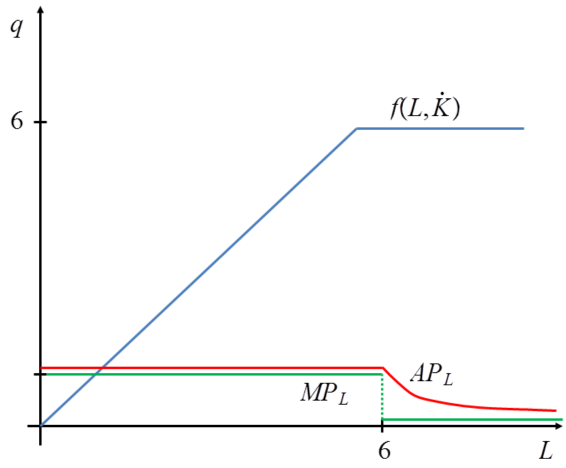
Answers to Chapter 6 Home Questions (page 201 of the Textbook)

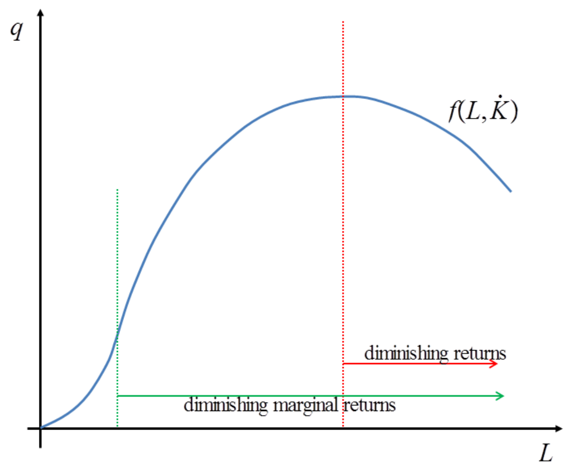
1. Please, refer to the end of the textbook for a detailed explanation.

2. If each extra worker produces additional unit of output, it means that by increasing the number of labor input by 1 the output is also increased by 1. Therefore the production function has a linear form $q = L$. However, if we increase labor beyond $L = 6$, the output does not change in any way. This means the production function becomes flat. On the interval $0 < L \leq 6$ the average product is $q/L = 1$. After $L = 6$ the average product will start to gradually diminish and slowly approach zero. On the interval $0 < L \leq 6$ marginal product equal to 1 but for $L > 0$ the marginal product is equal to zero and overlaps with the L-axis.



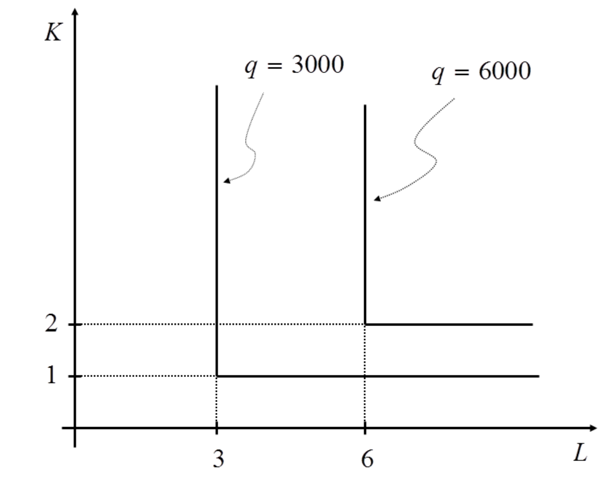
4. Because if an isoquant were thick, it would mean that you could find two input combinations on the same isoquant such that one of them would have more of more of both inputs - violating efficiency (monotonicity) assumption.

7. Diminishing marginal returns refers to decreasing marginal product – that is, an additional unit of input (e.g., worker) adds less to the total product than the unit just before it. Diminishing returns on the other hand refers to negative marginal product – that is, an additional unit of input actually reduces the total output. In case of Michelle's business, her production process clearly illustrated diminishing marginal returns.

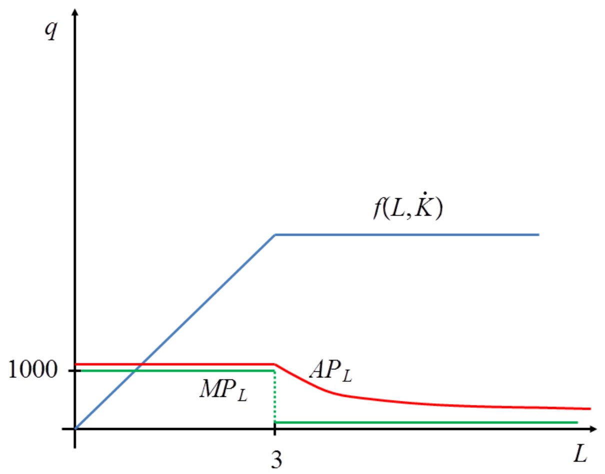


8. This problem has two parts:

a) $q = 1000 \times min(L, 3K)$. This is a a fixed proportions production function. We have seen similar function in class. For example, we can produce 3000 units with $L = 3$ and any $K \geq 1$ or $K = 1$ and $L \geq 3$



b) Let's fix level of capital at: $\dot{K} = 1$. The product curves are shown on the picture below.



10. Please, refer to the end of the textbook for a detailed explanation. But there is a typo in the problem! The production fucntion should be: $q = B + G/2$. (My thanks to the student who pointed this out to me.)

16. Suppose 1 old machine required 10 workers to operate and this combination produced 100 belts. Now 1 new machine requires only 5 workers to operate. $AP_L = \frac{q}{L} \rightarrow \frac{100}{10} < \frac{100}{5}$ which means the AP_L went up. However, we cannot tell anything more specific about the marginal product and returns to scale without knowing more about the shape of the production function.

17. Cobb- Douglas function is a good example here: for example take $q = L^{1/4}K^{3/4}$. Then,

$$MP_L = \frac{1}{4}\left(\frac{K}{L}\right)^{3/4} \quad \text{and} \quad MP_K = \frac{1}{4}\left(\frac{L}{K}\right)^{1/4}.$$

To check the returns to scale let us double the inputs:

$$(2L)^{1/4}(2K)^{3/4} = 2^{1/4+3/4}L^{1/4}K^{3/4} = 2(L^{0,25}K^{0,75})$$

which means that the output exactly doubled. We have constant returns to scale.

20. No, it does not, we have done an in-class exercise dealing with exactly this issue. Consider the following example:

	Firm 1		Firm 2	
	MP	AP	MP	AP
Worker 1	100	100	60	60
Worker 2	50	75	60	60
Worker 3	0	50	60	60

Notice that if we had only observed the two firms both employing only 2 workers then we would conclude that Firm 1 is more productive because it has a higher average product. But if both firms hired another (third) worker, then the Firm 2 would suddenly become more productive. The reason is that the third worker is useless to Firm 1 but quite useful for Firm 2.

21. In this problem a production function is substituted by a grade function with two inputs:

$$G^{Will} = 2,5A^{0,36}R^{0,64} \quad \text{and} \quad G^{David} = 2,5A^{0,25}R^{0,75}$$

a) We are asked to find MP_A and MP_R for Will and David, which are the partial derivatives of conforming functions with respect to the used argument:

$$MP_A^{Will} = \frac{dG_W}{dA} = 2,5 \times 0,36A^{0,36-1}R^{0,64} = 0,9\left(\frac{R}{A}\right)^{0,64}$$

$$MP_R^{Will} = \frac{dG_W}{dR} = 2,5 \times 0,64A^{0,36}R^{0,64-1} = 1,6\left(\frac{A}{R}\right)^{0,36}$$

$$MP_A^{David} = \frac{dG_W}{dA} = 2,5 \times 0,25A^{0,25-1}R^{0,75} = 0,625\left(\frac{R}{A}\right)^{0,75}$$

$$MP_R^{David} = \frac{dG_W}{dR} = 2,5 \times 0,75A^{0,25}R^{0,75-1} = 1,875\left(\frac{A}{R}\right)^{0,25}$$

b) Since $MRTS = -\frac{\Delta R}{\Delta A} = -\frac{MP_A}{MP_R}$ and we know all marginal products for both guys, we can calculate MRTS for Will and David by the following way:

$$MRTS_{Will} = -\frac{MP_A^{Will}}{MP_R^{Will}} = -\frac{0,9\left(\frac{R}{A}\right)^{0,64}}{1,6\left(\frac{A}{R}\right)^{0,36}} = -0,5625\frac{R}{A}$$

$$MRTS_{David} = -\frac{MP_A^{David}}{MP_R^{David}} = -\frac{0,625\left(\frac{R}{A}\right)^{0,75}}{1,875\left(\frac{A}{R}\right)^{0,25}} = -0,33\frac{R}{A}$$

c) Yes, it is possible, for example we could have

$$G^{Will} = 2A^{0,5}R^{0,5} \quad \text{and} \quad G^{David} = 3A^{0,5}R^{0,5}.$$

You verify that MP functions are different but $MRTS$s are the same.

22. Please, refer to the end of the textbook for a detailed explanation.

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25. In a short-run capital is not a variable – it is a constant $\dot{K}$.

   a) $q = 10L + \dot{K}$ is a linear production function that has constant marginal returns: $\frac{dq}{dL} = 10$.

   b) For a Cobb-Douglas type function $q = L^{0,5}\dot{K}^{0,5}$ the marginal product is decreasing in L

$$\frac{dq}{dL} = 0,5\left(\frac{\dot{K}}{L}\right)^{0,5}$$

   which means we have diminishing marginal returns.

26. Let us test each function for the case when we double the amount of inputs. If $q = f(L, K)$, then if $f(2L, 2K) > 2q$ we have increasing returns; if $f(2L, 2K) = 2q$ we have constant returns; and if $f(2L, 2K) < 2q$ we have decreasing returns to scale.

   a) $q = L + K$ is a linear function which has constant returns to scale: $2q = 2L + 2K = 2(L + K)$.

   b) $q = AL^{\alpha}K^{\beta}$. The return depends on values of α and β: $2q = A(2L)^{\alpha}(2K)^{\beta} = 2^{\alpha+\beta}AL^{\alpha}K^{\beta}$. More specifically:

$$\alpha + \beta < 1 - \textit{decreasing returns to scale}$$

$$\alpha + \beta = 1 - \textit{constant returns to scale}$$

$$\alpha + \beta > 1 - \textit{increasing returns to scale}$$

   c) $q = L + L^{\alpha}K^{\beta} + K$. The linear part of the function $(L + K)$ gives constant returns to scale; the middle part is the Cobb-Douglas function which corresponds to part b of this problem. Therefore, the answer will be the same as in part b.

   d) $q = (aL^p + bK^p)^{d/p}$. Notice that if $p = d = 1$ we will have constant returns to scale since the function is linear. Decreasing and increasing returns depend on value of d. If $d > p$ then we have increasing returns. If $d < p$ then we have decreasing returns.

28. $Q = L^{\alpha}K^{\beta}$ Electronics and Equipment (EE) $Q = AL^{0,49}K^{0,53}$:

$$MP_L^{EE} = \frac{dQ}{dL} = 0,49\frac{K^{0,53}}{L^{0,51}};\ MP_K^{EE} = \frac{dQ}{dK} = 0,53\frac{L^{0,49}}{K^{0,47}}$$

Following the same logic for tobacco and primary metals we can see that their marginal products have diminishing marginal returns. But the returns to scale are increasing.