

Answers to Chapter 7 Homework Questions (pages 201-204 in the textbook)

1. Solution is in the textbook at the end of Chapter 7.
2. Solution is in the textbook at the end of Chapter 7.
4. Solution is in the textbook at the end of Chapter 7.
9. Problem canceled.
15. Solution is in the textbook at the end of Chapter 7.
17. Problem setup:

- q is “TVs sold per hour”
- s is “number of skilled sales-reps per hour”
- u is “number of unskilled sales-reps per hour”
- 1 skilled sales-rep sells 6 TVs per 8-hour shift
- 1 unskilled sales-rep sells 4 TVs per 8-hour shift
- $w_s = 26$ is the wage of the skilled sales-rep
- $w_u = 16$ is the wage of the unskilled sales-rep

Skilled sales-rep sells $6/8$ TVs per hour (h). Unskilled sales-rep sells $4/8$ TVs per hour (h). Assuming that the marginal productivity does not change over time (e.g., the sales-rep is as productive in the morning as during the lunch time, afternoon and the evening hours) we have that

$$\begin{aligned} \text{the marginal product of the skilled sales-rep} & : \quad MP_s = \frac{6}{8} = \frac{3}{4} \\ \text{the marginal product of the unskilled sales-rep} & : \quad MP_u = \frac{4}{8} = \frac{1}{2} \end{aligned}$$

(MP_L in the context of this problem refers to the # of TVs sold per additional hour).

- a) We have to find an isoquant corresponding to hourly sales of 4 TVs: $q = 4$. The first step is to formulate the production function. This is simple, right? It is just the number of TVs that all sales-reps on the floor sell per hour: that is, the sales of skilled sales-reps plus the sales of unskilled sales-reps:

$$q = f(s, u) = MP_s \times s + MP_u \times u.$$

Then, plugging in for MP_s , MP_u , and q we get

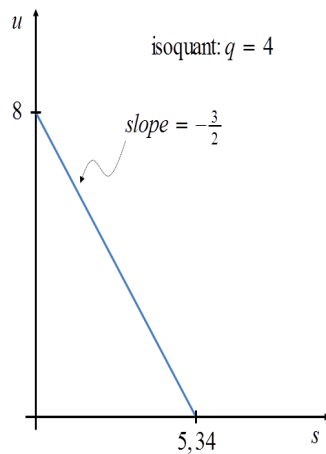
$$4 = \frac{3}{4}s + \frac{1}{2}u.$$

Then, by rearranging we get

$$u = 8 - \frac{3}{2}s$$

which is our isoquant: the intercept with the u -axis is at 8, the slope is $-3/2 = -1.5$, and it intersects the s -axis at

$$0 = 8 - \frac{3}{2}s \quad \Rightarrow \quad s = \frac{16}{3} = 5,33.$$



The skilled and unskilled sales-reps are not perfect substitutes for one another – that is, I cannot take one skilled rep out, plug in an unskilled rep in his place and sell as many TVs per hour as before. But they are substitutable in the sense that I can replace 3 unskilled sales-reps with 2 skilled sales-reps and not affect my TV sales in any way.

- b) Here we have to draw an isocost line for expenditure $e = 104$. The amount we spend on inputs is given by

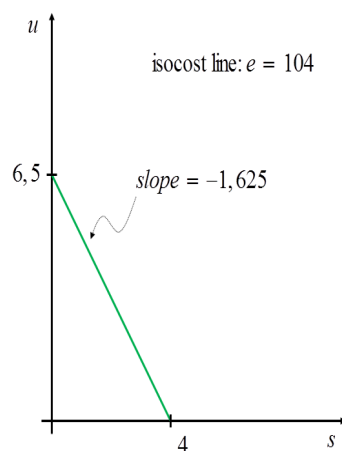
$$\begin{aligned} e &= w_s \times s + w_u \times u \\ 104 &= 26 \times s + 16 \times u \end{aligned}$$

rearranging gives us

$$u = \frac{104}{16} - \frac{26}{16}s$$

which is the equation for the isocost line. Now we can plot it: the intercept with the u -axis is at $104/16 = 6,5$, the slope is $-26/16 = -1,625$, and it intersects the s -axis at

$$0 = \frac{104}{16} - \frac{26}{16}s \Rightarrow s = \frac{104}{26} = 4$$



- c) Here we should find the cost-minimizing combination of inputs for the hourly output of 4 TVs per hour. We have seen in the class that if the cost minimizing combination of inputs involves positive amounts of both inputs, i.e., $s^* > 0$ and $u^* > 0$, then the

marginal optimality condition has to hold at (s^*, u^*) . So let's assume that both s^* and u^* are greater than zero. Marginal optimality condition is:

$$\frac{w_s}{w_u} = \frac{MP_s}{MP_u}$$

but in our case it never holds because when we plug in values for w_s , w_u , MP_s , and MP_u we get

$$\frac{26}{16} \neq \frac{3/4}{1/2} \text{ which is just } \frac{13}{8} \neq \frac{3}{2}$$

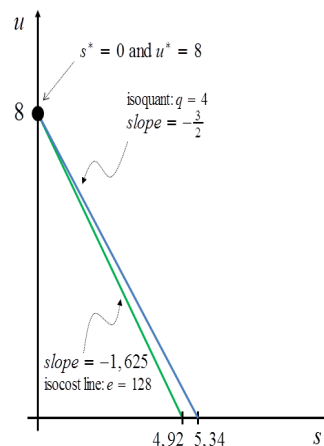
after simplification. Since marginal optimality condition never holds there is no combination of inputs $s^* > 0$ and $u^* > 0$ that minimizes the cost. This leaves us with only those input combinations where we either hire just skilled sales-reps ($u^* = 0$ and $s^* = 5,34$) or just unskilled sales-reps ($s^* = 0$ and $u^* = 8$). We can easily verify that hiring just unskilled sales-reps is cheaper: the cost is

$$26 \times 0 + 16 \times 8 = 128$$

for hiring just unskilled sales-reps and

$$26 \times 5,34 + 16 \times 0 = 138,84$$

for hiring just skilled sales-reps. And we have an answer. The cost is minimized by hiring only unskilled sales-reps (we hire 8 of them and they will produce 4 TV sales per hour). Here is how it looks on the picture



And here is additional intuition. We know that at the cost minimizing input combination: the last dollar spent on s must be as productive as the last dollar spent on u (this is just another way of stating the marginal optimality condition). That is,

$$\frac{MP_s}{w_s} = \frac{MP_u}{w_u}$$

but in our case this is never true because when we plug in the numbers we get

$$\frac{3/4}{26} \neq \frac{1/2}{16}$$

This happens because the last dollar spent on skilled sales-reps produces $(3/4)/26 = 0.029$ sales per hour but the last dollar spent on unskilled sales-reps always produces $(1/2)/16 = 0.031$ sales per hour. In other words, for the same money hiring an unskilled sales-rep is simply more productive. Therefore, the cheapest way to sell 4 TVs per hour is to hire 8 unskilled sales reps and 0 skilled sales reps.

25 Problem setup:

- Gail is able to make 10 floral arrangements per hour
- $w = 10$ for the first 8 hours
- $w = 15$ for each additional (overtime) hour.

The first step is to find Gale's production function: because she makes 10 floral arrangements per hour (h) we have

$$q = f(h) = 10 \times h.$$

Now we find the total cost function

$$C(q) = w \times h(q)$$

that is, the firm's cost is equal the number of hours spent working, $h(q)$, times wage per hour (w). First we take an inverse of the production function to get $h(q)$

$$h = q/10$$

then we plug back into the cost function to get

$$C(q) = w \times q/10.$$

Next we plug in for the wage w . But recall that Gail gets paid two different wages depending on whether she works overtime or not. She gets 10 per hour during normal 8-hour working time and 15 per hour for overtime. Therefore, her cost function is:

$$C(q) = \begin{cases} 10 \times q/10 & \text{if } q \leq 80 \\ 10 \times 80/10 + 15 \times (q - 80)/10 & \text{if } q > 80 \end{cases}$$
$$C(q) = \begin{cases} q & \text{if } q \leq 80 \\ 80 + \frac{3}{2}(q - 80) & \text{if } q > 80 \end{cases} .$$

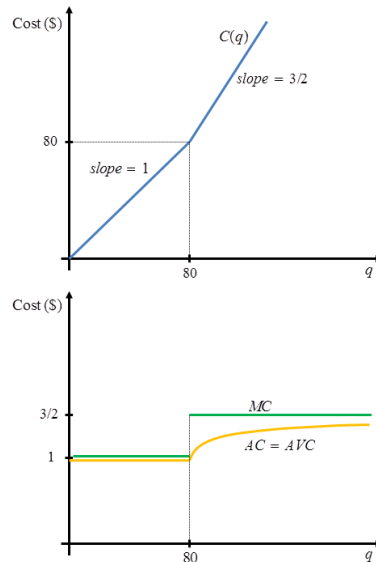
Now it is straight forward to calculate the AC , AVC , and MC . Because there are no fixed costs we have that $AC = AVC$. (For example, if the problem stated that Gail gets paid say 20 dollars for just showing up for work then there would have been a fixed cost of 20 and AC would be then different form AVC .)

$$AC = AVC = \begin{cases} q/q & \text{if } q \leq 80 \\ (80 + \frac{3}{2}(q - 80)) / q & \text{if } q > 80 \end{cases}$$
$$= \begin{cases} 1 & \text{if } q \leq 80 \\ \frac{3}{2} - \frac{40}{q} & \text{if } q > 80 \end{cases}$$

The marginal cost is

$$MC = \begin{cases} 1 & \text{if } q \leq 80 \\ 3/2 & \text{if } q > 80 \end{cases} .$$

Here is how the curves look in the picture.



32. This problem is almost the same as the example we worked in class. Problem setup:

- production function is

$$q = f(L, K) = 10\sqrt{L}\sqrt{K}.$$

- $w_L = 1$ per hour
- $w_K = 4$ per hour.

a) Suppose we pick a target $q = 30$. Then, cost minimizing input combination satisfies the marginal optimality condition

$$\frac{w_L}{w_K} = \frac{MP_L}{MP_K}$$

which is in our case

$$\frac{1}{4} = \frac{K}{L}$$

which can be rewritten as

$$K = \frac{1}{4}L \text{ this is our equation (1).}$$

In addition the production target has to be met, which means that we must have

$$30 = 10\sqrt{L}\sqrt{K} \text{ this is our equation (2).}$$

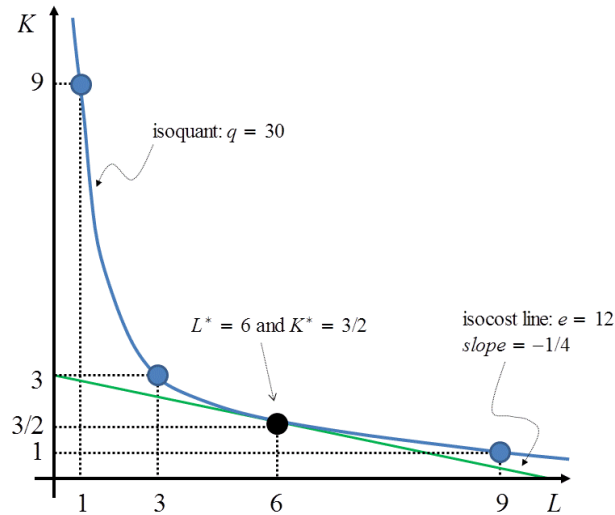
Plugging for K form (1) into (2) and solving for L (just like I did in class) gives

$$\begin{aligned} 30 &= 10\sqrt{\frac{1}{4}L} \\ L^* &= 6 \end{aligned}$$

and plugging that into (1) gives us K^*

$$K^* = 3/2.$$

The picture shows you what we just did.



- b) “Expansion path” is just a fancy word for what I did in class when I derived L^* and K^* for an arbitrary target q . We can easily do that here. As in part a) write down the marginal optimality condition

$$\frac{1}{4} = \frac{K}{L} \text{ and call it eq. (1)}$$

and then the production function

$$q = 10\sqrt{L}\sqrt{K} \text{ call this the eq. (2).}$$

Now solving for L^* and K^* we get

$$L^* = q/5 \quad \text{and} \quad K^* = q/20.$$

L^* and K^* tell us the “expansion path” – that is, what are the cost-minimizing combinations of inputs as we “expand” (vary) q .

- c) Deriving the long run total cost is also simple. Recall that total cost in the long run is the expenditure on inputs needed to produce a certain level of output q

$$C(q) = w_L \times L^* + w_K \times K^*.$$

Notice that L^* and K^* are both functions of q . So plugging in from part b) of the problem we get

$$C(q) = w_L \times \frac{q}{5} + w_K \times \frac{q}{20}.$$

Next, plugging in the wages from the problem setup we get

$$\begin{aligned} C(q) &= 1 \times \frac{q}{5} + 4 \times \frac{q}{20} \\ &= \frac{q}{5} + \frac{q}{5} \\ &= \frac{2}{5}q \end{aligned}$$

which is the long run cost curve.

35. Solution is in the textbook at the end of Chapter 7.

41. Problem setup:

- team production function is

$$q = f(h) = \sqrt{h}.$$

- $w = 150$ per hour.

a) We have done this before. The cost function is: wage times how many hours it takes to teach q swimmers

$$C(q) = w \times h(q).$$

But function $h(q)$ is just the inverse of $f(h)$: $h(q) = f^{-1}(q)$. To get the inverse we just rearrange the production function

$$h(q) = q^2.$$

Now we can plug into the cost function and get

$$C(q) = w \times q^2$$

and plugging in for the wage we get

$$C(q) = 150q^2.$$

The marginal cost is just the derivative of C with respect to q :

$$MC(q) = 300q.$$

b) We just repeat all the steps for the new production function and get

$$C^{New}(q) = 150q^{4/3}$$

and the marginal cost is

$$\begin{aligned} MC^{New}(q) &= \frac{4}{3}150q^{4/3-1} \\ &= 200q^{1/3}. \end{aligned}$$

c) The MC^{New} is clearly lower than the old MC because

$$300q > 200q^{1/3} \text{ for all } q > 1.$$

With the new teaching technique the coach Rob can train the same number of swimmers at fraction of the time that it took him before.