ADM 2304 **Winter 2013**

Solutions to Midterm Exam and Marking Guide

1.
(a)
-The samples are independent since there is no indication of pairing (plus the sample sizes are different).
[2 marks]
-1 mark for "independent" and 1 mark for reason
- 0 marks for "paired" samples answer.

(b)
-Since the samples are independent, we examine the technology and retail boxplots as the "difference" boxplot is meaningless; the retail boxplot is slightly skewed but the tech boxplot is highly skewed. Since they are both small samples, we require that both samples come from a normal distribution for a parametric test. Therefore the non-parametric test is appropriate.
[2 marks]
-1 mark for referring to the "tech" boxplot as skewed;
-1 mark for explaining that the parametric test requires sampling from a normal distribution.

If the answer to (a) is "paired samples", then (b) must refer to the "difference" boxplot which is slightly skewed, and therefore the parametric test may be reasonable or not reasonable, depending on how stringent one is.

(c)
Accept either equal variance or unequal variance 2 sample t tests:

Two-Sample T-Test and CI: loan, sector

```
Two-sample T for loan

sector   N   Mean  StDev  SE Mean
retail   8  95782  56289    19901
tech    10  52074  80772    25542

Difference = mu (retail) - mu (tech)
Estimate for difference:  43708.5
95% CI for difference:  (-25307.6, 112724.6)
T-Test of difference = 0 (vs not =): T-Value = 1.35  P-Value = 0.197  DF = 15
```

Two-Sample T-Test and CI: loan, sector

```
Difference = mu (retail) - mu (tech)
Estimate for difference:  43708.5
95% CI for difference:  (-27792.1, 115209.1)
T-Test of difference = 0 (vs not =): T-Value = 1.30  P-Value = 0.213  DF = 16
Both use Pooled StDev = 71105.3121
```

Ho: mu(retail) = mu(tech); Ha: not equal
t = 1.3 or 1.35, assuming unequal or equal variances, respectively.
Rejection region is |t| > 2.13 or 2.12, based on 15 or 16 df, respectively.
We cannot conclude the loan sizes are different.

[4 marks]
-1 mark for hypotheses (0 marks for 1-sided test here since the direction is suggested only by the data)
-1 mark for t-statistic calculation
-1 mark for rejection region (give mark for 1-sided rejection region t < -1.75)
-1 mark for decision and conclusion (deduct 0.5 marks for missing conclusion)

Manual calculations for equal variance test:

sample difference = 52074 – 95782 = -43708.

Pooled SD = $\sqrt{\dfrac{9 \times 80772^2 + 7 \times 56289^2}{16}}$ = 71105.31

SE = $71105.31 \times \sqrt{\dfrac{1}{10} + \dfrac{1}{8}}$ = 33728.21

$\dfrac{-43708 - 0}{33728.21}$ = -1.3, greater than critical t (-2.12) with df = 16, do not reject the null hypothesis.

(d)

Ho: PopMed(retail) = PopMed(tech) or θ(retail)= θ(tech); Ha: they are unequal

p-value is 0.0368 for the Mann-Whitney test

Reject null H since p-value < 0.05. Conclude they are different.

[3 marks]
-1 mark for hypotheses about the population medians.
-1 mark for p-value from MW test
-1 mark for decision and conclusion

Comment: The parametric test is inappropriate as the outliers lead to a wrong conclusion.

2.

(a)

Test and CI for One Proportion

```
Test of p = 0.15 vs p < 0.15


                               95% Upper
Sample   X    N  Sample p       Bound  Z-Value  P-Value
1       16  140  0.114286   0.158515    -1.18    0.118
```

Ho: p=0.15; Ha: p < 0.15
Z = (0.1143 – 0.15) / sqrt (.15*.85/140) = -1.18
Rejection region for 0.05 test is Z < -1.645
Do not reject null H, conclude there is insufficient evidence to conclude spoilage reduced to below 15%.

[4 marks]
-1 for hypotheses (2-sided test is acceptable)
-1 for z-statistic based on p=0.15(deduct 0.5 for use of SE based on p-hat=.114)
-1 for rejection region (for 2-sided test, require |z| > 1.96)
-1 for decision and conclusion (deduct 0.5 for missing conclusion)

(b) Require z = 1.96, M = 0.03.
Use p-hat = 16/140

n = (1.96/0.03)^2 * (16/140)*(124/140) = 432.07
or 432.12 (based on p-hat = .1143) or 431.13 (based on p-hat = .114)

accept any answer in the 433 or 432 range, even 431.

[2 marks]
-1 mark for using z value of 1.96 and M of 0.03
-1 mark for use of formula correctly based on p-phat of 16/140.

(c)

For the above calculations, it is assumed that **the sample proportion or p-hat** has a normal distribution. Obviously, the "sample" or the "population" are incorrect.

[1 mark}

3.

(a)

Based on unequal variance assumption, we have:

```
Two-sample T for Chronic Care vs Home for the Aged

                    N  Mean  StDev  SE Mean
Chronic Care       60  13.9   10.4      1.3
Home for the Aged  60  15.9   10.8      1.4


Difference = mu (Chronic Care) - mu (Home for the Aged)
Estimate for difference:  -2.05
95% CI for difference:  (-5.89, 1.79)
T-Test of difference = 0 (vs not =): T-Value = -1.06  P-Value = 0.292  DF = 117
```

Assuming equal population variances, we have:

Ho: mu(CC) = mu(home for aged); Ha: they are different.
Pooled variance is (59*10.4^2 + 59*10.8^2)/118 = 112.4 and pooled stdev is 10.6
t = (13.9 – 15.9) / (sqrt(112.4*2/60) = -1.03
rejection region is | t | > 1.96 (1.98 from t-table based on 100 df).
Do not reject null H, conclude there is no difference in the average length of stay.

[5 marks]
-1 for hypotheses (0 for 1-sided test)
-1 for pooled variance or stdev
-1 for t-statistic
-1 for rejection region (accept 1-sided rejection region if hypotheses are 1-sided)
-1 for decision and conclusion (deduct 0.5 for missing conclusion)

(b)

The test in (a) assumes that **<u>the sample means</u>** are normally distributed. Given the boxplots, it is reasonable to assume that **<u>the population data</u>** are normally distributed.

Do not accept “the samples are normally distributed”.

[1 mark]

1 mark for correct answer

(c)

Mann-Whitney test or Wilcoxon Rank Sum Test

Ho: PopMed(CC) = PopMed(home_aged) or $\theta 1 = \theta 2$; Ha: they are unequal

[1 mark]

Must identify test and state hypotheses about the population medians.

4. (a)

"paired samples" because each CC patient at hospital X is matched with a "home for the aged" patient from the same patient.

[2 marks]

- *1 for "paired samples" and 1 mark for reason*
- *0 marks for "independent samples"*

(b)

Since the samples are matched, we do not look at the two samples, only at the "differences" boxplot. Since this boxplot is symmetric (with no outliers), it is appropriate to do a parametric test. Actually, as long as the boxplot does not show extreme skewness, the parametric test is appropriate since the sample of differences is large.

[2 marks]

-1 mark for referring to the "differences" boxplot, 0 marks for not specifying this boxplot.

-1 mark for commenting on the assumption that the population is symmetric or not extremely skewed.

If the answer in (a) was "independent", they lost 2 marks there already. In this case, they can refer to the two boxplots, CC and "home for the aged", and say that since they are skewed, it is still reasonable to assume the two populations are not extremely skewed.

(c)

parametric test is paired t-test:

```
Paired T for Chronic Care - Home for the Aged
                      N     Mean   StDev   SE Mean
Chronic Care         45    27.44   24.81      3.70
Home for the Aged    45    41.44   31.57      4.71
Difference           45   -14.00   34.39      5.13

95% CI for mean difference: (-24.33, -3.67)
T-Test of mean difference = 0 (vs not = 0): T-Value = -2.73  P-Value = 0.009
```

Ho: mu(CC) = mu(home_aged); Ha: they are different

$t = (-14 / 5.13) = -2.73$

p-value is $2 P(t < -2.73) = 2 * 0.0032 = 0.0064$ (based on z-table)

From t-table, $0.001 < P(t < -2.73) < 0.005$, so p-value between 0.002 and 0.01 (closer to .01)

Reject Ho since p-value < 0.05, conclude there is evidence of a difference in the average lengths of stay.

[4 marks]

-1 for hypotheses

-1 for t-statistic

-1 for approximate p-value

-1 for decision and conclusion (deduct 0.5 if missing conclusion)

(d)

95% CI is $\pm 14 \pm 2.02 * 5.13 = \pm 14 \pm 10.4$ = (3.6, 24.4) or (-24.4, -3.6)

Accept 1.96 instead of 2.02 to obtain $\pm 14 \pm 10$ = (4, 24) or (-24, -4).

[2 marks]

-1 for z-value and standard error of 5.13

-1 for proper use of formula and final calculation.