

Solution Assignment #2

CVG 3148 - Winter 2013

(Solved by Leonardo Cortés)

Q1) $SD = 20 \text{ KN/m}$ $D = SD + Sw$
 $L = 25 \text{ KN/m}$
 $S = 10 \text{ KN/m}$

a) $A_c = (2.5 \times 0.2)(0.55 \times 0.35) = 0.6925 \text{ m}^2$; $\gamma_c = 23.5 \text{ KN/m}^3$
 $Sw = (23.5)(0.6925) = 16.27 \text{ KN/m}$

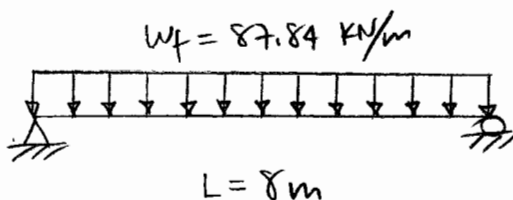
Case 1 : $1.4 D = (1.4)(20 + 16.27) = 50.78 \text{ KN/m}$

Case 2 : $1.25 D + 1.5 L + 0.5 S = 1.25(20 + 16.27) + 1.5(25) + 0.5(10)$
 $= 87.84 \text{ KN/m} \leftarrow \text{Governs}$

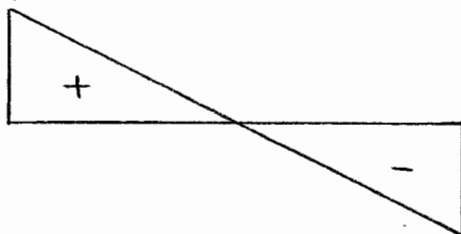
Case 3 : $1.25 D + 1.5 S + 0.5 L = 1.25(20 + 16.27) + 1.5(10) + 0.5(25)$
 $= 72.84 \text{ KN/m}$

$\therefore W_f = 87.84 \text{ KN/m}$

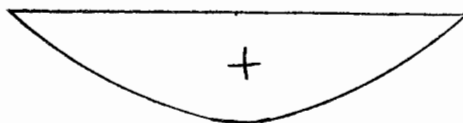
b)



$V_{fmax} =$



$V_{fmax} = 351.4 \text{ KN}$



$M_{fmax} = 702.7 \text{ KN-m}$

$$V_{fmax} = \frac{W_f L}{2}$$

$$= \frac{(87.84)(8)}{2}$$

$$= 351.4 \text{ KN}$$

$$M_{fmax} = \frac{W_f L^2}{8} = \frac{(87.84)(8)^2}{8}$$

$$= 702.7 \text{ KN-m}$$

Q2) Relationship between N and Δ

$$f'_c = 40 \text{ MPa}; f_y = 400 \text{ MPa}; f_{cr} = 2.5 \text{ MPa}; L = 5000 \text{ mm}$$

$$E_c = 5500 \sqrt{f'_c} = 34785.1 \text{ MPa}$$

$$\epsilon_{cr} = \frac{f_{cr}}{E_c} = \frac{2.5 \text{ MPa}}{34785.1 \text{ MPa}} = 72 \times 10^{-6} \text{ mm/mm}$$

$$A_s = 8(500 \text{ mm}^2) = 4000 \text{ mm}^2$$

$$A_c = A_g - A_s = (600)(600) - 4000 = 356000 \text{ mm}^2$$

Response in Tension

i) At cracking (Right before cracking) $\epsilon_{cf} = \epsilon_{cr} = 72 \times 10^{-6} \text{ mm/mm}$

$$\epsilon_{st} = \epsilon_{ct} = \epsilon_{st0}$$

$$\epsilon_{ct} = \epsilon_{cf} + \epsilon_{cth} + \epsilon_{ctsh} + \epsilon_{crp} = \epsilon_{cf} = 72 \times 10^{-6} \text{ mm/mm}$$

$$\epsilon_{st} = \epsilon_{sf} + \epsilon_{sth} = \epsilon_{sf} = 72 \times 10^{-6} \text{ mm/mm}$$

$$f_c = f_{cr} = 2.5 \text{ MPa}$$

$$f_s = E_s \epsilon_{sf} \leq f_y = 400 \text{ MPa}$$

$$= (200000)(72 \times 10^{-6}) = 14.4 \text{ MPa}$$

$$N = A_c f_c + A_s f_s$$

$$= (356000)(2.5) + (4000)(14.4) = 947.6 \text{ kN}$$

$$\Delta = \epsilon_{st} L = (72 \times 10^{-6})(5000) = 0.36 \text{ mm}$$

ii) At cracking (Right after cracking) $\epsilon_{cf} = \epsilon_{cr} = 72 \times 10^{-6} \text{ mm/mm}$

$$f_c = 0 \text{ MPa (concrete cracked)}$$

$$f_s = E_s \epsilon_{sf} \leq f_y = 400 \text{ MPa}$$

$$= (200000)(72 \times 10^{-6}) = 14.4 \text{ MPa}$$

$$N = A_c f_c + A_s f_s = (0)(2.5) + (4000)(14.4) = 57.6 \text{ kN}$$

$$\Delta = \epsilon_{st} L = (72 \times 10^{-6})(5000) = 0.36 \text{ mm}$$

iii) At yielding $\epsilon_{sf} = \epsilon_y$

$$\epsilon_{sf} = \epsilon_y = \frac{f_y}{E_s} = \frac{400}{200000} = 0.002 \text{ mm/mm}$$

$$\epsilon_{cf} = \epsilon_{sf} = 2.0 \times 10^{-3} > \epsilon_{cr} \therefore f_c = 0 \text{ MPa}$$

$$f_s = f_y = 400 \text{ MPa}$$

$$N = A_c f_c + A_s f_s$$

$$= (356000)(0) + (4000)(400) = 1600 \text{ kN}$$

$$\Delta = \epsilon_{cf} L = (2.0 \times 10^{-3})(5000) = 10 \text{ mm}$$

Response in Compression

i) At $\epsilon_{tt} = -1.0 \times 10^{-3} \text{ mm/mm}$

$$\epsilon_{cf} = \epsilon_{tt} = -1.0 \times 10^{-3} \text{ mm/mm}$$

$$\epsilon_{sf} = \epsilon_{tt} = -1.0 \times 10^{-3} \text{ mm/mm}$$

$$f_c = -f'_c \left[2 \left(\frac{\epsilon_{cf}}{\epsilon'_c} \right) - \left(\frac{\epsilon_{cf}}{\epsilon'_c} \right)^2 \right]$$

$$f_c = -40 \left[2 \left(\frac{-1.0}{-2.0} \right) - \left(\frac{-1.0}{-2.0} \right)^2 \right] = -30 \text{ MPa} \quad ; \quad \epsilon'_c = -2.0 \times 10^{-3} \text{ mm/mm}$$

$$f_s = E_s \epsilon_{sf} \geq f_y = -400 \text{ MPa}$$

$$= (200000)(-1.0 \times 10^{-3}) = -200 \text{ MPa}$$

$$N = A_c f_c + A_s f_s$$

$$= (356000)(-30) + (4000)(-200) = -11480 \text{ kN}$$

$$\Delta = \epsilon_{tt} L = (-1.0 \times 10^{-3})(5000) = -5 \text{ mm}$$

ii) At peak (concrete) / yielding (steel)

$$\epsilon_{tt} = -2.0 \times 10^{-3}$$

$$\epsilon_{cf} = \epsilon'_c = \epsilon_{tt} = -2.0 \times 10^{-3} \text{ mm/mm}$$

$$\epsilon_{sf} = \epsilon_y = \epsilon_{tt} = -2.0 \times 10^{-3} \text{ mm/mm}$$

$$f_c = -40 \left[2 \left(\frac{-2.0}{-2.0} \right) - \left(\frac{-2.0}{-2.0} \right)^2 \right] = -40 \text{ MPa}$$

$$f_s = E_s \epsilon_{sf} \geq f_y = -400 \text{ MPa}$$

$$= (200000)(-2.0 \times 10^{-3}) = -400 \text{ MPa}$$

$$N = A_c f_c + A_s f_s$$

$$= (356000)(-40) + (4000)(-400) = -15840 \text{ kN}$$

$$\Delta = \epsilon_{tt} L = (-2.0 \times 10^{-3})(5000) = -10 \text{ mm}$$

iii) At ultimate (concrete crushing) $\epsilon_{cf} = -4 \times 10^{-3} \text{ mm/mm}$

$$\epsilon_{cf} = \epsilon_{tt} = -4 \times 10^{-3} \text{ mm/mm}$$

$$\epsilon_{sf} = \epsilon_{tt} = -4 \times 10^{-3} \text{ mm/mm}$$

$$f_c = -40 \left[2 \left(\frac{-4.0}{-2.0} \right) - \left(\frac{-4.0}{-2.0} \right)^2 \right] = 0$$

$$f_s = E_s \epsilon_{sf} \geq f_y = -400 \text{ MPa}$$

$$= (200000)(-4.0 \times 10^{-3}) = -800 \text{ MPa} \therefore f_s = f_y = -400 \text{ MPa}$$

$$N = A_c f_c + A_s f_s$$

$$= (356000)(0) + (4000)(-400) = -1600 \text{ kN}$$

$$\Delta = \epsilon_{tt} L = (-4.0 \times 10^{-3})(5000) = -20 \text{ mm}$$

LOAD-DEFORMATION RESPONSE

| | | | |
|------------------|----------------------|-------------------|------------------------|
| L= | 5000 mm | f _c = | 40 Mpa |
| | | ε' _c = | -0.002 mm/mm |
| E _s = | 200000 Mpa | E _{ct} | 34785.05 MPa |
| ε _y = | 0.002 mm/mm | f _{cr} = | 2.5 Mpa |
| A _s = | 4000 mm ² | ε _{cr} = | 7.19E-05 mm/mm |
| | | A _c = | 356000 mm ² |

| ε _{ctot} (mm/mm) | ε _{cf} (mm/mm) | f _c (Mpa) | ε _{sf} (mm/mm) | f _s (Mpa) | Δ (mm) | N (kN) | Notes |
|------------------------------|----------------------------|-------------------------|----------------------------|-------------------------|-----------|-----------|-------------------------|
| Tension | | | | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| 7.19E-05 | 7.19E-05 | 2.50 | 7.19E-05 | 14.37 | 0.36 | 947.50 | Cracking of concrete |
| 7.19E-05 | 7.19E-05 | 0.00 | 7.19E-05 | 14.38 | 0.36 | 57.53 | Just after cracking |
| 2.00E-04 | 2.00E-04 | 0.00 | 2.00E-04 | 40.00 | 1.00 | 160.00 | |
| 2.00E-03 | 2.00E-03 | 0.00 | 2.00E-03 | 400.00 | 10.00 | 1600.00 | Yielding of Reinf. |
| 4.00E-03 | 4.00E-03 | 0.00 | 4.00E-03 | 400.00 | 20.00 | 1600.00 | |
| Compression | | | | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | |
| -1.00E-03 | -1.00E-03 | -30.00 | -1.00E-03 | -200.00 | -5.00 | -11480.00 | |
| -1.25E-03 | -1.25E-03 | -34.38 | -1.25E-03 | -250.00 | -6.25 | -13237.50 | |
| -1.50E-03 | -1.50E-03 | -37.50 | -1.50E-03 | -300.00 | -7.50 | -14550.00 | |
| -1.75E-03 | -1.75E-03 | -39.38 | -1.75E-03 | -350.00 | -8.75 | -15417.50 | |
| -2.00E-03 | -2.00E-03 | -40.00 | -2.00E-03 | -400.00 | -10.00 | -15840.00 | Peak/Yielding of Reinf. |
| -2.25E-03 | -2.25E-03 | -39.38 | -2.25E-03 | -400.00 | -11.25 | -15617.50 | |
| -2.50E-03 | -2.50E-03 | -37.50 | -2.50E-03 | -400.00 | -12.50 | -14950.00 | |
| -2.75E-03 | -2.75E-03 | -34.38 | -2.75E-03 | -400.00 | -13.75 | -13837.50 | |
| -3.00E-03 | -3.00E-03 | -30.00 | -3.00E-03 | -400.00 | -15.00 | -12280.00 | |
| -4.00E-03 | -4.00E-03 | 0.00 | -4.00E-03 | -400.00 | -20.00 | -1600.00 | Crushing of concrete |

