University of Ottawa
MAT 1332 Midterm Exam
Mar 21, 2012. Duration: 80 Minutes

Family Name: _____

First Name: _____

Instructor (circle one): Alexander Hoffnung Jason Levy Olga Vasilyeva

DGD 1 DGD 2 DGD 3 DGD 4

Do **not** write your student ID number on this front page. Please write your student ID number in the space provided on the second page.

Take your time to read the entire paper before you begin to write, and read each question carefully. Remember that certain questions are worth more points than others. Make a note of the questions that you feel confident you can do, and then do those first: you do not have to proceed through the paper in the order given.

- You have 80 minutes to complete this exam.
- This is a closed book exam, and no notes of any kind are allowed. The use of cell phones, pagers or any text storage or communication device **is not permitted.**
- Only the Faculty approved TI-30 calculator is allowed.
- The correct answer requires justification written legibly and logically: you must convince me that you know why your solution is correct. Answer these questions in the space provided. Use the backs of pages if necessary.
- Where it is possible to check your work, do so.
- Good Luck!

Student number:_____, Total marks: _____ out of 30

Problem	1	2	3	4	5	6	bonus
Marks							

Question 1. [5 points] Consider the following differential equation

$$y' = (e^y - 1)(y - 2).$$

(a) Find the equilibrium points of the differential equation.

(b) Classify the equilibrium points in (a) (stable or unstable).

(c) Draw the phase-line diagram of the differential equation.

(d) Sketch in the same coordinate system the equilibrium solutions, the solution with initial condition $y(0) = -10$, the solution with initial condition $y(0) = 1$, and the solution with initial condition $y(0) = 10$.

BLANK PAGE

Question 2. [4 points] Consider the following equation

$$z^2 - 2z + 7 = 0.$$

(a) Solve this equation in $\mathbb{C}$. Write the solutions in the form $a+bi$ and in polar coordinates.

(b) Sketch the solutions in the complex plane.

(c) Express the quotient

$$\frac{3+2i}{2+i}$$

in the form $a + bi$.

BLANK PAGE

Question 3. [4 points] Consider the matrix:

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

(a) Find the eigenvalues of A.

(b) Provide an eigenvector for each of the eigenvalues found in (a).

BLANK PAGE

Question 4. [7 points]

(a) Find all values of a such that the matrix below is invertible.

$$\begin{bmatrix} 1 & a & 2 \\ 3 & 4 & a \\ 1 & 2 & -3 \end{bmatrix}$$

(b) Determine for which values of b and c the system of equations

$$\begin{aligned} 3x + 4y &= 2 \\ 2x + by &= c \end{aligned}$$

has

(i) no solutions

(ii) infinitely many solutions

(iii) exactly one solution

BLANK PAGE

Question 5. [4 points] Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

(a) Use the characteristic equation to verify that $\lambda = 2$ is an eigenvalue of A.
(b) Find an eigenvector corresponding to $\lambda = 2$.

BLANK PAGE

Question 6. [6 points] Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 0 \\ 1 & 0 & 4 \end{bmatrix}.$$

(a) Determine the matrix A^{-1}.
(b) Solve the equation

$$A\mathbf{x} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}.$$

BLANK PAGE

Bonus. [2 points] Express $(1+i)^9$ in the form $a+bi$.