Intermediate Microeconomics: 2020, Sec. H
Date: 02/05/2013

Name: _

Exam 1

Always show your work and/or provide explanation for your answers. (I need to know how you arrived at the correct answer. In parts where I say "explain" or "show work" or "justify your answer" stating just the correct answer without any further explanation will not earn you a full credit.)

All three problems use the following information:

- The production function is

$$f(L,K)=\sqrt{L}+\sqrt{K}.$$

- The wage is $w=20$ and the rent is $r=10$.

Problem 1: The firm is operating in the short run. The level of capital is fixed at $\dot{K}=4$. Please answer the following parts: write down

a) (2pt) short run production function

$$\begin{aligned} f(L,\dot{K}) &= \sqrt{L}+\sqrt{\dot{K}} \\ &= 2+\sqrt{L}. \end{aligned}$$

b) (2pt) marginal product MP_L

$$MP_L=\frac{df(L,4)}{dL}=\frac{1}{2\sqrt{L}}$$

c) (2pt) average product AP_L

$$AP_L=\frac{f(L,4)}{L}=\frac{2}{L}+\frac{1}{\sqrt{L}}$$

d) (2pt) cost function

$$\begin{aligned} C(q) &= r\dot{K}+wf^{-1}(q,4) \\ &= 40+20(q-2)^2 \end{aligned}$$

f) (2pt) marginal cost MC

$$\begin{aligned} MC &= \frac{dC(q)}{dq} \\ &= 40(q-2) \end{aligned}$$

or

$$\begin{aligned} MC &= w\frac{1}{MP_L} \\ &= 20 \times 2\sqrt{L(q)} = 20 \times 2\sqrt{(q-2)^2} = 40(q-2) \end{aligned}$$

g) (3pt) Is the MC greater than, equal to, or less than the AC at the output level where the AC is minimized? Explain.

We know that MC crosses AVC at its lowest point and therefore they are the same. Or, draw picture. Or, explicitly calculate.

h (3pt) Does L have diminishing returns or diminishing marginal returns? Explain.

diminishing returns:

$$MP_L = \frac{df(L,4)}{dL} = \frac{1}{2\sqrt{L}} \quad \text{is always positive: so no}$$

diminishing marginal returns:

$$MP_L = \frac{df(L,4)}{dL} = \frac{1}{2\sqrt{L}} \quad \text{is always decreasing iin } L\text{: so yes}$$

Problem 2: Now the firm is operating in the long run. Please answer the following parts:

a) (3pt) Write down the expression for an isoquant corresponding to the output level $q = 4$.

$$L = (4 - \sqrt{K})^2$$

b) (3pt) Plot the isoquant from part a. When drawing the picture first find three points that lie on the isoquant (please label their coordinates on axes) and then draw the curve that passes through them.

Plot three points: here are 5 points easy to draw (4,4), (1,9), (9,1), (16,0), (0,16). Then draw a curve passing through them...

c) In class we have made three restrictions about the shape of the isoquant. What are they?

   1. isoquants have to be thin (monotonicity), 2. isoquants have to be downward sloping (monotonicity), 3. isoquants have to have increasing slope / decreasing slope when in absolute value or slope of the isoquant has to be getting flatter or diminishing MRTS. We also talked about: "isoquants can't cross" (transitivity). Strictly speaking, this is not related to the shape of the isoquant but if you wrote this as one of your three restrictions I will count it as correct.

d) (3pt) Does the production function have decreasing returns to (doubling the) scale? Explain

$$f(2L, 2K) = \sqrt{2L} + \sqrt{2K} = \sqrt{2}(\sqrt{L} + \sqrt{K}) = \sqrt{2}f(L, K).$$

e) (3pt) Suppose that $L = 4$ and $K = 16$. You have an extra 20 to spend on inputs. Which input will you buy (L or K)?

   First realize that you can purchase 2 units of K but only one unit of L

$$\text{compare } \frac{MP_L(4,16)}{20} \text{ with } \frac{MP_K(4,16)}{10} + \frac{MP_K(4,17)}{10}$$

   this is

$$\frac{1}{4 \times 20} \text{ vs. } \frac{1}{8 \times 10} + \text{ something positive}$$
$$\frac{1}{80} \text{ vs. } \frac{1}{80} + \text{ something positive,}$$

   so buy 2 units of K.

Problem 3: The firm is operating in the long run. Suppose the target output level is $q = 4$. Please answer the following parts:

a) (3pt) State the law of diminishing marginal rate of technical substitution (MRTS) and verify that it holds for the production function f.

   First the law of diminishing MRTS says that the slope of the isoquant is getting flatter as we move along side horizontal axis:

$$MRTS(L, K) = -\frac{\sqrt{K}}{\sqrt{L}}$$

   notice that it is an increasing (or in abs. value decreasing) function of L. In other words, slope is getting flatter with L. As we have more L we are increasingly less willing to substitute K for L...

b) (3pt) Find the cost-minimizing input combination L^* and K^*. (Show work.)

first state the marginal optimality condition

$$\begin{aligned} \frac{w}{r} &= MRTS(L^*, K^*) \\ 2 &= \sqrt{\frac{K}{L}} \\ K &= 4L \text{ this is eq. (1)} \end{aligned}$$

then also we have to meet the target output $q = 4$, so

$$q = 4 = \sqrt{L} + \sqrt{K} \text{ this is eq. (2).}$$

Plugging from (1) into (2) we get

$$\begin{aligned} 4 &= \sqrt{L} + \sqrt{4L} = 3\sqrt{L} \\ L^* &= 16/9 \end{aligned}$$

and plugging back to (1) we get

$$K^* = 64/9.$$