1 A parallel vector to the line of intersection will be orthogonal to the normal vectors to both planes. So, to find one, we need to cross product the normal vectors:

4

$$\langle 2,-1,-1\rangle \times \langle 4,3,-1\rangle = \langle 4,-2,10\rangle$$

And to find a point of intersection, we solve:

3 $2x-y-3=4x+3y-3$. In particular for $x=0$, we obtain $y=0$ and $z=-3$.

So, the symmetric equations are:

$$\frac{x}{4}=\frac{y}{-2}=\frac{z+3}{10}.$$

2

2 The plane contains the direction vector of the line, $\vec{v}_1 = \langle 1, -2, 2 \rangle$. To find another vector on the plane, we find a point on L:

4 $x = 1 \Rightarrow y = -1, \ z = 3$.

Then $\vec{v}_2 = (1,3,2) - (1,-1,3) = \langle 0, 4, -1 \rangle$ is on the plane, so

3 $\vec{n} = \vec{v}_1 \times \vec{v}_2 = \langle -6, 1, 4 \rangle$ is normal to the plane. Hence, the equation of the plane is

3 $$-6(x-1) + (y-3) + 4(z-2) = 0$$

(3) From the definition of the osculating circle, the equation of the osculating circle at the given point $(1,-1,1)$ is

4 $$\vec{r}(t) = \frac{1}{k}\cos t\,\vec{T} + \frac{1}{k}\sin t\,\vec{N} + C_0$$

where k, $\vec{T}$ and $\vec{N}$ are the curvature, the unit tangent vector, and the unit normal vector, at the point $(1,-1,1)$.

And C_0 is the center of the osculating circle which is obtained as

$$C_0 = (1,-1,1) + \frac{1}{k}\vec{N}$$ 2

Now we find k, $\vec{T}$, $\vec{N}$ and C_0 at the given point:

continued:

3 A vector function for the curve of intersection is $\vec{r}(t) = \langle t^2, t, t^4 \rangle$. So,

1

$$\vec{r}'(t) = \langle 2t, 1, 4t^3 \rangle \Rightarrow \vec{r}'(-1) = \langle -2, 1, -4 \rangle$$

$$\Rightarrow \vec{T}(-1) = \frac{1}{\sqrt{21}} \langle -2, 1, -4 \rangle.$$

1

$$|\vec{r}'(t)| = \sqrt{4t^2 + 1 + 16t^6}$$

$$\Rightarrow \vec{T}'(t) = \frac{4t + 48t^5}{\sqrt{4t^2+1+16t^6}} \cdot \vec{r}'(t) + \frac{1}{|\vec{r}'(t)|} \vec{r}''(t) =$$

$$\frac{1}{|\vec{r}'(t)|} \langle 8t^2 + 96t^6 + 2, \ 4t + 48t^5, \ (4t+48t^5)4t^3 + 12t^2 \rangle$$

$$\Rightarrow \vec{T}'(-1) = \frac{1}{\sqrt{21}} \langle 106, -52, -196 \rangle$$

$$\Rightarrow \vec{N}(-1) = \frac{1}{\sqrt{13089}} \langle 53, -26, -98 \rangle$$

On the other hand, we have

1

$$k(-1) = \frac{|\vec{r}'(-1) \times \vec{r}''(-1)|}{|\vec{r}'(-1)|^3} = \frac{118\sqrt{2}}{21\sqrt{21}}$$

1

which means the radius of the osculating circle is $\frac{21\sqrt{21}}{118\sqrt{2}}$. Then its center is

③ Continued:

$$C_0 = (1,-1,1) + \frac{21\sqrt{21}}{118\sqrt{2}}\vec{N}(-1) = \Big\langle 1 + \frac{(21\sqrt{21})(53)}{118\sqrt{2}\sqrt{13089}},$$

$$-1 - \frac{(21\sqrt{21})(26)}{118\sqrt{2}\sqrt{13089}}, 1 - \frac{(21\sqrt{21})(98)}{118\sqrt{2}\sqrt{13089}}\Big\rangle$$

- Then the vector function for the osculating circle is:

$$\vec{r}(t) = \frac{1}{k(-1)}\cos t\,\vec{T}(-1) + \frac{1}{k(-1)}\sin t\,\vec{N}(-1) + C_0$$

④ Using Newton's second law, we obtain the acceleration of the ball:

From gravity: $\vec{a}_1 = -9.8\,\vec{k}$

from the wind: $\vec{a}_2 = -10\,\vec{i}$

So, $\vec{a}(t) = -10\,\vec{i} - 9.8\,\vec{k}$

2

$\Rightarrow \vec{v}(t) = -10t\,\vec{i} - 9.8t\,\vec{k} + \vec{v}_0$

$= -10t\,\vec{i} - 9.8t\,\vec{k} + 20\,\vec{j} + 20\sqrt{3}\,\vec{k}$

$= -10t\,\vec{i} + 20\,\vec{j} + (20\sqrt{3} - 9.8t)\,\vec{k}$

1

$\Rightarrow \vec{r}(t) = -5t^2\,\vec{i} + 20t\,\vec{j} + (20\sqrt{3}\,t - 4.9t^2)\,\vec{k}$

1

2

To find when it lands, we solve:

$20\sqrt{3}\,t - 4.9t^2 = 0 \Rightarrow t = 0$ or $t = \frac{20\sqrt{3}}{4.9}$

2

And at $t = \frac{20\sqrt{3}}{4.9}$, the coordinates of the landing point are: $\vec{r}\left(\frac{20\sqrt{3}}{4.9}\right) = \left(\frac{-6000}{(4.9)^2}, \frac{400\sqrt{3}}{4.9}, 0\right)$

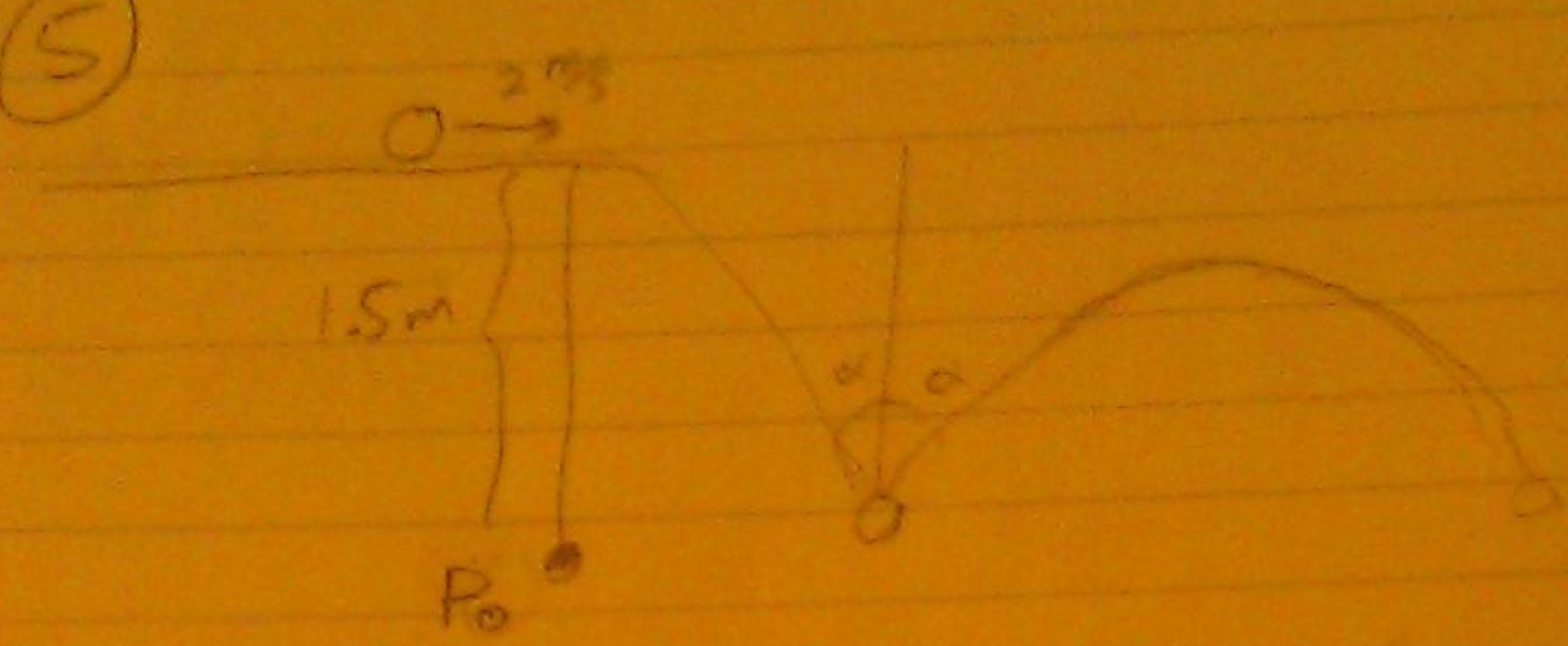
2

And at that moment, the speed is

$v\left(\frac{20\sqrt{3}}{4.9}\right) = \left|\vec{v}\left(\frac{20\sqrt{3}}{4.9}\right)\right| = \left|\left\langle \frac{-200\sqrt{3}}{4.9}, 20, 20\sqrt{3} - 40\sqrt{3}\right\rangle\right|$

$= \sqrt{\frac{12 \times 10^4}{(4.9)^2} + 400 + 1200}$

5



Placing the origin of the coordinate system at P_0 we have:

$$\vec{r}(0) = 1.5\vec{j} \quad , \quad \vec{v}_0 = 2\vec{i}$$

$$\vec{a}(t) = -9.8\vec{j} \Rightarrow \vec{v}(t) = 2\vec{i} - 9.8t\vec{j}$$

$$\Rightarrow \vec{r}(t) = 2t\vec{i} + (1.5 - 4.9t^2)\vec{j}$$

2

a) To find the time of impact, we solve

$$1.5 - 4.9t^2 = 0 \Rightarrow t = \frac{\sqrt{15}}{7}$$

2

So, at that instant we have

3 $\vec{r}\left(\frac{\sqrt{15}}{7}\right) = \left(\frac{2\sqrt{15}}{7}, 0\right)$, and the speed at impact is

$$v_1 = v\left(\frac{\sqrt{15}}{7}\right) = \left|\vec{v}\left(\frac{\sqrt{15}}{7}\right)\right| = \sqrt{4 + \left(\frac{9.8\sqrt{15}}{7}\right)^2} = \sqrt{4+3(9.8)}$$

b) Since $\vec{v}\left(\frac{\sqrt{15}}{7}\right) = \left\langle 2, \frac{-(9.8)\sqrt{15}}{7}\right\rangle$ is the velocity

2 at the time of impact, we get $\alpha = \operatorname{Arctan}\left(\frac{4.9\sqrt{15}}{7}\right) \approx 70°$

2 So, the distance to the origin of the second point of impact is $\frac{2\sqrt{15}}{7} + \frac{v_1^2 \sin 2\alpha}{-9.8} \approx \frac{2\sqrt{15}}{7} + \frac{(33.4)(0.65)}{-9.8}$

⑥ Along the line $x=-1$ we obtain:

5

$$\lim_{y\to 2} \frac{-2y+2+2}{1+y^2-2y-1} = \lim_{y\to 2} \frac{-2y+4}{y^2-2y} = \lim_{y\to 2} \frac{-2}{2y-2}$$

$$= -1$$

We get similar result along the line $y=2$.

So, next we consider the line $y=x+3$, then we get

$$\lim_{x\to -1} \frac{2x(x+3)-2x+2}{x^2+(x+3)^2-2(x+3)-1} =$$

$$\lim_{x\to -1} \frac{2x^2+6x-2x+2}{x^2+x^2+6x+9-2x-6-1} =$$

5

$$\lim_{x\to -1} \frac{2x^2+4x+2}{2x^2+4x+2} = 1$$