Chapter 22 Electric Fields

(HRW p580-596)

Fields

Temperature throughout a room gives the temperature field.

Similarly pressure throughout the room.

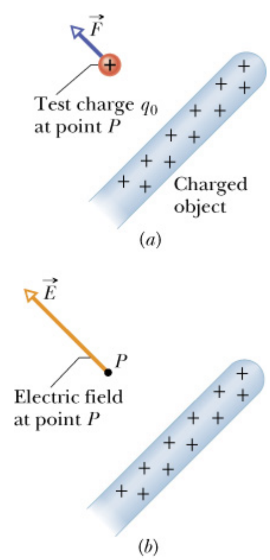
They are both scalar fields (no direction involved).

The electric field is a vector field. A distribution of vectors, one for every point in the volume of interest.

Electric Fields

Place a +ve test charge q_o at a given point P. Measure the electrostatic force on q_o and define the electric field

$$\vec{E} = \frac{\vec{F}}{q_o}$$



Magnitude of electric field at P is

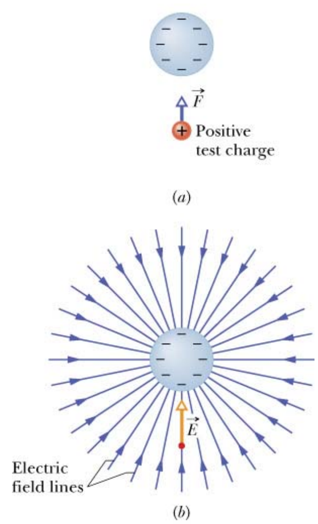
F/q_o and direction is that of the force acting on a +ve charge

Units: N/C newtons/coulomb

The field is there with or without the test charge being there.

Electric field lines

Michael Faraday introduced the concept of electric field in the 19th century. He thought of the field filled with lines of force.



Field lines and electric field vectors

i) the tangent to the field lines at any point gives the direction of E at that point

ii) field lines are drawn so that number per unit area, in a plane perpendicular to them, gives the magnitude of E.

Electric fields extend away from +ve charges and towards -ve charges.

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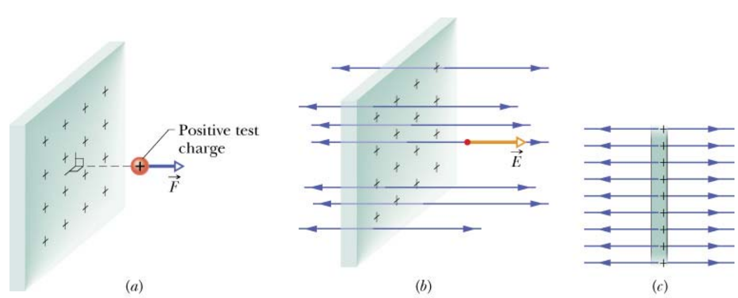
Electric field lines



Example: infinitely large non-conducting sheet or plane with a uniform distribution of +v charge on one side.

By symmetry, force acting on test charge is perpendicular to (⊥) the plate and away from the sheet on both sides.

Creates a uniform electric field.

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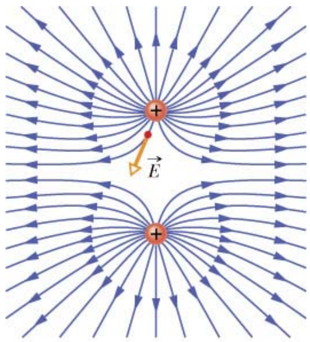
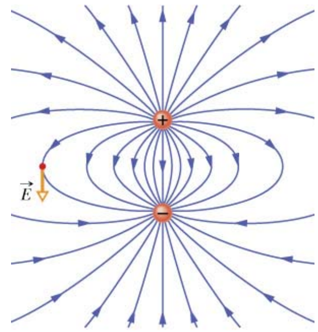
Electric field lines





Equal +ve point charges

-lines terminate on distant -ve charges

-rotationally symmetric

Equal opposite point charges

-rotationally symmetric

-an electric dipole

Note the electric field vectors which are tangent

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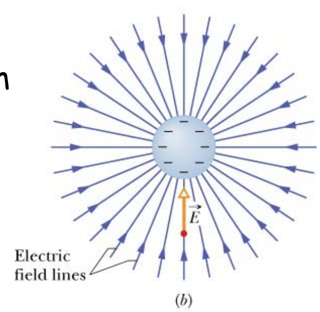
Electric field lines: sample problem 22-1

How does the magnitude of the electric field vary with distance from the centre of a uniformly charged sphere? Use an argument based on electric field lines.



Key idea: field lines are uniformly distributed and extend out without interruption. Place a sphere of radius r around it. All N lines must pass thru sphere of area $4\pi r^2$ so number of lines/area = $N/4\pi r^2$.

Key Idea: magnitude E of electric field is proportional to #lines/area perpendicular to the lines. The shell is perpendicular, so E is prop to $N/4\pi r^2$, ie E varies as $1/r^2$.

Which we know from the following argument as well.

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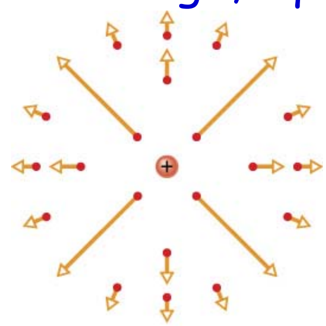
Electric field due to a point charge, +q

Magnitude of electrostatic force on q_o

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q||q_o|}{r^2}$$

Direction is directly away.



$$E = \frac{F}{q_o} = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$$

Figure shows electric field vectors (not lines)

Direction of electric field is same as direction of force.

Applies at all points.

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Electric field due multiple point charges

Net force is

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \cdots + \vec{F}_{0n}$$

Thus:

$$\vec{E} = \frac{\vec{F}_0}{q_o} = \frac{\vec{F}_{01}}{q_o} + \frac{\vec{F}_{02}}{q_o} + \frac{\vec{F}_{03}}{q_o} + \cdots + \frac{\vec{F}_{0n}}{q_o}$$
$$= \vec{E}_{01} + \vec{E}_{02} + \vec{E}_{03} + \cdots + \vec{E}_{0n}$$

i.e. the principle of superposition applies here as well.

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Electric fields

Use demonstration physlets at

http://www.phas.ucalgary.ca/physlets/

--->electric fields:A review and the next two.

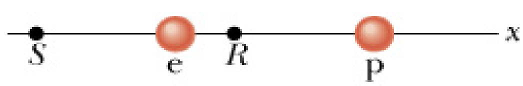
The link on my course page

www.physics.carleton.ca/~drogers/phy1004

Requires java enabled.

Ch22-9/39

Electric fields



proton and electron. What is the direction of the electric field due to the electron alone at

(a) point S (left or right)

(b) point R "

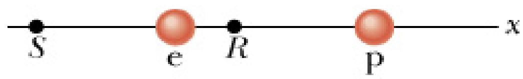
What is the direction of the net electric field at

(c) point R

(d) point S

Ch22-10/39

Electric fields



proton and electron. What is the direction of the electric field due to the electron alone at

(a) point S (left or right) right (-ve attracts +ve test)

(b) point R " left (-ve attracts)

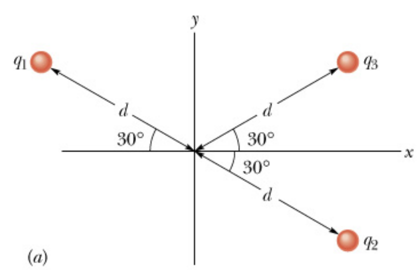
What is the direction of the net electric field at

(c) point R left(both left)

(d) point S right(-ve closer)

Sample problem 22-2

What is the net E at the origin when q_1=+2Q, q_2=-2Q and q_3=-4Q

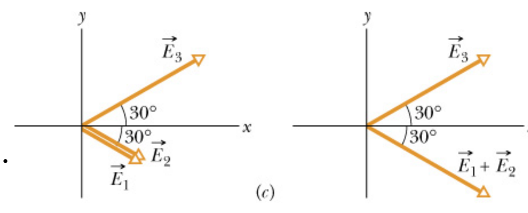


Key idea: Use symmetry

In the y direction, E1, E2

exactly offset E3 .

We need to add up the x-component of each of them.



Book shows the details based on $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$

Aside on binomial theorem pA-10

$$(1+x)^n = 1 + \frac{nx}{1!} + \frac{n(n-1)x^2}{2!} + \cdots \quad (x^2 < 1)$$

$$\frac{1}{(1+x)} = 1 - x + x^2 - x^3 + \cdots \quad (x^2 < 1)$$

$$\frac{1}{(1-x)} = 1 + x + x^2 + x^3 + \cdots \quad (x^2 < 1)$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots \quad (x^2 < 1)$$

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + \cdots \quad (x^2 < 1)$$

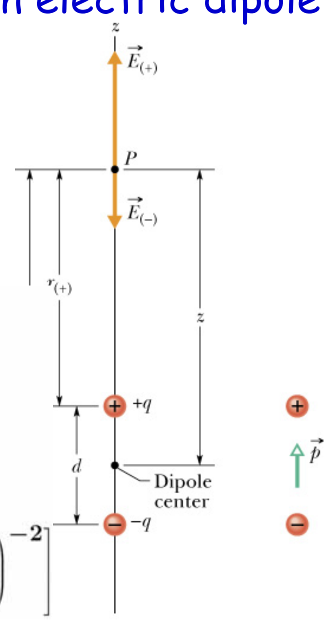
Ch22-13/39

On-axis electric field due to an electric dipole

Dipole: 2 charged particles, magnitude q, distance d.

Find electric field at point P at a distance z from the mid-point on the dipole axis.

$$\begin{aligned} E &= E_{(+)} - E_{(-)} \\ &= \frac{1}{4\pi\epsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_{(-)}^2} \\ &= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(z-\frac{1}{2}d)^2} - \frac{1}{(z+\frac{1}{2}d)^2}\right] \\ &= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right] \end{aligned}$$



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On-axis electric field due to an electric dipole (cont)

$$E = \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-2}\right]$$

$\frac{1}{(1-x)^2} = 1 + 2x$

$\frac{d}{2z} \ll 1$ => use 2 terms of binomial theorem

$$= \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+\frac{2d}{2z}+\cdots\right) - \left(1-\frac{2d}{2z}+\cdots\right)\right]$$

$$= \frac{q}{4\pi\epsilon_0 z^2}\frac{2d}{z}$$

$$= \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3} \qquad \left[\frac{N\cdot m^2}{C^2}\frac{C\cdot m}{m^3}\right] = \left[\frac{N}{C}\right]$$

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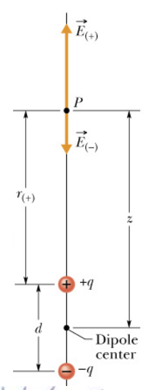
On-axis electric field due to an electric dipole (cont)

Electric dipole moment p of the dipole.

It is a **vector quantity**

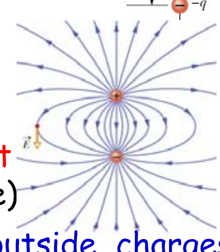
Magnitude is product qd [units C-m]

Directed from negative to positive end of the dipole.



$$E = \frac{1}{2\pi\epsilon_0}\frac{qd}{z^3} = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3}$$

Eqn holds only for distant points on the axis of the dipole.



E for a dipole varies as $1/r^3$ for all distant points, even off the axis (not proven here)

Direction of E on axis is same as p when outside charges.

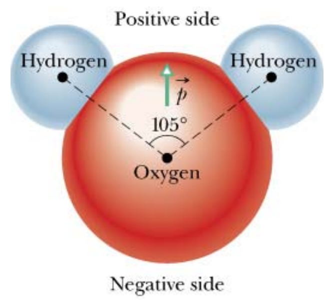
Ch22-16/39

Why do we care about electric dipoles?

Many molecules have an electric dipole moment, eg water.

The relative location of the atoms was based on the work of physicist Gerhard Herzberg who worked at NRC on Sussex Drive from 1948 to 1994. He won the Nobel prize for chemistry in 1971.

The 10 electrons stay on the oxygen side more often
=> net -ve charge on oxygen side
net +ve side on hydrogen side
=> dipole moment with direction as shown.



Ch22-17/39

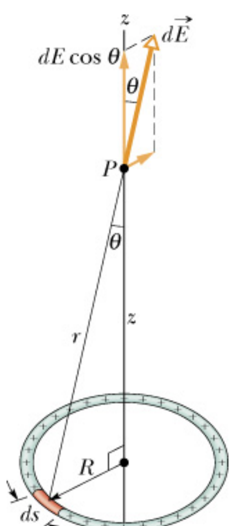
Electric field due to a line(ring) of charge

Think of charge as continuous rather than discrete (an approximation, but a very good one).

Convenient to talk of a charge density. Linear charge density λ (charge per unit length)-   SI unit: coulomb/meter.

We consider a thin ring of radius R with a uniform +ve λ on an insulator. What is electric field E at point P on the axis?

Think of ring as a series of point charges $dq = \lambda\, ds$ where ds is the length of a differential element of ring



Ch22-18/39

Electric field due to a line(ring) of charge (cont)

Magnitude of electric field due to dq?

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{r^2}$$

Rewrite from the geometry

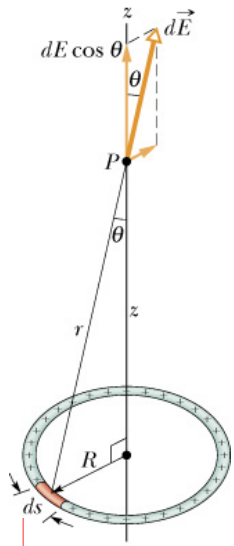
$$dE = \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{(z^2+R^2)}$$

dE is at θ to the axis=> 2 components.

Key idea: perpendicular points all cancel by symmetry consideration.

Consider dEcosθ.

$$cos\theta = \frac{z}{r} = \frac{z}{(z^2+R^2)^{1/2}}$$



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Electric field due to a line(ring) of charge (cont)

$$dEcos\theta = \frac{z\lambda}{4\pi\epsilon_0(z^2+R^2)^{3/2}}ds$$

$$E = \int dEcos\theta = \frac{z\lambda}{4\pi\epsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds$$

$$= \frac{z\lambda(2\pi R)}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$$

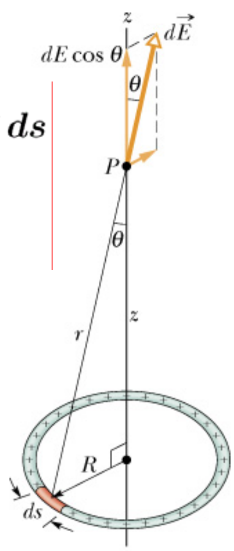
By definition $\lambda 2\pi R = q$

$$E = \frac{zq}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$$

Distant limit? z>>R

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{z^2}$$ i.e. looks like a point

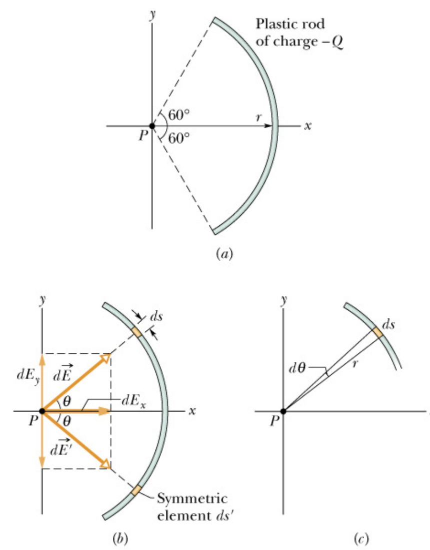
What happens for z=0? E = 0



Ch22-20/39

Sample problem 22-4

What is the electric field at point P due to an insulting rod of charge -Q?



Key ideas:

- y components cancel by symmetry considerations.
- integrate over length of arc using $ds = r d\theta$

Be sure to understand it.

This question or equivalent is on every exam

Ch22-21/39

A field guide for lines of charge (p590)

Summarizes in 6 steps the tactics to be used. Basically outlines the steps we took with the ring of charge calculation.

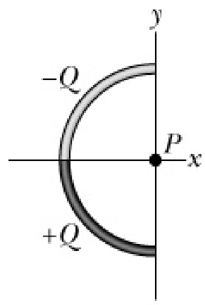
1) determine length element ds (=r dθ or dx usually)
2) relate dq to ds (dq=λds usually)
3) relate dE to dq using $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$
4) consider symmetry and look for cancellations of components or entire parts of the field. Add all of the non-cancelling components by integration
5) take constants outside the integral and reduce the integrals to being over a single variable
6) use integration limits that give a +ve result, and relate λ to total charge if possible.

Ch22-22/39

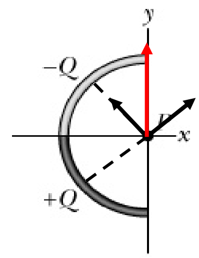
Questions

Uniform charge of Q along top half and same on bottom half, but opposite signs.

What is the direction of the net electric field at point P?



(a)



Up, down, left, right, other (specify)?

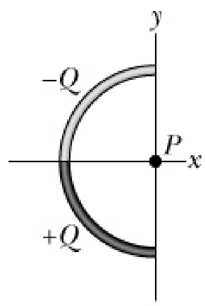
Hint: symmetry is important.

Ch22-23/39

Questions

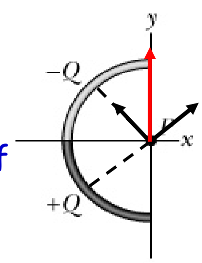
Uniform charge of Q along top half and same on bottom half, but opposite signs.

What is the direction of the net electric field at point P?



(a)

Ans: Electric field is up since x components cancel because of opposite charges



Up, down, left, right, other (specify)?

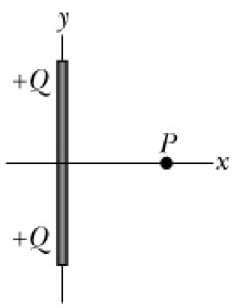
Hint: symmetry is important.

Ch22-24/39

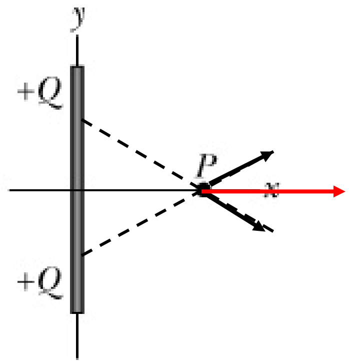
Questions

Uniform charge of Q along straight rod.

What is the direction of the net electric field at point P?



(b)



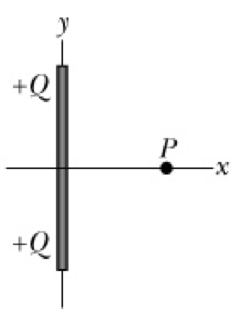
Up, down, left, right, other (specify)?

Ch22-25/39

Questions

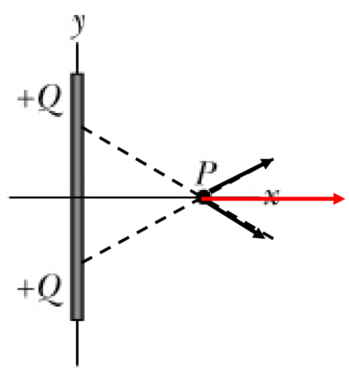
Uniform charge of Q along straight rod.

What is the direction of the net electric field at point P?



(b)

Ans: E field is to right since y components cancel thru symmetry



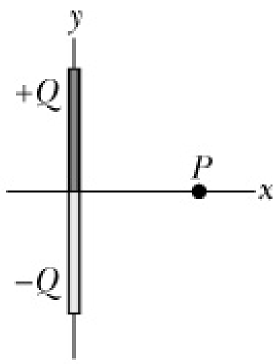
Up, down, left, right, other (specify)?

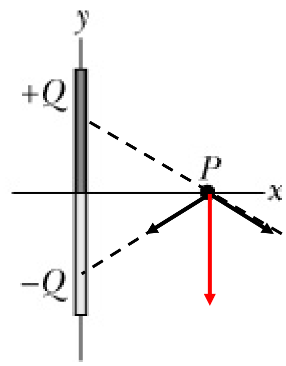
Ch22-26/39

Questions

Two non-conducting rods. Uniform charge of Q along top half and same on bottom half, but varying signs.

What is the direction of the net electric field at point P?





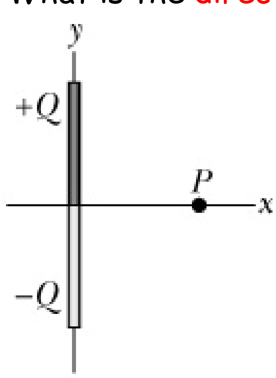
Up, down, left, right, other (specify)?

Ch22-27/39

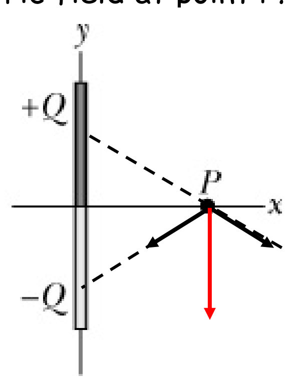
Questions

Two non-conducting rods. Uniform charge of Q along top half and same on bottom half, but varying signs.

What is the direction of the net electric field at point P?



Ans: E field is down since x-components cancel because of opposite charges

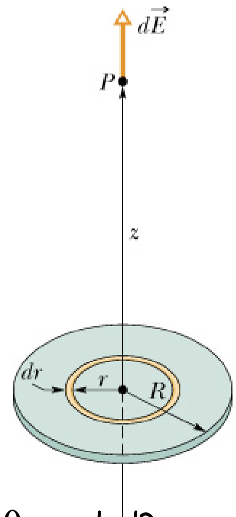


Up, down, left, right, other (specify)?

Ch22-28/39

Electric Field Due to a Charged Disk

Disk radius R with +ve uniform surface charge density on upper surface of σ (C/m^2). What is the electric field at point P a distance z from the disk on its axis?



Consider disk as a set of concentric rings and then add up all the contributions.

Charge on a ring radius r, thickness dr is (charge/area)x area = dq = σ(2πrdr).

Electric field from ring: $E = \dfrac{zq}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$

$$dE = \frac{z\sigma 2\pi r dr}{4\pi\epsilon_0(z^2+r^2)^{3/2}} = \frac{z\sigma}{4\epsilon_0}\frac{2rdr}{(z^2+r^2)^{3/2}}$$

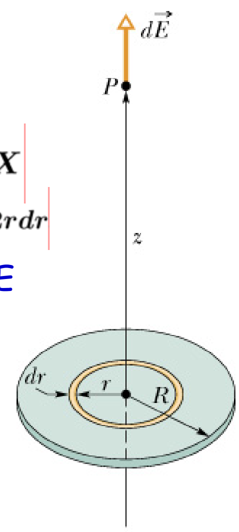
Why is no cosθ needed?

Ans: already included from before

Ch22-29/39

Electric Field Due to a Charged Disk (cont)

$$E = \int dE = \frac{z\sigma}{4\epsilon_0}\int_0^R \frac{2rdr}{(z^2+r^2)^{3/2}}$$



Solve this by getting it to look like $\int X^m dX$

by setting: $X = (z^2+r^2), \quad m = -\frac{3}{2}, \quad dX = 2rdr$

& knowing $\int X^m dX = \dfrac{X^{m+1}}{m+1}$ Appendix E page A-11

and hence
$$E = \frac{\sigma z}{4\epsilon_o}\left[\frac{(z^2+r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R = \frac{\sigma}{2\epsilon_o}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

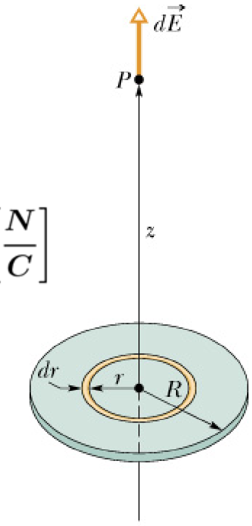
Ch22-30/39

Electric Field Due to a Charged Disk (cont)

$$E = \frac{\sigma}{2\epsilon_o}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

Let $R \to \infty \quad z$ finite

Then $E = \dfrac{\sigma}{2\epsilon_o} \quad \left[\dfrac{C/m^2}{C^2/(N \cdot m^2)}\right] = \left[\dfrac{N}{C}\right]$



i.e. this is the size of the field near a semi-infinite plate. Note it is constant, independent of distance.

Let $z \to 0 \quad R$ finite we get the same result (i.e., close to disk it is like a semi-infinite plate).

Ch22-31/39

Point charge in an electric field

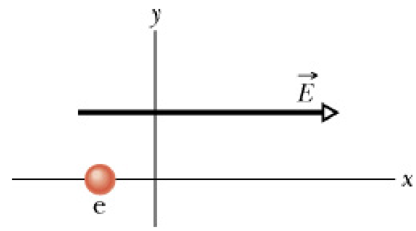
The force acting on a point charge q in an external electric field (as opposed to that set up by the charge itself) is:

$$\vec{F} = q\vec{E}$$

Note: for +ve charges this force is in the direction of E and for -ve charges, it is in the opposite direction.

Ch22-32/39

Question



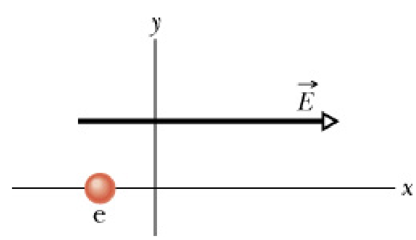
a)What is the direction of the force on the electron in the electric field shown above?

b) which direction will the electron accelerate if moving parallel to the y-axis before encountering E.

c) if the electron is initially moving rightward, will its speed increase, decrease or remain constant?

Ch22-33/39

Question



a)What is the direction of the force on the electron in the electric field shown above? Ans: to left since -ve

b) which direction will the electron accelerate if moving parallel to y-axis before encountering E.

Ans: to the left since F=ma

c) if electron initially moving rightward, will its speed increase, decrease or remain constant?
Ans: decrease since accelerates to left

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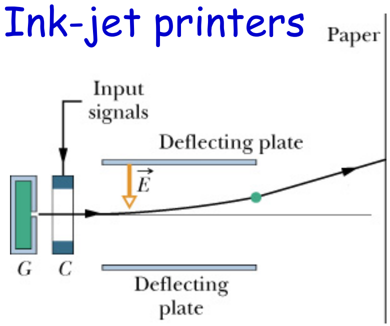
Ink-jet printers



High speed printing achieved by varying the amount of charge added to each drop of ink in the charging unit C after being ejected from the generator, G.

There is a constant electric field, **E**, between two plates.

The field bends the drop according to how much charge it has per unit mass.

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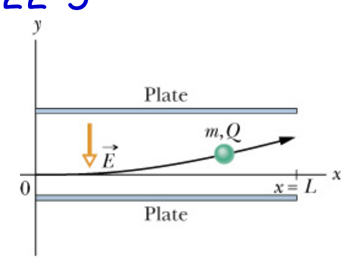
Sample problem 22-5



Two parallel plates create a uniform electric field E pointing down with magnitude 1.4E6 N/C. Ink drop of mass m=1.3E-10 kg and -ve charge Q=1.5E-13 C initially moving along the x-axis with speed v_x=18 m/s. Length of plates, L, is 1.6 cm. What is the vertical deflection at far end of plates?

Key Ideas: Constant force, magnitude Q·E acts upward.

$F=ma_y$ and $F=QE$ gives constant acceleration $a_y = QE/m$

We know $y = 1/2a_yt^2$ and $L = v_xt$. Eliminate t to give y.

$y= 1/2a_y(L/v_x)^2 = 1/2QE/m\ (L/v_x\ )^2 = 0.64\text{mm}$.

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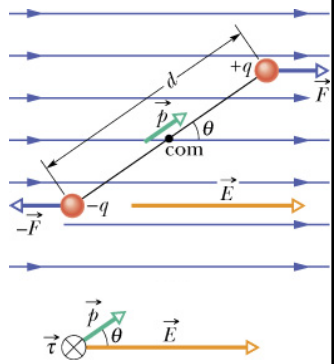
Dipole in electric field

Recall: defined the electric dipole moment, p, of a dipole as the vector with magnitude qd [C-m] directed from the -ve to +ve ends of dipole.

Effect of an external electric field is in terms of **E** and **p** alone.

Electrostatic field applies forces in opposite directions, magnitude F=qE.



Uniform field => net force 0 => centre of mass on axis of dipole does not move.

But there is a net torque (twisting motion), τ, from the force $\perp$ to dipole. $F_\perp$ is $F\sin\theta$ which is applied at x and d-x from the centre of mass. So magnitude of torque is

$\tau = Fx\sin\theta + F(d-x)\sin\theta = Fd\sin\theta = Eqd\sin\theta = Ep\sin\theta$

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Dipole in electric field(cont)

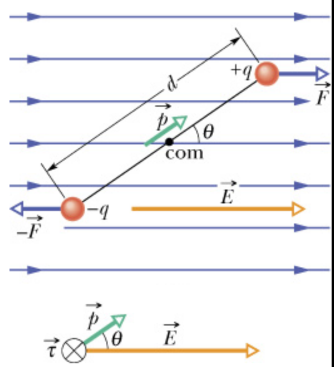
$\tau = pE\sin\theta$

We generalize this using the vector cross product

$$\vec{\tau} = \vec{p} \times \vec{E}$$

where the magnitude is $pE\sin\theta$

The direction of the torque is into the page from the right hand rule.



By convention, if the rotation is clockwise, the torque includes a negative sign with the magnitude: $\tau = -pE\sin\theta$

The torque rotates the dipole moment into the same direction as the electric field (when τ becomes 0) .

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Potential energy of an electric dipole

The book discusses the potential energy of the system, based on previous chapters. Read the section on p594/5.

Summary:

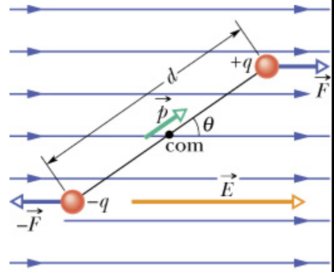
define potential as zero for θ = 90°

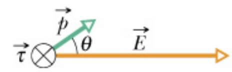
The potential energy is given by

$$U = -\vec{p} \cdot \vec{E} = -pEcos\theta$$

which is 0 for 90° and maximum (pE) for 180° and minimum (-pE) for 0° which is the equilibrium position.

(favourite exam question).





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