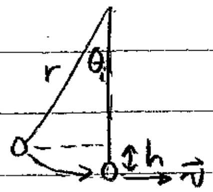
PHY 1124 Test #2 solutions 2011

(Using S.I. units throughout.)

1. 

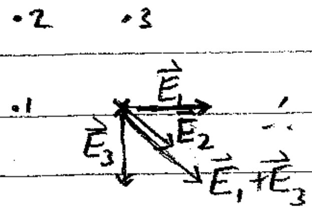
$K_i + U_i = K_f + U_f \Rightarrow \underbrace{U_i - U_f}_{mgh} = K_f - K_i$ $= \frac{1}{2}mv_f^2 \quad$ "0

$v_f^2 = 2gh = 2gr(1-\cos\theta_i) = 2(9.8)(10)\underbrace{(1-\cos 37°)}_{0.20} = 39.2 \frac{m^2}{s^2}$

$v_f = 6.3$ m/s (d)

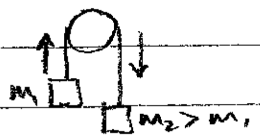
2. $a(t) = \frac{d^2x}{dt^2} = -\omega^2 x(t)$ so a_{max} when $x_{max} \Rightarrow U_{max}, K_{min}$ (b)

3. $F = \frac{k q_1 q_2}{r^2} = \frac{9\cdot 10^9 (1.6\cdot 10^{-19})^2}{(1.6\cdot 10^{-9})^2} = 9.0\cdot 10^{-11}$ N (e)

4. 

$|\vec{E}_2| = \frac{kQ}{(\sqrt{2}a)^2} = \frac{kQ}{2a^2}$ $\quad |\vec{E}_1| = |\vec{E}_3| = \frac{kQ}{a^2}$

$\therefore |\vec{E}_1 + \vec{E}_3| = \sqrt{2}\cdot\frac{kQ}{a^2}$ $\quad |\vec{E}_{net}| = \frac{kQ}{2a^2} + \sqrt{2}\frac{kQ}{a^2} = \underbrace{\left(\frac{1}{2}+\sqrt{2}\right)}_{1.9}\frac{kQ}{a^2}$ (a)

5. 

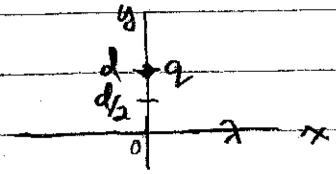
m_2 accelerates downwards & m_1 accelerates upwards so $\Delta K > 0$ and $\Delta U < 0$. (a)

6. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{79\text{ N/m}}{0.5\text{ kg}}} = 12.6\ s^{-1}$ $\quad -0.1\text{m} = x(0) = x_m\cos(\omega t + \phi)$ at $t=0$

~~and $v(0)=0$ so $x_m = 0.1$m and $\cos\phi = -1 \Rightarrow \phi = \pm\pi$~~ (c)

7. $E_z = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \underbrace{\frac{R}{\sqrt{R^2+R^2}}}_{1/\sqrt{2}}\right) = \frac{6\cdot 10^{-6}(C/m^2)}{2(8.85\cdot 10^{-12} C^2/Nm^2)}\left(1-\frac{1}{\sqrt{2}}\right)$

$= 9.9\cdot 10^4$ N/C (c)

8. 

$E_y = \frac{1}{4\pi\varepsilon_0}\frac{q}{(d/2)^2}(-1) + \frac{\lambda}{2\pi\varepsilon_0 d/2} = \frac{1}{\pi\varepsilon_0 d^2}(-q + \lambda d)$

(e)

1. A child is sitting on the seat of a swing with ropes 10 m long. Her father pulls the swing back until the ropes make a 37° angle with the vertical and then releases the swing. If air resistance is neglected, what is the speed of the child at the bottom of the arc of the swing when the ropes are vertical? (in m/s)

   a. 11   b. 8.8   c. 14   d. 6.3   e. 12

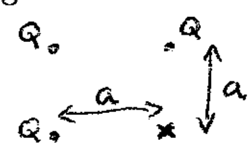
2. A body moving in simple harmonic motion has maximum acceleration when

   a. it has maximum velocity   b. it has minimum kinetic energy   c. it has zero displacement
   d. its kinetic energy and potential energy are equal   e. its displacement is half of the maximum

3. Two electrons are separated by a distance of 1.6 nm. (1 nm $= 10^{-9}$ m) Each electron experiences an electrostatic force (in N) of approximately

   a. 9.0×10^{9} b. 1.1×10^{-32} c. 9.0×10^{20} d. 1.0×10^{-22} e. 9.0×10^{-11}

4. Three identical charges Q are arranged at three corners of a square as shown. Let the width of the square be a, and the Coulomb constant be $k = 1/4\pi\epsilon_0$. The magnitude of the electric field at "×", the fourth corner of the square, is approximately



   a. $1.9kQ/a^2$ b. $0.5kQ/a^2$ c. $1.4kQ/a^2$
   d. $1.0kQ/a^2$ e. $2.5kQ/a^2$

5. Two unequal masses hang from either end of a massless cord that passes over a frictionless pulley. Which of the following is true about the gravitational potential energy (U) and kinetic energy (K) after the masses are released from rest?

   a. $\Delta U < 0$ and $\Delta K > 0$ b. $\Delta U = 0$ and $\Delta K > 0$ c. $\Delta U < 0$ and $\Delta K = 0$
   d. $\Delta U = 0$ and $\Delta K = 0$ e. $\Delta U > 0$ and $\Delta K < 0$

6. A 0.5 kg mass is suspended from a massless spring that has a force constant of 79 N/m. The mass is displaced 0.1 m down from its equilibrium position and released from rest. If the downward direction is negative, the displacement (in m) of the mass as a function of time (in s) is given by

   a. $y = 0.1\cos(158t - \pi)$ b. $y = 0.2\cos(158t - \pi)$ c. $y = 0.1\cos(12.6t - \pi)$
   d. $y = 0.2\cos(12.6t + \pi)$ e. $y = 0.1\cos(2t + \pi)$

7. A disc of radius 10 cm carries a uniform surface charge density of 6.0×10^{-6} C/m^2. The electric field on the axis of the disc at a distance of 10 cm is approximately (in N/C)

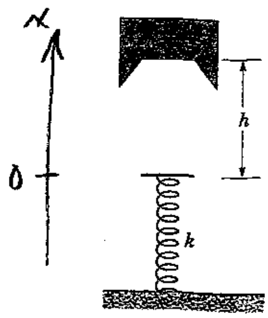
   a. 3.4×10^{5} b. 6.8×10^{4} c. 9.9×10^{4} d. 5.4×10^{5} e. 1.8×10^{4}

8. An infinite line of linear charge density λ lies along the x-axis, and a point charge q is located at position $y = d$ on the y-axis. The y-component of the net electric field at a point $y = d/2$ on the y-axis is

   a. $(q + \lambda d)/(4\pi\epsilon_0 d^2)$ b. $(q - \lambda d)/(4\pi\epsilon_0 d^2)$ c. $(q/d + \lambda)/(\pi\epsilon_0 d)$
   d. $(q - \lambda d)/(2\pi\epsilon_0 d^2)$ e. $(\lambda d - q)/(\pi\epsilon_0 d^2)$

PART B: ANSWER QUESTIONS 9 AND 10 IN THE SPACE PROVIDED, SHOWING ALL YOUR WORK

9. A block of mass $m = 2.0$ kg is dropped from a height $h = 0.30$ m onto a spring of spring constant $k = 800$ N/m as shown. Find the maximum distance the spring is compressed.



$$K_i + U_i = K_f + U_f$$

$$0 + mgh = 0 + mgx + \tfrac{1}{2}kx^2$$

$$\tfrac{1}{2}kx^2 + mgx - mgh = 0$$

$$\therefore \quad x = \frac{1}{2(\frac{1}{2}k)}\left(-mg \pm \sqrt{(mg)^2 - 4(\tfrac{1}{2}k)(-mgh)}\right)$$

$$= \frac{1}{k}\left(-mg \pm \sqrt{(mg)^2 + 2mgkh}\right)$$

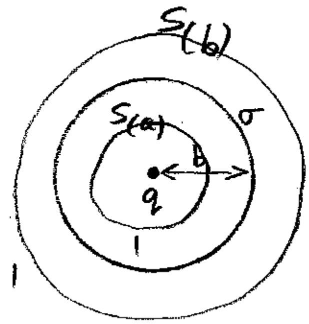
↑ lowest point (−ve sign) gives max. distance

$$= \frac{1}{800}\left(-2 \cdot 9.8 - \underbrace{\sqrt{(2 \cdot 9.8)^2 + 2(2 \cdot 9.8)(800)(0.3)}}_{98.9\text{ N}}\right)$$

$$= -0.148 \text{ m}$$

max. distance $|x| \cong 0.15$ m

10. A point mass q is at the origin. A thin, spherical shell of radius b, centred on the origin, has a uniform surface charge density σ. (a) Use Gauss' law to find the electric field $\vec{E}(\vec{r})$ in the region $r < b$, showing your work and stating any assumptions you make based on symmetry. Draw the Gaussian surface you use, on the figure shown. (b) Use Gauss' law to find the electric field in the region $r > b$. Draw the Gaussian surface you use, on the figure shown.



a) let $\vec{E} = E(r)\hat{r}$ by symmetry

Gauss' law $\oint_S \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0}\underbrace{q_{enc}}_{q}$

Let S be sphere of radius r.

$\therefore \oint_S E(r)\hat{r}\cdot(\hat{r}\,dA) = E(r)\underbrace{\oint_S dA}_{4\pi r^2}$

$\therefore 4\pi r^2 E(r) = \frac{1}{\varepsilon_0} q$

$\therefore E(r) = \dfrac{q}{4\pi\varepsilon_0 r^2}$ and $\vec{E} = E(r)\hat{r}$

b) $\oint_S \vec{E}\cdot d\vec{A} = \frac{1}{\varepsilon_0} q_{enc}$ with surface $S(b)$ shown

$\Rightarrow 4\pi r^2 E(r) = \frac{1}{\varepsilon_0}\left(q + \underbrace{4\pi b^2\sigma}_{\text{charge on shell}}\right)$

$\therefore E(r) = \dfrac{q + 4\pi b^2\sigma}{4\pi\varepsilon_0 r^2}$ and $\vec{E} = E(r)\hat{r}$ as above.