) and 3)

1), 2) and 3)

(e) None of the above

23. The radius of convergence R of the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x-3)^n}{5^n\, n!}$ is

(a) $R = 0$

(b) $R = 2$

(c) $R = 3$

(d) $R = 6$

(e) $R = \infty$

24. The interval of convergence I of the series $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{\sqrt{n}\, 3^n}$ is

(a) $I = (-5,\ 1)$

(b) $I = (-5,\ 1]$

(c) $I = [-5,\ 1]$

(d) $I = [-5,\ 1)$

(e) $I = [1,\ 5]$

25. A power series representation of the function $f(x) = \frac{x^2}{1-x^3}$, along with its interval of convergence I, is

(a) $\sum_{n=0}^{\infty} x^{3n-2}$ $\quad I = (-1,\ 1)$

(b) $\sum_{n=0}^{\infty} (-1)^n x^{3n+2}$ $\quad I = [0,\ 1)$

(c) $\sum_{n=0}^{\infty} x^{3n+2}$, $\quad I = (-1,\ 1)$

(d) $\sum_{n=0}^{\infty} (-1)^n x^{3n+1}$, $\quad I = (-1,\ 1)$

(e) $\sum_{n=0}^{\infty} (-1)^n x^{3n+1}$ $\quad I = [0,\ 1)$

26. In the power series representation of $\int \frac{dx}{1+x^5}$, the coefficient of x^{16} is

(a) $\frac{-1}{16!}$ (b) $\frac{1}{16!}$ (c) $\frac{1}{16}$ (d) $\frac{-1}{16}$ (e) $\frac{5}{16}$

27. The first four nonzero terms of the binomial series for $f(x) = \frac{1}{\sqrt[4]{1-x}}$ are

(a) $1 + \frac{1}{4}x + \frac{1}{16}x^2 + \frac{1}{64}x^3$

(b) $1 + \frac{1}{4}x + \frac{5}{32}x^2 + \frac{15}{128}x^3$

(c) $1 + \frac{1}{4}x + \frac{5}{32}x^2 + \frac{15}{64}x^3$

(d) $1 - \frac{1}{4}x + \frac{3}{64}x^2 - \frac{5}{128}x^3$

(e) $1 - \frac{1}{4}x + \frac{3}{32}x^2 - \frac{5}{64}x^3$

28. What is the coefficient of x^5 in the Maclaurin series expansion of $f(x) = e^{2x}$?

(a) $\frac{15}{4}$ (b) $\frac{4}{15}$ (c) 4 (d) 15 (e) $\frac{2}{15}$

29. What is the coefficient of $\left(x-\frac{\pi}{2}\right)^4$ in the Taylor series expansion of the function $f(x)=\sin x$ at $a=\frac{\pi}{2}$?

(a) $\frac{-1}{24}$ (b) $\frac{1}{24}$ (c) $\frac{-1}{48}$ (d) $\frac{1}{48}$ (e) $\frac{\pi}{24}$

30. In the power series solution $y=\sum_{n=0}^{\infty} c_n x^n$ of the differential equation $(1-x^2)y''+2y=0$, the coefficients c_n satisfy the recursion formula

$c_{n+2}=\frac{1}{(n+2)}\,c_n$

$c_{n+2}=(n+2)\,c_n$

$c_{n+2}=\frac{n+2}{n-2}\,c_{n+1}$

$c_{n+2}=\frac{n-2}{n+2}\,c_n$

(e) $c_{n+2}=-\frac{1}{n-2}\,c_n$

ANSWERS

1	a	11	c	21	a
2	d	12	d	22	c
3	b	13	b	23	e
4	e	14	e	24	b
5	b	15	c	25	c
6	d	16	b	26	d
7	b	17	c	27	b
8	d	18	a	28	b
9	a	19	e	29	b
10	e	20	b	30	d

Have fun

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4. Consider the Bernoulli differential equation $y' + y\tan x - y^4 \sin x$

The transformation used to obtain the general solution is

(a) $u = \frac{1}{y}$
(b) $u = y^2$
(c) $u = \frac{1}{y^2}$
(d) $u = y^3$
(e) $u = \frac{1}{y^3}$

5. Let $f(x, y) = \ln\left(\frac{y}{x}\right)$. What is $f_x(2,3)$?

(a) $\frac{1}{6}$
(b) $-\frac{1}{2}$
(c) $-\frac{1}{6}$
(d) $\frac{1}{3}$
(e) -6

6. Let $ye^x + xz + ze^y = 0$. What is $\frac{\delta z}{\delta x}$?

(a) $-\left(\frac{e^x}{x+e^y}\right)$
(b) $\frac{ye^x + z}{x+e^y}$
(c) $\frac{e^x + xe^y}{x+e^y}$
(d) $-\left(\frac{ye^x + z}{x+e^y}\right)$
(e) $-\left(\frac{e^x + ze^y}{x+e^y}\right)$