

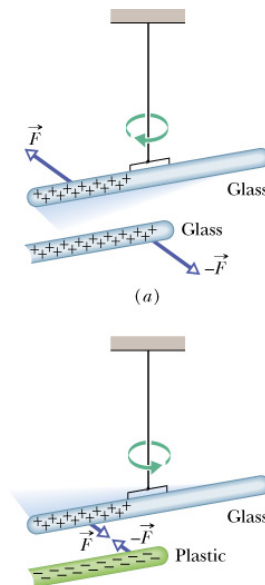
Chapter 21 p561-572 Electric Charge & Coulomb's Law

- 2 types of electric charge
 - Benjamin Franklin arbitrarily defined as +v and -ve
- Like charges repel each other, unlike charges attract
- Will quantify with Coulombs law of electrostatic force
 - Charges stationary or close to stationary
- Electrostatic force is very strong but most objects are neutral charge, ie there is no net charge \Rightarrow no force

Ch21-1/36

electrostatic forces

- Same sign \Rightarrow repulse
glass+silk \Rightarrow +ve
- Opposite sign \Rightarrow attract
plastic+fur \Rightarrow -ve



Ch21-2/36

Conductors/insulators

- **conductors**
 - Charge can move relatively freely
- **nonconductors/insulators**
 - Charge cannot move freely
- **semiconductors**
 - intermediate (silicon, germanium)
- **superconductors**
 - no hindrance at all
- Our interest here is the first two

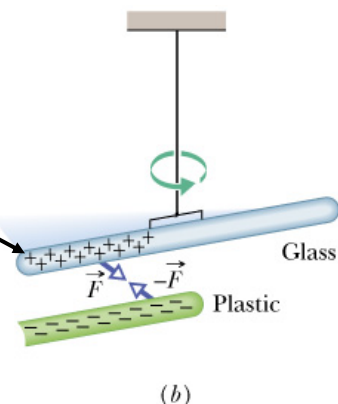
Ch21-3/36

Conductors/insulators

What happens if we replace the glass rod with a neutral copper rod?

Will the rods

- 1) move apart
- 2) move together
- 3) stay in the same place?



Ch21-4/36

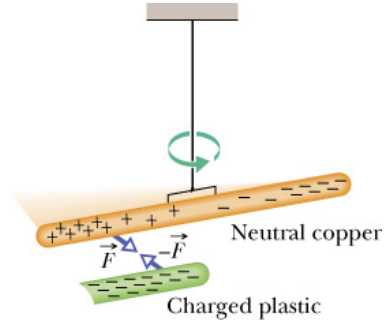
conductors/insulators

The charges in the copper are free to move

The negative charges are repelled to the far end because they are like charges

There is an attractive force between the +ve and -ve charges

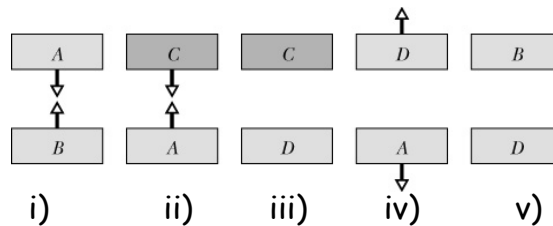
=> They move towards each other



Ch21-5/36

Question

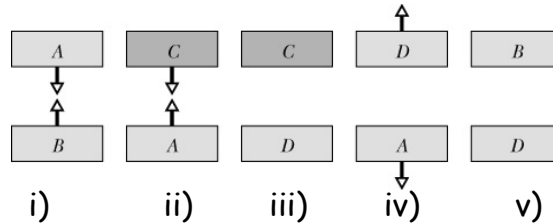
A,B,D **charged** plastic insulators; C a **neutral** conductor;
Will the remaining pairs attract or repulse each other?



Ch21-6/36

Question

A, B, D **charged** plastic insulators; C a **neutral** conductor;
Will the remaining pairs attract or repulse each other?



C & D attract each other for the same reason as A&C; which is because the like charges were repelled to the far side and hence the remaining opposite charges attract more.

A&B have opposite charges based on i). **A&D have the same charges**, based on iv). Therefore **B&D have opposite charges and will attract**.

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Coulomb's Law of Electrostatic Force

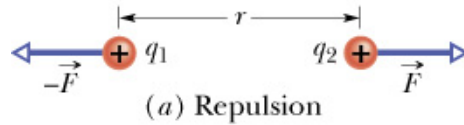
- two charged particles (points) have charges q_1 and q_2
- separated by distance r
- electrostatic force of **attraction or repulsion** has the magnitude

$$F = k \frac{|q_1||q_2|}{r^2}$$

- where k is the **electrostatic constant** = $8.99 \times 10^9 \text{ N m}^2/\text{C}^2$
- each exerts a force of this magnitude on the other
- a Newton's third-law force pair
- directed towards or away from the other particle

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Coulomb's Law (cont)



Ch21-9/36

Coulomb's Law (cont)

- Charles Augustin Coulomb
- expts in 1785 led to the equation.
- Still known to hold in all circumstances.
- cf Newton's eqn for the **gravitational force between two bodies**



$$F = k \frac{|q_1||q_2|}{r^2} \qquad F = G \frac{|m_1||m_2|}{r^2}$$

- gravity always attractive - both $1/r^2$
- gravitational constant $G = 6.673 \times 10^{-11} \text{ N m}^2/\text{kg}^2$

Ch21-10/36

Coulomb's Law (cont)

- SI unit of charge is the **coulomb** (C)
- a derived unit (from the **ampere** (A) the unit of current)
- current is the rate at which charge moves past a point

$$i = \frac{dq}{dt}$$

- $dq = i dt$ $1C = (1A)(1s)$
- for historical and simplification reasons (later) we write

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$

where ϵ_0 is the **permittivity constant**

Ch21-11/36

Coulomb's Law (cont)

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 N \cdot m^2/C^2$$

where k is the electrostatic constant. Hence the **permittivity constant is**

$$\epsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2$$

Do the units make sense?

F is in **N**, q is in **C**, r is in **m** $\Rightarrow k = F r^2/(q_1 q_2)$ is in **N m²/C²**, and ϵ_0 is just the inverse

Ch21-12/36

Coulomb's Law (cont)

- as with gravity, electrostatic forces obey the principle of superposition

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \dots + \vec{F}_{1n}$$

- these are **vectors**
- F_{14} is the force on particle 1 by 4, etc

Aside

What is a Newton in terms of base SI units?

$$F=ma \quad [F] = \text{N} \quad [m] = \text{m} \quad [a] = \text{m/s}^2, \text{ so } 1 \text{ N} = 1 \text{ m}^2/\text{s}^2$$

Ch21-13/36

Shell theorem for Coulomb's Law

In analogy to two theorems for the gravitational force, (stated without proof)

- A shell of uniform charge attracts or repels a charged particle that is outside the shell **as if all the shell's charge were concentrated at its centre**
- If a charged particle is located **inside a shell** of uniform charge, there is **no net electrostatic force on the particle from the shell**
- First theorem assumes that the charge on the shell is not disturbed by that on the particle
- In 2nd, emphasis is on **no net force**, lots of forces cancel.

Ch21-14/36

Spherical Conductors

Another theorem without proof (we prove it later):

Excess charges reside on the outer surface of a spherical conductor in order to maximize the distance between the excess charges.

Shell theorem tells us that the shell acts as if all charge were at the centre

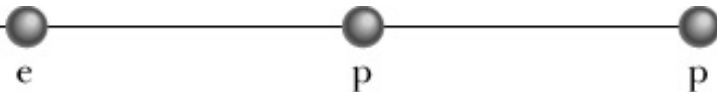
Same applies for +ve or -ve charge

Useful implication:

- stay in your car if hit by lightning

Ch21-15/36

Question



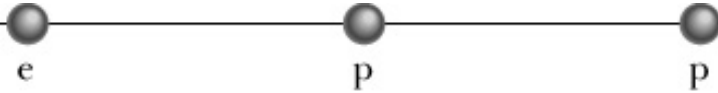
Two protons (p) and one electron (e) are on an axis.

What is the direction of

- electrostatic force on central p from e:
- electrostatic force on central p from other p:
- net electrostatic force on central p:

Ch21-16/36

Question



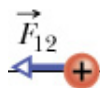
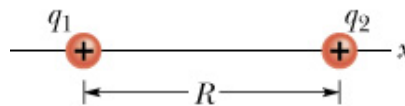
Two protons (p) and one electron (e) are on an axis.

What is the **direction** of

- electrostatic force on central p from e: **to left**
- electrostatic force on central p from other p: **to left**
- net electrostatic force on central p: **to left**

Ch21-17/36

Sample Problem 21.1



Key Ideas

both +ve \Rightarrow repel

\Rightarrow force on q_1 is to left

Coulomb's law applies

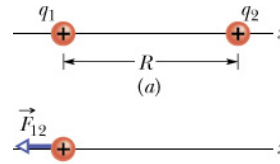
- $q_1 = 1.60 \times 10^{-19} \text{ C}$ and $q_2 = 3.20 \times 10^{-19} \text{ C}$
- $R = 0.0200 \text{ m}$
- What is magnitude and direction of \vec{F}_{12} , the force on particle 1 due to particle 2?

Ch21-18/36

Sample Problem 21.1a (cont)

Coulomb's Law states:

$$\begin{aligned}
 F_{12} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2} \\
 &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \\
 &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(0.0200 \text{ m})^2} \\
 &= 1.15 \times 10^{-24} \text{ N}.
 \end{aligned}$$



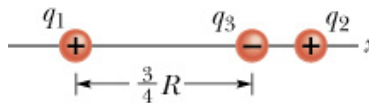
- So force has magnitude and direction

$$1.15 \times 10^{-24} \text{ N} \quad \text{and} \quad 180^\circ$$

- or, in vector notation $\vec{F}_{12} = -(1.15 \times 10^{-24} \text{ N})\vec{i}$

Ch21-19/36

Sample Problem 21.1b



A third particle is introduced with charge $q_3 = -3.20 \times 10^{-19} \text{ C}$ at distance $3/4R$ from particle 1

What is net electrostatic force on particle 1?

Is the problem more complex? **Ans:** Not really - effects are **treated separately** and then added up as vectors

Is additional force stronger? **Ans:** Force is now **attractive** (opposite sign) and **stronger** since it is closer (recall $q_2 = 3.20 \times 10^{-19} \text{ C}$)

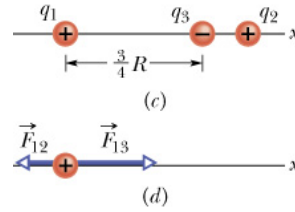
& which way? Net force will be algebraic difference



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Sample Problem 21.1b(cont)

- **Key Idea** particle 3 does not affect the force on particle 1 from particle 2 nor does particle 2 affect the interaction of 1 and 3
- new force attractive since charges have opposite signs

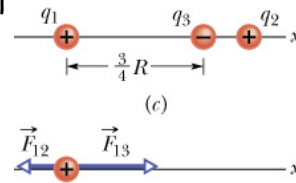


$$\begin{aligned}
 F_{13} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_3|}{\left(\frac{3}{4}R\right)^2} \\
 &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)}{\left(\frac{3}{4}\right)^2 (0.0200\text{m})^2} \\
 &\quad \times (1.60 \times 10^{-19}\text{C})(3.20 \times 10^{-19}\text{C}) \\
 &= 2.05 \times 10^{-24}\text{N}.
 \end{aligned}$$

Ch21-21/36

Sample Problem 21.1b(cont)

- **Key Idea** net force is vector sum of individual forces

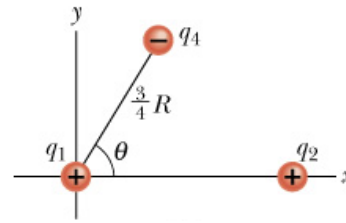


$$\begin{aligned}
 \vec{F}_{1,\text{net}} &= \vec{F}_{12} + \vec{F}_{13} \\
 &= -(1.15 \times 10^{-24}\text{N})\vec{i} + (2.05 \times 10^{-24}\text{N})\vec{i} \\
 &= (9.00 \times 10^{-24}\text{N})\vec{i}
 \end{aligned}$$

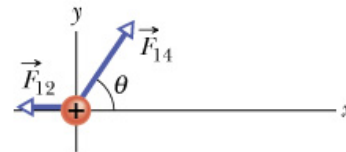
Ch21-22/36

Sample Problem 21.1c

Now add a different 3rd particle, with charge $q_4 = -3.20 \times 10^{-19} \text{ C}$ at distance $\frac{3}{4}R$ from particle 1 at an angle of 60° with the x-axis
 What is net force from particles 2 and 4 on 1?



F_{14} has the same magnitude as F_{13} in previous problem $= 2.05 \times 10^{-24} \text{ N}$ but now we must add vectors

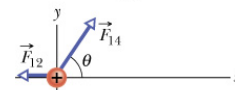
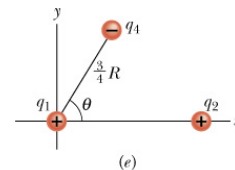


Ch21-23/36

Sample Problem 21.1c(cont)

- **Key idea** as before about the net force being the vector sum, but the vectors are no longer lined up.

$$\begin{aligned}
 F_{14} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_4|}{\left(\frac{3}{4}R\right)^2} \\
 &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \\
 &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{\left(\frac{3}{4}\right)^2 (0.0200 \text{ m})^2} \\
 &= 2.05 \times 10^{-24} \text{ N}.
 \end{aligned}$$



- Note this **magnitude** is same as F_{13} so we didn't need to repeat the calculation!
- but must now add the vectors

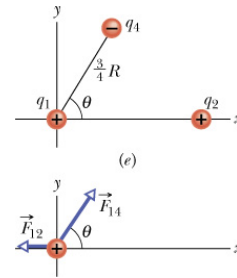
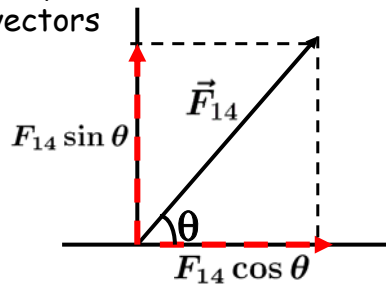
Ch21-24/36

Sample Problem 21.1c(cont)

vector addition:

$$\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{14}$$

Text lists 3 ways: here we use one, we sum unit vectors

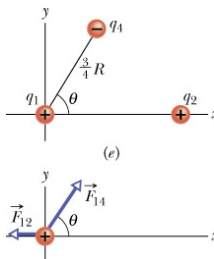


$$\vec{F}_{14} = (F_{14} \cos \theta)\vec{i} + (F_{14} \sin \theta)\vec{j}$$

$$\cos 60^\circ = 1/2 = \sin 30^\circ \quad \sin 60^\circ = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$$

$$\vec{F}_{14} = (1.025 \times 10^{-24} \text{ N})\vec{i} + (1.775 \times 10^{-24} \text{ N})\vec{j} \quad \text{Ch21-25/36}$$

Sample Problem 21.1c(cont)

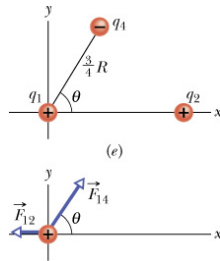


$$\begin{aligned} \vec{F}_{1,net} &= \vec{F}_{12} + \vec{F}_{14} \\ &= -(1.15 \times 10^{-24} \text{ N})\vec{i} \\ &\quad + (1.025 \times 10^{-24} \text{ N})\vec{i} + (1.775 \times 10^{-24} \text{ N})\vec{j} \\ &= (-1.25 \times 10^{-25} \text{ N})\vec{i} + (1.78 \times 10^{-24} \text{ N})\vec{j} \end{aligned}$$

What is the magnitude and direction of this resultant force?

Ch21-26/36

Sample Problem 21.1c(cont)

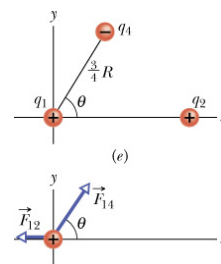
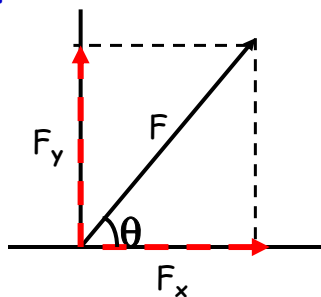


What is the magnitude and direction of the resultant force?

$$F_{1,net} = \sqrt{F_{1,net,x}^2 + F_{1,net,y}^2} = 1.78 \times 10^{-24} \text{ N.}$$

Ch21-27/36

Sample Problem 21.1c(cont)



$$\theta = \tan^{-1} \frac{F_{1,net,y}}{F_{1,net,x}} = \tan^{-1} \frac{1.78}{-0.125} = \tan^{-1} -14.24 = -86.0^\circ$$

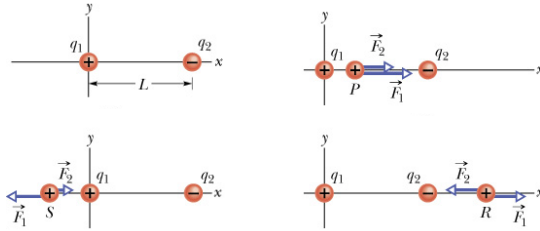
This makes no sense because it would mean the force was down and it must be upwards because the new charge is negative, so need the other solution of \tan^{-1}

$$-86.0^\circ + 180^\circ = 94.0^\circ$$

i.e. - we need to use common sense!!

Ch21-28/36

Sample Problem 21.2



$$q_1 = +8q \quad q_2 = -2q$$

Proton is placed on same axis

Where is the force on the proton 0?

Is it stable there?

Start the looking for the solution by qualitatively seeing which region the proton must be in.

Can it be between, to left, to right? (use figures above)

Use algebra done in the book to figure out where $F_1 = F_2$.

Stable: if moved away slightly, it is forced back to equilibrium (ball in a well)

Unstable: a small movement causes situation to continue changing (ball on top of mountain)

Which is it here?

Unstable Ch21-29/36

Charge is quantized

- In Franklin's day, charge was thought of as a continuous fluid
- but **charge is quantized**
- $q = ne$ $n = +/-1, +/-2, +/-3 \dots$
 - e is the **elementary charge** which is a constant of nature (electron, protons, positrons etc)
 - $e = 1.602 \times 10^{-19} \text{ C}$ (note problem used this)
- "**charge on a sphere**" is a common but somewhat misleading phrase- charge is not a substance: the particles are the substance and the charge is **one of their properties**

(note quarks have charges $+/-e/3$ or $+/-2e/3$ but they are never seen individually)

Ch21-30/36

Question

- Two conducting spheres of the same size
 - A has $-50e$ charge
 - B has $+20e$ charge
- they touch
- What is the charge on A?
 - $-30e$
 - $-15e$
 - $-50e$
 - $+30e$

Ch21-31/36

Question

- Two conducting spheres of the same size
 - A has $-50e$ charge
 - B has $+20e$ charge
 - they touch
 - What is the charge on A?
 - $-30e$
 - $-15e$
 - $-50e$
 - $+30e$
- $-15e$ since net charge $-30e$ is equally shared

Ch21-32/36

How strong is the gravitational force compared to coulomb force between protons?

The charge of the proton is $1.602 \times 10^{-19} \text{ C}$.

The mass of the proton is $1.67 \times 10^{-27} \text{ kg}$.

Do we need to know the distance between them?

No, since the $1/r^2$ terms cancel out.

$$F = k \frac{|q_1||q_2|}{r^2} \qquad F = G \frac{|m_1||m_2|}{r^2}$$

$$\frac{F_{el}}{F_{gr}} = \frac{kq^2}{Gm^2} = \frac{8.99 \times 10^9 (1.602 \times 10^{-19})^2}{6.67 \times 10^{-11} (1.67 \times 10^{-27})^2} = 1.24 \times 10^{36}$$

Would the ratio be larger or smaller for an electron?

So what holds the nucleus together?

Ch21-33/36

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Would the ratio be larger or smaller for an electron?

Larger since the proton is 1837 times heavier: 4.2×10^{42}
So what holds the nucleus together?

The nuclear or so-called strong force (very short range).

Ch21-34/36

Charge is conserved

- Rub silk on a glass rod => rod becomes charged but the charge goes to the silk and then to your hand
- Charge is conserved (again proposed by Franklin)
- no exceptions known!
- **nuclear decay**, the final charge of all particles is a constant $^{238}\text{U}_{92} \rightarrow ^{234}\text{Th}_{90} + ^4\text{He}_2$
- **positron annihilation** (e^+ & e^- annihilate each other and give off gamma-rays of about 511 keV) - charge conserved
- **pair production**: gamma-ray produces e^+ & e^- pair when interacting in the field of the nucleus

Ch21-35/36

